Improvements to A-Priori

Park-Chen-Yu Algorithm
Multistage Algorithm
Approximate Algorithms
Compacting Results
PCY Algorithm

- Hash-based improvement to A-Priori.
- During Pass 1 of A-priori, most memory is idle.
- Use that memory to keep counts of buckets into which pairs of items are hashed.
  - Just the count, not the pairs themselves.
- Gives extra condition that candidate pairs must satisfy on Pass 2.
Picture of PCY

Item counts

Hash table

Pass 1

Frequent items

Bitmap

Counts of candidate pairs

Pass 2
PCY Algorithm --- Before Pass 1
Organize Main Memory

- Space to count each item.
  - One (typically) 4-byte integer per item.
- Use the rest of the space for as many integers, representing buckets, as we can.
PCY Algorithm --- Pass 1

FOR (each basket) {
    FOR (each item)
        add 1 to item’s count;
    FOR (each pair of items) {
        hash the pair to a bucket;
        add 1 to the count for that bucket
    }
}
Observations About Buckets

1. If a bucket contains a frequent pair, then the bucket is surely frequent.
   - We cannot use the hash table to eliminate any member of this bucket.

2. Even without any frequent pair, a bucket can be frequent.
   - Again, nothing in the bucket can be eliminated.
Observations --- (2)

3. But in the best case, the count for a bucket is less than the support $s$.
   - Now, all pairs that hash to this bucket can be eliminated as candidates, even if the pair consists of two frequent items.
PCY Algorithm --- Between Passes

◆ Replace the buckets by a bit-vector:
  ♦ 1 means the bucket count exceeds the support \( s \) (frequent bucket); 0 means it did not.

◆ Integers are replaced by bits, so the bit-vector requires little second-pass space.

◆ Also, decide which items are frequent and list them for the second pass.
PCY Algorithm --- Pass 2

- Count all pairs \( \{i,j\} \) that meet the conditions:
  1. Both \( i \) and \( j \) are frequent items.
  2. The pair \( \{i,j\} \), hashes to a bucket number whose bit in the bit vector is 1.

- Notice all these conditions are necessary for the pair to have a chance of being frequent.
Memory Details

- Hash table requires buckets of 2-4 bytes.
  - Number of buckets thus almost 1/4-1/2 of the number of bytes of main memory.

- On second pass, a table of \((\text{item, item, count})\) triples is essential.
  - Thus, hash table must eliminate 2/3 of the candidate pairs to beat a-priori.
Multistage Algorithm

- **Key idea**: After Pass 1 of PCY, rehash only those pairs that qualify for Pass 2 of PCY.
- On middle pass, fewer pairs contribute to buckets, so fewer *false positives* --- frequent buckets with no frequent pair.
Multistage Picture

- Item counts
- First hash table
- Bitmap 1
- Second hash table
- Bitmap 1
- Counts of Candidate pairs
- Bitmap 2
- Freq. items
- Bitmap 1
- Freq. items
Multistage --- Pass 3

Count only those pairs \{i,j\} that satisfy:

1. Both \(i\) and \(j\) are frequent items.
2. Using the first hash function, the pair hashes to a bucket whose bit in the first bit-vector is 1.
3. Using the second hash function, the pair hashes to a bucket whose bit in the second bit-vector is 1.
Important Points

1. The two hash functions have to be independent.
2. We need to check both hashes on the third pass.
   - If not, we would wind up counting pairs of frequent items that hashed first to an infrequent bucket but happened to hash second to a frequent bucket.
Multihash

◆ **Key idea**: use several independent hash tables on the first pass.

◆ **Risk**: halving the number of buckets doubles the average count. We have to be sure most buckets will still not reach count $s$.

◆ If so, we can get a benefit like multistage, but in only 2 passes.
Multihash Picture

- Item counts
  - First hash table
  - Second hash table
- Freq. items
  - Bitmap 1
  - Bitmap 2
  - Counts of Candidate pairs
Extensions

- Either multistage or multihash can use more than two hash functions.
- In multistage, there is a point of diminishing returns, since the bit-vectors eventually consume all of main memory.
- For multihash, the bit-vectors total exactly what one PCY bitmap does, but too many hash functions makes all counts $\geq s$. 
All (Or Most) Frequent Itemsets In $\leq 2$ Passes

- Simple algorithm.
- SON (Savasere, Omiecinski, and Navathe).
- Toivonen.
Simple Algorithm --- (1)

- Take a main-memory-sized random sample of the market baskets.
- Run a-priori or one of its improvements (for sets of all sizes, not just pairs) in main memory, so you don’t pay for disk I/O each time you increase the size of itemsets.
  - Be sure you leave enough space for counts.
The Picture

Copy of sample baskets

Space for counts

20
Simple Algorithm --- (2)

◆ Use as your support threshold a suitable, scaled-back number.
  ♦ E.g., if your sample is 1/100 of the baskets, use $s/100$ as your support threshold instead of $s$. 


Simple Algorithm --- Option

Optionally, verify that your guesses are truly frequent in the entire data set by a second pass.

But you don’t catch sets frequent in the whole but not in the sample.
  - Smaller threshold, e.g., $s/125$, helps.
SON Algorithm --- (1)

- Repeatedly read small subsets of the baskets into main memory and perform the first pass of the simple algorithm on each subset.

- An itemset becomes a candidate if it is found to be frequent in any one or more subsets of the baskets.
SON Algorithm --- (2)

- On a second pass, count all the candidate itemsets and determine which are frequent in the entire set.

- Key “monotonicity” idea: an itemset cannot be frequent in the entire set of baskets unless it is frequent in at least one subset.
Toivonen’s Algorithm --- (1)

◆ Start as in the simple algorithm, but lower the threshold slightly for the sample.
  - **Example**: if the sample is 1% of the baskets, use $s/125$ as the support threshold rather than $s/100$.
  - Goal is to avoid missing any itemset that is frequent in the full set of baskets.
Toivonen’s Algorithm --- (2)

- Add to the itemsets that are frequent in the sample the *negative border* of these itemsets.
- An itemset is in the negative border if it is not deemed frequent in the sample, but *all* its immediate subsets are.
Example: Negative Border

$ABCD$ is in the negative border if and only if it is not frequent, but all of $ABC$, $BCD$, $ACD$, and $ABD$ are.
Toivonen’s Algorithm --- (3)

- In a second pass, count all candidate frequent itemsets from the first pass, and also count the negative border.

- If no itemset from the negative border turns out to be frequent, then the candidates found to be frequent in the whole data are exactly the frequent itemsets.
Toivonen’s Algorithm --- (4)

- What if we find something in the negative border is actually frequent?
- We must start over again!
- Try to choose the support threshold so the probability of failure is low, while the number of itemsets checked on the second pass fits in main-memory.
Theorem:

If there is an itemset frequent in the whole, but not frequent in the sample, then there is a member of the negative border frequent in the whole.
Proof:

- Suppose not; i.e., there is an itemset $S$ frequent in the whole, but not frequent or in the negative border in the sample.
- Let $T$ be a \textit{smallest} subset of $S$ that is not frequent in the sample.
- $T$ is frequent in the whole (monotonicity).
- $T$ is in the negative border (else not “smallest”).
Compacting the Output

1. **Maximal Frequent itemsets**: no immediate superset is frequent.
2. **Closed itemsets**: no immediate superset has the same count.
   - Stores not only frequent information, but exact counts.
**Example: Maximal/Closed**

<table>
<thead>
<tr>
<th></th>
<th>Count</th>
<th>Maximal s=3</th>
<th>Closed</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>AB</td>
<td>4</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>AC</td>
<td>2</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>BC</td>
<td>3</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>ABC</td>
<td>2</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>