CS109A Second Class Quiz
Friday, March 3, 1995, 3:15–4:05PM

Directions

The exam is *open book*; any written materials may be used. Answer all 7 questions on the exam paper itself. The total number of points is 100.

If you do not wish to do heavy arithmetic, you may leave answers as a product of integers or as a quotient of products of integers. You **may not** leave answers in terms of factorials, permutations (Π), combinations $\binom{n}{m}$, or other formulas. Also, you must **cancel common factors** from numerator and denominator.

You may make additional comments at the bottoms of pages or at the end if you feel the need to clarify or justify your answers.

Do not forget to **sign the pledge** below.

I acknowledge and accept the honor code.

_____

Print your name here: _____

1	2	3	4
5	6	7	

Problem 1: (10 points) There are 22 teams in the Palo Alto Junior Basketball League. How many possible matches (set of two teams) are there? _____ Explain your reasoning below for partial credit.

Problem 2: (10 points) In my pocket are 3 nickels, 2 dimes, and 4 quarters. In how many ways can I line these 9 coins in a row? _____ Assume that the coins of each denomination are indistinguishable. *Note*: Please see comment on appropriate forms of answer in the exam directions.
Explain your reasoning below for partial credit.

Problem 3: (20 points) In the card game of "Go-Fish," we start off with a hand of 4 cards dealt at random. We wish to compute the probability that a hand consists of cards of 4 different ranks. There are several ways you might approach this problem. *Note*: Please see comment on appropriate forms of answer in the exam directions.

a) Describe the probability space that you will use for your calculation.

b) How many points are there in your probability space?

Explain your reasoning for partial credit.

c) How many points are in the event "the 4 cards are of different ranks"?

Explain your reasoning for partial credit.

d) What is the probability of this event?

Problem 4: (15 points) Here is a function `lookup` that tests whether x is on a list L.

```
(1) fun lookup(x,nil) = false
(2) |   lookup(x,y::ys) =
(3)         if x=y then true
(4)         else lookup(x,ys);
```

We wish to prove that `lookup` returns true if and only if x is on list L. The proof must be an induction on n, the length of list L. *Note*: There will be no credit given for proofs that are not inductions. In particular, your proof of the inductive step *must* make use of the inductive hypothesis.

a) Write the statement $S(n)$ to be proved here.

b) What value of n is the appropriate basis? _____

c) Prove the basis case here:

d) Prove the inductive part here:

Problem 5: (15 points) An *odd-tree* is a tree whose every interior node has an odd number of children. We wish to prove the statement

$S(T)$: if T is an odd-tree, then T has an odd number of leaves

by structural induction. *Note*: There will be no credit awarded for proofs that are not structural inductions. In particular, you *must* use the inductive hypothesis, and you *must not* attempt an induction on some numerical parameter of the tree such as its height or number of nodes.

a) Prove the basis case.

b) Prove the inductive case.

Problem 6: (15 points) We would like to find the longest common subsequences of the strings `abca` and `baca`.

a) In the space below, create a table giving the length of the LCS between each pair of prefixes, one from each of the strings. Please use columns to correspond to prefixes of `abca` and rows to represent prefixes of `baca`.

b) What is the length of the LCS of these strings? _____

c) What are all the LCS's of this length? Just give the LCS's; you need not indicate the positions of the two original strings from which they come.

Problem 7: (15 points) Write an ML function `philter(f,x,L)` that takes a function f and an element x, and returns the list of all those elements y on list L such that $f(x,y)$ is true.