

## Multivalued Dependencies

Fourth Normal Form

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## A New Form of Redundancy

- ◆ Multivalued dependencies (MVD's) express a condition among tuples of a relation that exists when the relation is trying to represent more than one many-many relationship.
- ◆ Then certain attributes become independent of one another, and their values must appear in all combinations.

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## Example

Drinkers(name, addr, phones, beersLiked)

- ◆ A drinker's phones are independent of the beers they like.
- ◆ Thus, each of a drinker's phones appears with each of the beers they like in all combinations.
- ◆ This repetition is unlike redundancy due to FD's, of which name->addr is the only one.

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## Tuples Implied by Independence

If we have tuples:

name	addr	phones	beersLiked
sue	a	p1	b1
sue	a	p2	b2
sue	a	p2	b1
sue	a	p1	b2

Then these tuples must also be in the relation.

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## Definition of MVD

- ◆ A *multivalued dependency* (MVD)  $X \twoheadrightarrow Y$  is an assertion that if two tuples of a relation agree on all the attributes of  $X$ , then their components in the set of attributes  $Y$  may be swapped, and the result will be two tuples that are also in the relation.

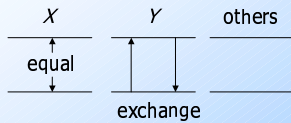
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## Example

- ◆ The name-addr-phones-beersLiked example illustrated the MVD  
name->>phones  
and the MVD  
name->>beersLiked.

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## Picture of MVD $X \twoheadrightarrow Y$



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## MVD Rules

- ◆ Every FD is an MVD.
  - ◆ If  $X \rightarrow Y$ , then swapping  $Y$ 's between two tuples that agree on  $X$  doesn't change the tuples.
  - ◆ Therefore, the "new" tuples are surely in the relation, and we know  $X \twoheadrightarrow Y$ .
- ◆ *Complementation* : If  $X \twoheadrightarrow Y$ , and  $Z$  is all the other attributes, then  $X \twoheadrightarrow Z$ .

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## Splitting Doesn't Hold

- ◆ Like FD's, we cannot generally split the left side of an MVD.
- ◆ But unlike FD's, we cannot split the right side either --- sometimes you have to leave several attributes on the right side.

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## Example

- ◆ Consider a drinkers relation:  
Drinkers(name, areaCode, phone, beersLiked, manf)
- ◆ A drinker can have several phones, with the number divided between areaCode and phone (last 7 digits).
- ◆ A drinker can like several beers, each with its own manufacturer.

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## Example, Continued

- ◆ Since the areaCode-phone combinations for a drinker are independent of the beersLiked-manf combinations, we expect that the following MVD's hold:

name  $\twoheadrightarrow$  areaCode phone  
name  $\twoheadrightarrow$  beersLiked manf

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## Example Data

Here is possible data satisfying these MVD's:

name	areaCode	phone	beersLiked	manf
Sue	650	555-1111	Bud	A.B.
Sue	650	555-1111	WickedAle	Pete's
Sue	415	555-9999	Bud	A.B.
Sue	415	555-9999	WickedAle	Pete's

But we cannot swap area codes or phones by themselves. That is, neither name  $\twoheadrightarrow$  areaCode nor name  $\twoheadrightarrow$  phone holds for this relation.

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## Fourth Normal Form

- ◆ The redundancy that comes from MVD's is not removable by putting the database schema in BCNF.
- ◆ There is a stronger normal form, called 4NF, that (intuitively) treats MVD's as FD's when it comes to decomposition, but not when determining keys of the relation.

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## 4NF Definition

- ◆ A relation  $R$  is in 4NF if whenever  $X \twoheadrightarrow Y$  is a nontrivial MVD, then  $X$  is a superkey.
  - ◆ "Nontrivial means that:
    1.  $Y$  is not a subset of  $X$ , and
    2.  $X$  and  $Y$  are not, together, all the attributes.
  - ◆ Note that the definition of "superkey" still depends on FD's only.

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## BCNF Versus 4NF

- ◆ Remember that every FD  $X \rightarrow Y$  is also an MVD,  $X \twoheadrightarrow Y$ .
- ◆ Thus, if  $R$  is in 4NF, it is certainly in BCNF.
  - ◆ Because any BCNF violation is a 4NF violation.
- ◆ But  $R$  could be in BCNF and not 4NF, because MVD's are "invisible" to BCNF.

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## Decomposition and 4NF

- ◆ If  $X \twoheadrightarrow Y$  is a 4NF violation for relation  $R$ , we can decompose  $R$  using the same technique as for BCNF.
  1.  $XY$  is one of the decomposed relations.
  2. All but  $Y - X$  is the other.

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## Example

Drinkers(name, addr, phones, beersLiked)

FD: name  $\rightarrow$  addr

MVD's: name  $\twoheadrightarrow$  phones  
name  $\twoheadrightarrow$  beersLiked

- ◆ Key is {name, phones, beersLiked}.
- ◆ All dependencies violate 4NF.

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## Example, Continued

- ◆ Decompose using name  $\rightarrow$  addr:
  1. Drinkers1(name, addr)
    - ◆ In 4NF, only dependency is name  $\rightarrow$  addr.
  2. Drinkers2(name, phones, beersLiked)
    - ◆ Not in 4NF. MVD's name  $\twoheadrightarrow$  phones and name  $\twoheadrightarrow$  beersLiked apply. No FD's, so all three attributes form the key.

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## Example: Decompose Drinkers2

◆ Either MVD name  $\rightarrow$  phones or name  $\rightarrow$  beersLiked tells us to decompose to:

- ◆ Drinkers3(name, phones)
- ◆ Drinkers4(name, beersLiked)