# More Stream Mining

## Bloom Filters
## Sampling Streams
## Counting Distinct Items
## Computing Moments

Jeffrey D. Ullman
Stanford University

# Filtering Stream Content

- To motivate the Bloom-filter idea, consider a web crawler.
- It keeps, centrally, a list of all the URL’s it has found so far.
- It assigns these URL’s to any of a number of parallel tasks; these tasks stream back the URL’s they find in the links they discover on a page.
- It needs to filter out those URL’s it has seen before.

# Role of the Bloom Filter

- A Bloom filter placed on the stream of URL's will declare that certain URL's have been seen before.
- Others will be declared new, and will be added to the list of URL's that need to be crawled.
- Unfortunately, the Bloom filter can have false positives.
  - It can declare a URL seen before when it hasn't.
- But if it says "never seen," then it is truly new.
- So we need to restart the filter periodically.

# Example: Filtering Chunks

- Suppose we have a database relation stored in a DFS, spread over many chunks.
- We want to find a particular value v, looking at as few chunks as possible.
- A Bloom filter on each chunk will tell us certain values are there, and others aren't.
  - As before, false positives are possible.
- But now things are exactly right: if the filter says v is not at the chunk, it surely isn't.
  - Occasionally, we retrieve a chunk we don't need, but can't miss an occurrence of value v.

# How a Bloom Filter Works

- A *Bloom filter* is an array of bits, together with a number of hash functions.
- The argument of each hash function is a stream element, and it returns a position in the array.
- Initially, all bits are 0.
- When input x arrives, we set to 1 the bits h(x), for each hash function h.

# Example: Bloom Filter

- Use N = 11 bits for our filter.
- Stream elements = integers.
- Use two hash functions:
  - $h_1(x)$ =
    - Take odd-numbered bits from the right in the binary representation of x.
    - Treat it as an integer i.
    - Result is i modulo 11.
  - $h_2(x)$ = same, but take even-numbered bits.

# Example – Continued

| Stream element | $h_1$ | $h_2$ | Filter contents |
|---|---|---|---|
| | | | 00000000000 |
| 25 = 11001 | 5 | 2 | 00100100000 |
| 159 = 10011111 | 7 | 0 | 10100101000 |
| 585 = 1001001001 | 9 | 7 | 10100101010 |

Note: bit 7 was already 1.

# Bloom Filter Lookup

- Suppose element y appears in the stream, and we want to know if we have seen y before.
- Compute h(y) for each hash function y.
- If all the resulting bit positions are 1, say we have seen y before.
  - We could be wrong.
    - Different inputs could have set each of these bits.
- If at least one of these positions is 0, say we have not seen y before.
  - We are certainly right.

# Example: Lookup

- Suppose we have the same Bloom filter as before, and we have set the filter to 10100101010.
- Lookup element y = 118 = 1110110 (binary).
- $h_1(y)$ = 14 modulo 11 = 3.
- $h_2(y)$ = 5 modulo 11 = 5.
- Bit 5 is 1, but bit 3 is 0, so we are sure y is not in the set.

# Performance of Bloom Filters

- Probability of a false positive depends on the density of 1's in the array and the number of hash functions.
  - = (fraction of 1's)$^{\text{\# of hash functions}}$.
- The number of 1's is approximately the number of elements inserted times the number of hash functions.
  - But collisions lower that number slightly.

# Throwing Darts

- Turning random bits from 0 to 1 is like throwing *d* darts at *t* targets, at random.
- How many targets are hit by at least one dart?
- Probability a given target is hit by a given dart = 1/t.
- Probability none of d darts hit a given target is $(1-1/t)^d$.
- Rewrite as $(\underbrace{\boxed{1-1/t)^{t}}}_{\sim= 1/e}{}^{(d/t)} \sim= e^{-d/t}$.

# Example: Throwing Darts

- Suppose we use an array of 1 billion bits, 5 hash functions, and we insert 100 million elements.
- That is, $t = 10^9$, and $d = 5*10^8$.
- The fraction of 0's that remain will be $e^{-1/2} = 0.607$.
- Density of 1's = 0.393.
- Probability of a false positive = $(0.393)^5 = 0.00937$.

# Sampling a Stream

## What Doesn't Work
## Sampling Based on Hash Values

# When Sampling Doesn't Work

- Suppose Google would like to examine its stream of search queries for the past month to find out what fraction of them were unique – asked only once.
- But to save time, we are only going to sample 1/10th of the stream.
- The fraction of unique queries in the sample != the fraction for the stream as a whole.
  - In fact, we can't even adjust the sample's fraction to give the correct answer.

# Example: Unique Search Queries

- The length of the sample is 10% of the length of the whole stream.
- Suppose a query is unique.
  - It has a 10% chance of being in the sample.
- Suppose a query occurs exactly twice in the stream.
  - It has an 18% chance of appearing exactly once in the sample.
- And so on … The fraction of unique queries in the stream is unpredictably large.

# Sampling by Value

- My mistake: I sampled based on the position in the stream, rather than the value of the stream element.
- The right way: hash search queries to 10 buckets 0, 1,…, 9.
- Sample = all search queries that hash to bucket 0.
  - All or none of the instances of a query are selected.
  - Therefore the fraction of unique queries in the sample is the same as for the stream as a whole.

# Controlling the Sample Size

- Problem: What if the total sample size is limited?
- Solution: Hash to a large number of buckets.
- Adjust the set of buckets accepted for the sample, so your sample size stays within bounds.

# Example: Fixed Sample Size

- Suppose we start our search-query sample at 10%, but we want to limit the size.
- Hash to (say) 100 buckets, 0, 1,..., 99.
  - Take for the sample those elements hashing to buckets 0 through 9.
- If the sample gets too big, get rid of bucket 9.
- Still too big, get rid of 8, and so on.

# Sampling Key-Value Pairs

- This technique generalizes to any form of data that we can see as tuples (K, V), where K is the "key" and V is a "value."
- Distinction: We want our sample to be based on picking some set of keys only, not pairs.
  - In the search-query example, the data was "all key."
- Hash keys to some number of buckets.
- Sample consists of all key-value pairs with a key that goes into one of the selected buckets.

# Example: Salary Ranges

- Data = tuples of the form (EmpID, Dept, Salary).
- Query: What is the average range of salaries within departments?
- Key = Dept.
- Value = (EmpID, Salary).
- Sample picks some departments, has salaries for all employees of that department, including its min and max salaries.
- Result will be an unbiased estimate of the average salary range.

# Counting Distinct Elements

## Applications
## Flajolet-Martin Approximation Technique
## Generalization to Moments

# Counting Distinct Elements

- Problem: a data stream consists of elements chosen from a set of size *n*. Maintain a count of the number of distinct elements seen so far.
- Obvious approach: maintain the set of elements seen.

# Applications

- How many different words are found among the Web pages being crawled at a site?
  - Unusually low or high numbers could indicate artificial pages (spam?).
- How many unique users visited Facebook this month?
- How many different pages link to each of the pages we have crawled.
  - Useful for estimating the PageRank of these pages.
    - Which in turn can tell a crawler which pages are most worth visiting.

# Estimating Counts

- Real Problem: what if we do not have space to store the complete set?
  - Or we are trying to count lots of sets at the same time.
- Estimate the count in an unbiased way.
- Accept that the count may be in error, but limit the probability that the error is large.

# Flajolet-Martin Approach

- Pick a hash function *h* that maps each of the *n* elements to at least $\log_2 n$ bits.
- For each stream element *a*, let *r*(*a*) be the number of trailing 0's in *h*(*a*).
  - Called the *tail length*.
- Record *R* = the maximum *r*(*a*) seen for any a in the stream.
- Estimate (based on this hash function) = $2^R$.

# Why It Works

- The probability that a given $h(a)$ ends in at least $i$ 0's is $2^{-i}$.
- If there are $m$ different elements, the probability that $R \geq i$ is $1 - \boxed{(\boxed{1 - 2^{-i}})^m}$.

Prob. all h(a)'s end in fewer than $i$ o's.

Prob. a given h(a) ends in fewer than $i$ o's.

# Why It Works – (2)

- Since $2^{-i}$ is small, $1 - (1-2^{-i})^m \approx 1 - e^{-m2^{-i}}$.
- If $2^i >> m$, $1 - e^{-m2^{-i}} \approx 1 - (1 - m2^{-i}) \approx m/2^i \approx 0$.
- If $2^i << m$, $1 - e^{-m2^{-i}} \approx 1$.
- Thus, $2^R$ will almost always be around $m$.

First 2 terms of the Taylor expansion of $e^x$

Same trick as "throwing darts." Multiply and divide m by $2^{-i}$.

# Why It Doesn't Work

- $E(2^R)$ is, in principle, infinite.
  - Probability halves when $R$ -> $R$+1, but value doubles.
- Workaround involves using many hash functions and getting many samples.
- How are samples combined?
  - Average? What if one very large value?
  - Median? All values are a power of 2.

# Solution

- Partition your samples into small groups.
  - O(log n), where n = size of universal set, suffices.
- Take the average within each group.
- Then take the median of the averages.

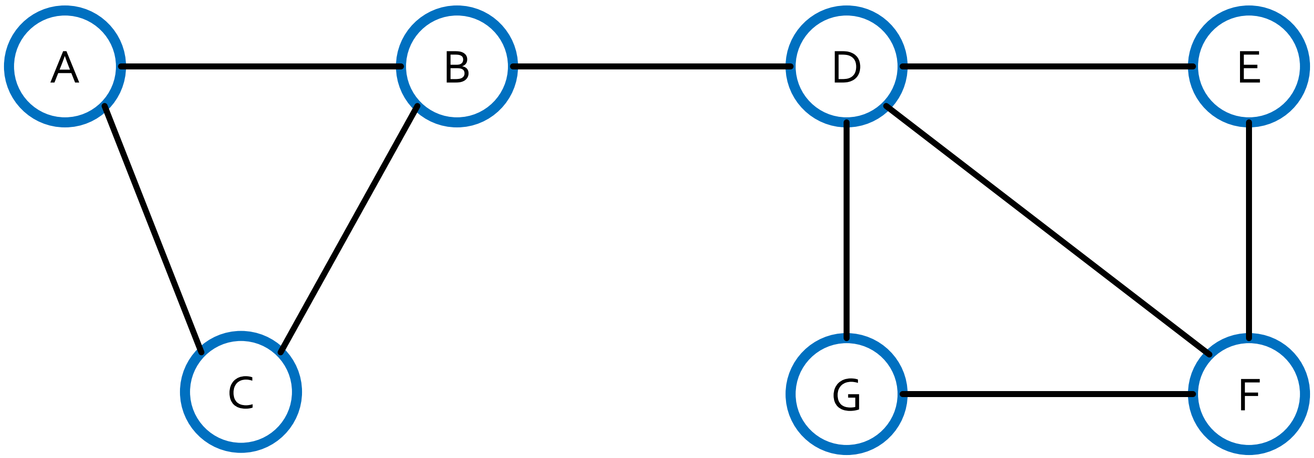
# Application:Neighborhoods

- Neighborhood of Distance d
- Recursive Algorithm for Neighborhoods
- Approximate Neighborhood Count

# Neighbors and Neighborhoods

- If there is an edge between nodes u and v, then u is a *neighbor* of v and vice-versa.
- The *neighborhood* of node u at distance d is the set of all nodes v such that there is a path of length at most d from u to v.
  - Denoted n(u,d).
- Notice that if there are N nodes in a graph, then n(u,N-1) = n(u,N) = n(u,N+1) = … = all nodes reachable from u.

# Example: Neighborhoods




n(E,0) = {E}; n(E,1) = {D,E,F}; n(E,2) = {B,D,E,F,G};
n(E,3) = {A,B,C,D,E,F,G}.

# Why Neighborhoods?

- The sizes of neighborhoods of small distance measure the “influence” a person has in a social network.
  - Note it is the size of the neighborhood, not the exact members of the neighborhood that is important here.

# Algorithm for Finding Neighborhoods

- n(u,0) = {u} for every u.
- n(u,d) is the union of n(v, d-1) taken over every neighbor v of u.
- Not really feasible for large graphs, since the neighborhoods get large, and taking the union requires examining the neighborhood of each neighbor.
  - To eliminate duplicates.
- Note: Another approach where we take the union of neighbors of members of n(u, d-1) presents similar problems.

# Approximate Algorithm for Neighborhood Sizes

- The idea behind Flajolet-Martin lets you estimate the number of distinct elements in the union of several sets.
- Pick several hash functions; let h be one.
- For each node u and distance d compute the maximum tail length among all nodes in n(u,d), using hash function h.

# Approximate Algorithm – (2)

- Remember: if R is the maximum tail length in a set of values, then $2^R$ is a good estimate of the number of distinct elements in the set.
- Since n(u,d) is the union of all neighbors v of u of n(v,d-1), the maximum tail length of members of n(u,d) is the largest of
  1. The tail length of h(u), and
  2. The maximum tail length for all the members of n(v,d-1) for any neighbor v of u.

# Approximate Algorithm – (3)

- Thus, we have a recurrence (on d) for the maximum tail length of any neighbor of any node u, using any given hash function h.
- Repeat for some chosen number of hash functions.
- Combine estimates to get an estimate of neighborhood sizes, as for the Flajolet-Martin algorithm.

# Moments

## Surprise Numbers
## AMS Algorithm

# Generalization: Moments

- Suppose a stream has elements chosen from a set of $n$ values.
- Let $m_i$ be the number of times value $i$ occurs.
- The $k^{\text{th}}$ *moment* is the sum of $(m_i)^k$ over all $i$.

# Special Cases

- $0^{th}$ moment = number of different elements in the stream.
  - The problem just considered.
- $1^{st}$ moment = sum of counts of the numbers of elements = length of the stream.
  - Easy to compute.
- $2^{nd}$ moment = *surprise number* = a measure of how uneven the distribution is.

# Example: Surprise Number

- Stream of length 100; 11 values appear.
- Unsurprising: 10, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9. Surprise # = 910.
- Surprising: 90, 1, 1, 1, 1, 1, 1, 1 ,1, 1, 1. Surprise # = 8,110.

# AMS Method

- Works for all moments; gives an unbiased estimate.
- We'll talk about only the 2nd moment.
- Based on calculation of many random variables $X$.
  - Each requires a count in main memory, so number is limited.

# One Random Variable

- Assume stream has length $n$.
- Pick a random time to start, so that any time is equally likely.
- Let the chosen time have element $a$ in the stream.
- $X = n$ * ((twice the number of $a$'s in the stream starting at the chosen time) – 1).
  - Note: store $n$ once, store count of $a$'s for each $X$.

# Expected Value of $X$

- $2^{nd}$ moment is $\Sigma_a (m_a)^2$.
- $E(X) = (1/n)\big(\Sigma_{\text{all times } t}\ n$ * (twice the number of times the stream element at time $t$ appears from that time on) – 1$\big)$.
- $= \Sigma_a (1/n)(n)(1+3+5+\ldots+2m_a-1)$ .
- $= \Sigma_a (m_a)^2$.

Group times by the value seen

Time when the last $a$ is seen

Time when penultimate $a$ is seen

Time when the first $a$ is seen

# Problem: Streams Never End

- We assumed there was a number *n*, the number of positions in the stream.
- But real streams go on forever, so *n* changes; it is the number of inputs seen so far.

# Fixups

1. The variables *X* have *n* as a factor – keep *n* separately; just hold the count in *X*.
2. Suppose we can only store *k* counts.  We cannot have one random variable X for each start-time, and must throw out some start-times as we read the stream.
   - Objective: each starting time *t* is selected with probability *k*/*n*.

# Solution to (2)

- Choose the first *k* times for *k* variables.
- When the $n^{th}$ element arrives ($n > k$), choose it with probability $k/n$.
- If you choose it, throw one of the previously stored variables out, with equal probability.
- Probability of each of the first n-1 positions being chosen:

$$\underbrace{(n-k)/n}_{\text{n-th position not chosen}} * \underbrace{k/(n-1)}_{\text{Previously chosen}} + \underbrace{k/n}_{\text{n-th position chosen}} * \underbrace{k/(n-1)}_{\text{Previously chosen}} * \underbrace{(k-1)/k}_{\text{Survives}} = k/n$$

# Final Remarks

- Thus, each variable has the second moment as its expected value.
- Use many (e.g., 100) such variables.
- Combine them as for Flajolet-Martin: average of groups of size O(log n), and then take the median of the averages.