STeP
The Stanford Temporal Prover
Educational Release
Version 1.0

USER’S MANUAL

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Chapter 1

Introduction

The Stanford Temporal Prover\textsuperscript{1} supports the computer-aided formal verification of concurrent and reactive systems based on temporal specifications. Reactive systems maintain an ongoing interaction with their environment and their specifications are typically expressed as constraints on their behavior over time. \texttt{step} is not restricted to finite-state systems but combines \textit{model checking} with \textit{deductive methods} to allow for the verification of a broad class of systems including parameterized (\textit{N}-component) circuit designs, parameterized (\textit{N}-process) programs, and programs with infinite data domains.

Finite-state systems can be automatically verified using model checking alone. For large finite-state or for infinite-state systems\textsuperscript{1} for which model checking is not possible or feasible, \texttt{step} also uses deductive methods: \textit{Verification rules} [Manna and Pnueli 1995] reduce temporal properties to first-order verification conditions. \textit{Verification diagrams} [Manna and Pnueli 1994] are a visual language for guiding, organizing and displaying proofs. They can be used to construct proofs hierarchically starting from a high-level sketch and proceeding incrementally through layers of greater detail.

\texttt{step} also includes techniques for automatic \textit{invariant generation} [Björner et al. 1995]. Deductive verification almost always relies on finding for a given program and specification suitably strong invariants and intermediate assertions. The system generates bottom-up invariants automatically by analyzing the program text. Combining them with the property to be proved, sufficiently detailed invariants can often be obtained to carry through the verification process.

Finally, \texttt{step} provides a collection of simplification and decision procedures that automatically check the validity of a large class of first-order and temporal formulas. This degree of automated deduction can handle many of the verification conditions that arise in deductive verification.

Figure 1.1 shows an overview of \texttt{step}. The main inputs are a reactive system (which can be a description of hardware or software) and a property to be proven about the system represented by a temporal logic formula. Verification can be performed by the model checker or by deductive means. Model checking is automatic; deductive verification is to a large extent automatic for simple \textit{safety} properties while \textit{progress} properties require more user guidance usually given in the form of a verification diagram. In all cases, the automatic prover is responsible for generating and proving the required verification conditions. An interactive Gentzen-style theorem prover is used to establish those verification conditions.
that are not proved automatically.

**STEP: The Educational Version**

The current release of step is the *educational version* and is a companion to the textbook [Manna and Pnueli 1995]. Like the book, it concentrates on safety properties and is intended for use by students, instructors, and researchers of formal methods.

### 1.1 Reading this Manual

This manual is a complete reference guide for using step. For more background on step’s framework for temporal specification and program verification, see [Manna and Pnueli 1991] and [Manna and Pnueli 1995].

Following are some suggestions on reading this manual:

- **Getting started**: Section 1.2 describes how to get step up and running. Section 1.3 gives an overview of step and a flavor of how a typical session proceeds. Section 1.4 defines the main terms used throughout this manual.
1.2. STARTING STEP

- **The Tutorial**: This manual includes a basic tutorial on step in Chapter 9. This tutorial is the recommended method for readers to familiarize themselves with the step interface and the different styles of verification in step.

After the tutorial, you should be able to verify some of the other programs included in the examples directory as well as your own simple programs using the rest of this manual as a reference.

Some sections are marked with a "*" to indicate that they should be read only after the unmarked sections are understood. For instance, first-time users can postpone studying features such as tactics (Section 6.3) and user-defined rewrite and simplification rules (Section 2.4.4) until they feel comfortable with the rest of the system. Sections marked with "**" are intended for advanced users and may be safely omitted by the beginning student.

Chapters 2-8 are the main reference for using the system. For those readers unfamiliar with [Manna and Pnueli1995]\ref{manna1995}, Appendixes A and B summarize step's program and property specification languages. Appendix C presents the most important differences between step and [Manna and Pnueli1995].

### 1.2 Starting step

The following are the steps for starting step:

1. Set your `STEP_DIR` environment variable to the full directory path where step is installed.\(^1\) If this variable is not set, you will not be able to run step. (If you are installing the system, you can edit the `bin/STeP` file to correctly set this value for all users.)

2. The `DISPLAY` environment variable is used by X-windows to identify the screen where programs should run and is usually of the form `machine-name:0.0`. Due to the inner workings of eXene (the SML X-windows library), this should be changed to the form `ip-address:0.0` where `ip-address` is the corresponding numerical IP-address.\(^2\)

   Alternatively, the display can be specified with the `-display` command-line option when running step as in `STeP -display ip-address:0.0`.

3. Make sure that the machine where you plan to run step has permission to open windows on your display. To make sure this is the case, execute in your X-windows console the command `xhost +`.

4. Add `STEP_DIR` to your `path`. In this way, when executing step you don't have to type `STEP_DIR/STeP` but just `STeP`.

5. Execute step by running `STeP`. If your `DISPLAY` variable and `xhost` permissions are set correctly, the step main window will appear on your screen.

\(^1\)Environment variables are set with "setenv variable value" Unix commands, which you can add to your `.cshrc` file for convenience.

\(^2\)On UNIX systems, you can usually obtain this address with the commands `nslookup machine-name` or "ping -l machine-name".
A number of other environment variables can be optionally set according to each user’s preferences; these are described in Appendix E.

Appendix D gives details on the installation of \textsc{step}. If you have problems installing or running \textsc{step}, please send mail to \texttt{step-bugs@cs.stanford.edu}.

### 1.3 Interacting with \textsc{step}

\textsc{step} has three main interface components: the \textit{Top-level Prover} from which verification sessions are managed and verification rules invoked; the \textit{Interactive Prover} used to prove the validity of first-order and temporal-logic formulas; and the \textit{Verification Diagram Editor} for the creation of Verification Diagrams. Figure 1.2 shows all \textsc{step}’s interfaces with the program \textsc{mux-sem} loaded.

![Figure 1.2: Overview of \textsc{step}’s interfaces](image-url)
1.3. INTERACTING WITH STEP

Verification Session—An Overview

A verification session begins by loading a program or a transition system that describes the system of interest and entering a temporal-logic formula that expresses one or more system properties to be proved.

The formula becomes the top or root goal of a proof tree. There are now several ways to proceed. The most common route is to apply a verification rule (Section 4.2) which reduces the formula to simpler first-order or temporal formulas thus generating the first level of subgoals of the proof tree. These subgoals are numbered accordingly: 1.1, 1.2, 1.3, etc. If all subgoals are established automatically by the Simplifier (i.e., the Simplifier can reduce them to true) all branches are closed and the proof is finished.

If the Simplifier is not able to prove one of the subgoals, we have one of the following possibilities:

1. The subgoal is not valid for the given program. In this case the proof cannot succeed.

2. The property is valid for the program but not generally valid. In this case you will have to (a) strengthen the original formula or (b) first prove some auxiliary properties of the program relative to which the problematic verification condition becomes valid. In the latter case you may start a new proof tree for the auxiliary property and return to the original property after the auxiliary proof is finished. This approach reflects the incremental proof style advocated in [Manna and Pnueli, 1995].

Another way to establish auxiliary properties is through the automatic generation of invariants. step includes three such methods which generate properties that often help in proving verification conditions (Chapter 7).

3. The property is generally valid but the simplifier is unable to reduce it to true. In this case the Interactive Prover (Chapter 8) can be used to establish the given property with user guidance. If this is successful the given subgoal is closed. Partial results may also be returned to the Top-level Prover and incorporated in the current proof tree as a new subgoal.

Another way to verify a program property is to use a verification diagram. A verification diagram represents a set of verification conditions sufficient to establish a given property. Diagrams are created by the Verification Diagram Editor (Chapter 5) and subsequently transferred to the Top-level Prover.

Finally, any property or subgoal may be established by the Model Checker through the exploration of the state space of the system (Section 4.5). Although model checking is only guaranteed to succeed for finite-state programs whose state space is not too large, it is also generally useful to identify erroneous properties since it provides a counterexample computation if the property is not valid for the given system.3

step can also be used as a theorem prover independently of any particular system verification task. In this case you only need to load a specification and only the Simplifier and the Interactive Prover are used.

3Restrictions may be placed on the system to make model checking more feasible—see Section 4.5.
Verification Session—A Quick Tour

To make things more concrete we will give an example of an actual step session. For a step-by-step description of the commands needed to run this example see the basic step tutorial in Chapter 9.

\[
\begin{align*}
\text{local} & \quad y : \text{integer where } y = 1 \\

P_1 :: & \quad \begin{cases}
\ell_0: \text{loop forever do} \\
\ell_1: \text{noncritical} \\
\ell_2: \text{request } y \\
\ell_3: \text{critical} \\
\ell_4: \text{release } y
\end{cases} & \quad || & \quad P_2 :: \begin{cases}
m_0: \text{loop forever do} \\
m_1: \text{noncritical} \\
m_2: \text{request } y \\
m_3: \text{critical} \\
m_4: \text{release } y
\end{cases}
\end{align*}
\]

Figure 1.3: Program \texttt{mux-sem} (mutual exclusion by semaphores)

Consider program \texttt{mux-sem} shown in Figure 1.3. It implements mutual exclusion using the semaphore \texttt{y}'s \texttt{request} decrements the semaphore when it is positive and \texttt{release} increments the semaphore. We want to prove the following properties of this program:

\[
\varphi_1 : \Box (y \geq 0)
\]
\[
\varphi_2 : \Box (\neg (\ell_3 \land m_3))
\]

The first property stating that \( y \) is always greater than or equal to 0 is proved automatically using rule B-INV (Section 4.2) and decision procedures for linear arithmetic. The property is then added to the list of background properties and may be used in subsequent proofs.

The second property expresses mutual exclusion and is not inductive; that is, it is not preserved by all transitions. In particular the verification conditions for transitions \( \ell_2 \) and \( m_2 \) are not valid and indeed when we apply rule B-INV the Simplifier does not reduce them to true. We now have several options. Since this is a finite-state program with a small number of states model checking has a good chance of succeeding; indeed the Model Checker can quickly verify this property.

Another option is to generate some auxiliary invariants. One of the \textit{linear invariants} automatically generated for this system is:

\[
I : \Box (\ell_0 + \ell_1 + \ell_2 = m_3 + m_4 + y)
\]

Unfortunately property \( \varphi_2 \) is not inductive relative to this invariant so the Simplifier is still unable to prove the verification conditions for \( \ell_2 \) and \( m_2 \). Repeating the application of B-INV with all linear invariants active for simplification you will notice that simplification is significantly slower since the background properties are being used in simplification.

However property \( \varphi_2 \) is directly implied by the invariant \( I \) and the property \( \varphi_1 \) we proved earlier. Therefore we can apply rule MON-I (Section 4.4) directly to the top-level goal and finish the proof after one simplification step.
Another option is to use rule G-INV (Section 4.2) to supplement the verification with a strengthened assertion, e.g., $\ell_3 + \ell_4 + m_3 + m_4 + y = 1$

This assertion implies mutual exclusion and is inductive (the locations have been arithmetized, e.g., the value of $\ell_3$ is 1 if control resides at $\ell_3$ and 0 otherwise). All verification conditions are reduced to true by the Simplifier, thus completing the proof.

Yet another option is a proof by verification diagram. The lower left corner of Figure 1.2 shows a state-partition diagram for program \texttt{MUX-SEM} that implies mutual exclusion; the property $\square (\neg (\ell_3 \land m_3))$ is proved automatically by invoking the Verification Diagram Rule on this diagram. This produces 35 different verification conditions, all of which are reduced to true.

### 1.4 Terms and Definitions

**Step Sub-systems**

In step, we identify the following sub-systems, each of which operates relatively independently of the rest:

**Top-level Prover (Chapter 3)** The Top-level Prover is the main interface to a step session from which all other activities are initiated.

**Interactive Prover (Chapter 8)** The Interactive Prover is invoked from the Top-level Prover to prove a particular goal or subgoal. When the Interactive Prover is active, the Top-level Prover is suspended. The Interactive Prover provides a Gentzen-style proof system which is guided by the user. While the Interactive Prover is normally used to complete the proof of a given subgoal, partial results from the Interactive Prover can also be transferred back to the Top-level Prover.

**Verification Diagram Editor (Chapter 5)** The Verification Diagram Editor is also invoked from the Top-level Prover but can be run concurrently with it. It is used to create and edit verification diagrams which can be given to the Top-level Prover to use in a proof.

**Model Checker (Section 4.5)** The Model Checker can be invoked from the Top-level Prover on any given temporal subgoal. If the Model Checker establishes the $P$-validity of the formula, the corresponding goal is proved. Otherwise, a counterexample computation (i.e., one that violates the formula) is returned.

**Simplifier (Section 6.1)** The Simplifier is a collection of decision procedures and rewrite rules used to reduce and prove general verification conditions. The Simplifier may be invoked by the user both from the Top-level Prover and the Interactive Prover to simplify single goals or subgoals. In the Top-level Prover, the Simplifier uses all axioms, invariants, and previously proven properties that are currently active. The power of the Simplifier, i.e., the trade-off between power and speed, is controlled by a number of global settings.
STEP Activities

We identify a few (nested) levels of activities each with its own environment:

STEP session At the top level is the step session which lasts for the entire period step is running. Simplification settings are part of the step session environment. They can be changed at any time but are not affected by loading new systems or moving on to new goals.

System verification session The next level down is the system verification session which lasts from the time a program or system is loaded until a new one is loaded. Proven properties are accumulated as long as the session lasts. Thus both the system and all the properties proven about it as well as the axioms loaded for it are part of the system verification session environment. All properties proven are valid relative to the current system. A system verification session may be terminated explicitly by loading a new system or resetting step.

Theorem-proving session At the same level as the system verification session is the theorem-proving session. The difference is that a theorem-proving session environment does not include a system. Thus properties are proved to be generally valid in this case and only the Simplifier and the Interactive Prover can be used to prove formulas.

Goal session A system verification or theorem-proving session may contain multiple goal sessions. Each unfinished proof of a goal is represented by a tree with the goal as its root. At any given time only one tree is being worked on. The user can move from one tree to the next in the middle of a proof and later return to the unfinished proof e.g. when some auxiliary properties have been established. A goal session ends when the goal is proved valid or is abandoned by the user.

Properties

We use different terms for properties depending on their status in a verification or theorem-proving session. We list the terms here in alphabetical order:

Axiom An axiom is provided by the user and is assumed to be valid throughout a system verification or theorem-proving session. An axiom is considered to be a background property.

Background property A background property is assumed or proven to be valid for the program in a system verification session and assumed or proven to be generally valid in a theorem-proving session. The user controls their use in simplification by activating and de-activating them. Background properties may also be deleted altogether from a verification session.

If active background properties are used by the Simplifier in the simplification process of goals and subgoals at the Top-level Prover and can be added to the current sequent in the Interactive Prover.

Background properties include axioms entered by the user properties already proved in the same verification session and automatically generated invariants.
1.4. TERMS AND DEFINITIONS

Note that the set of background properties is lost upon termination of a system-verification or theorem-proving session.

**Goal** A *goal* is a formula that we want to prove generally valid or valid over a given program and becomes the root of a proof tree.

**Invariant** An *invariant* is a formula of the form $\neg p$ stating that $p$ is true at every state of a program or system computation. Invariants can be independently proved or automatically generated by one of three methods: local, linear, or polyhedral invariant generation. Proven invariants are added to the set of background properties and will be used by the simplifier when simplifying new formulas. They can also be used as axioms in the interactive prover.

**Subgoal** A *subgoal* is a formula that results from the application of a verification rule to a goal or another subgoal. It is a node of the proof tree. An *open* subgoal is a leaf of the proof tree that has not been reduced to *true*.

**Verification condition** A *verification condition* is a formula that results from the application of a verification rule to a goal or a subgoal. Verification conditions are a subset of the subgoals.

**System Description and Specification**

`step` accepts a system description in two different formats, namely as an SPL program or as a fair transition system. Specifications are entered as linear-time temporal logic formulas.

**SPL program** A system description may be given in the form of an SPL program (*Simple Programming Language* program). The syntax of well-formed SPL programs is given in Section 2.5. SPL programs are parsed into fair transition systems. The semantics of SPL are given in Appendix A.

**Fair transition system** An alternative way to provide a system description is by means of a fair transition system. A *fair transition system* is a tuple with the following components:

- $V$: a set of system variables. These include all program variables.
- $\Theta$: the initial condition.
- $T$: a set of transitions.

Each transition is a relation between states specifying how the system can move from one state to the next. Section 2.6 describes how transition systems are specified in `step`. Appendix A describes fair transition systems in more detail.

**Auxiliary and system variables** Auxiliary variables are declared in specifications. System variables get declared in system descriptions and participate in the transition relations of the described transition system.
**Flexible and rigid variables** The value of a *flexible* variable may change over time while that of a *rigid* variable is assumed to be fixed throughout a computation. Auxiliary variables in `step` can be declared to be flexible or rigid. System variables declared as `in` are rigid while those declared as `out` and `local` are flexible.

Note that the flexible or rigid status of variables has a significant effect on whether terms under the scope of temporal operators can be replaced by others during interactive and automatic theorem-proving.\(^4\)

**Future operators** Future temporal operators are □ (Henceforth), □ (Eventually), □ (Until), □ (Waiting-for also called Unless) and ◻ (Next).

In `step`'s ASCII representation these are □ Γ ◻ Γ Until Γ Awaits Γ and ◻ (see Appendix B) respectively.

**Past operators** Past operators are □ (So-far), □ (Once), □ (Since), □ (Back-to) and ◻ (Previously).

In `step`'s ASCII representation these are □ [ ] Γ ◻ Γ Since ◻ Γ Backto Γ and ◻ (see Appendix B) respectively.

**Past formula** A *past formula* is a temporal-logic formula that does not contain any future operators that is it only contains state formulas and past operators.

### 1.5 Feedback

If you have problems running `step` or find bugs in the system please send mail to

step-bugs@cs.stanford.edu.

If you have questions or feedback send mail to

step-comments@cs.stanford.edu.

There is a mailing list for `step` users which will be used to distribute updated information about the system. To be added or removed from the list send mail to

step-request@cs.stanford.edu.

### 1.6 Acknowledgements

`step` has been designed and implemented by Niko Bjørner, Anca Brevnol, Eddie Chang and Tomás Uribe. Henry Sipma and Arjun Kapur have collaborated with the development and documentation of `step`.

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\(^4\)For example it is in general not possible to instantiate an existentially bound rigid variable by a term containing flexible variables.
Harish Devarajan implemented the decision procedure for full Presburger arithmetic. Assistance and feedback has been provided by Anuchit Anuchitanukul, Luca de Alfaro, Jeff Kamerer, and Yoshihiro Yamada. We would also like to thank Paul Neves and the students of CS256L at Stanford. The step logo was designed by Anuchit Anuchitanukul.

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Chapter 2

Systems and Specifications

A system can be input to step as an SPL program or directly as a transition system; SPL programs are parsed into transition systems. The computational model associated with transition systems and SPL programs is presented in Appendix A. The specification language is linear-time temporal logic. Its semantics is presented in Appendix B.

An SPL program or transition system is always loaded from a file. The properties to be proven may be entered directly or loaded from a file.

This chapter presents the step format for describing systems and their specifications. Since systems and specifications share the same syntax for variable and type declarations and logical expressions we first describe the general syntax for types (Section 2.1) and expressions (Section 2.2). We then describe the overall style of specifications (Section 2.3) and the syntax for declarations (Section 2.4). Finally we present the syntax for describing systems either as SPL programs (Section 2.5) or as transition systems (Section 2.6).

We use extended Backus-Naur form (EBNF) to describe the syntax of step’s input. Terminal symbols are written in typefont non-terminals in italics. Alternatives are separated by |, optional parts enclosed in [], zero or more occurrences of a construct are indicated by enclosing in {} and {{}+ is used for one or more occurrences.

Comments can be included in a file using the % character (indicating a comment until the end of the line) or using (* and *) for multi-line comments.

2.1 Types

The basic types in step are booleans, integers, rationals, range types, underspecified types, and datatypes. New compound types can be created using type constructors. The type constructors in step listed below define types for arrays, channels, and tuples of types in the usual way. A type can be enclosed in parenthesis to distinguish for instance a tuple of three elements, e.g., int * rat * bool from a nested tuple of pairs (int * rat) * bool.

\[
\text{type} ::= \begin{cases} 
\text{id} & \text{type identifier} \\
\text{base\_ty} & \text{base type} \\
\text{array [ range \{, range \} ] of type} & \text{array type} \\
\text{channel of type} & \text{synchronous channel} \\
\text{channel [i..] of type} & \text{unbounded asynchronous channel} 
\end{cases}
\]
The type constructor * has higher precedence than all other type constructors. Thus, the type

```
channel [1..] of bool * int * int
```

is parsed as

```
channel [1..] of (bool * int * int).
```

Note that (bool * int * int) \Gamma (bool * int) * int) and (bool * (int * int)) are three different types.

New underspecified types, datatypes and enumeration types can be declared in specification files; see Section 2.4.1.

### 2.2 Expressions

Expressions are constructed from constants, identifiers, function application, array references, tuple expressions, binary infix expressions, prefix expressions and quantification. The following well-formedness constraints are imposed:

- The boolean type is treated as a subtype of the integers \( \Gamma \) where the boolean value \( \text{false} \) corresponds to the integer value 0 and \( \text{true} \) corresponds to 1. Therefore any expression of boolean type can be used in an integer context. This is referred to as \textit{arithmetization}. \(^1\) Similarly, the integers (type int) are treated as a subtype of rationals (type rat).

- The operators \( \wedge \) (conjunction), \( \vee \) (disjunction), \( \rightarrow \) (implication), \( \leftrightarrow \) (bi-implication), \( \neg \) (negation) \( \Gamma \) and the temporal operators Until, Awaits, Since, Backto, ==>, <=>, O, [], <>, (-), [-], <-> take boolean arguments \( \Gamma \) and the resulting expression is of type boolean.

- The operators +, -, * require integer or rational arguments and produce a value of the corresponding type. Division / requires arguments of type integer or rational \( \Gamma \) and results in a value of type rational. The operators mod and div require arguments of type integer and result in a value of type integer.

\(^1\)Users should note that this arithmetization is very convenient, but at times confusing, see Section 2.7.
2.2. **EXPRESSIONS**

- Variables bound by quantifiers may not be array types.

The full grammar for expressions that can be used in specifications is as follows:

\[
\begin{align*}
\text{expn} & ::= \quad \text{bool-const} \quad \text{boolean value: true or false} \\
| & \quad \text{int-const} \quad \text{integer value} \\
| & \quad \text{id} \quad \text{identifier} \\
| & \quad \text{id \{ \text{index \{ , index \}} \}} \quad \text{disjunction of locations} \\
| & \quad \text{id(expns)} \quad \text{id can be an array identifier} \\
| & \quad \text{expr[expns]} \quad \text{function application} \\
| & \quad (\text{expn}) \quad \text{array reference} \\
| & \quad (\text{expn},\text{expns}) \quad \text{parenthesized expression} \\
| & \quad \# \text{int-const} \text{expn} \quad \text{int-const}'th projection of tuple} \\
| & \quad \text{expn infix expn} \quad \text{infix operator} \\
| & \quad \text{prefix expn} \quad \text{unary prefix operator} \\
| & \quad \text{if expn then expn else expn} \quad \text{if then else} \\
| & \quad \text{bind bv . expn} \quad \text{binding} \\
| & \quad \text{expr'} \quad \text{primed expression}
\end{align*}
\]

\[
\begin{align*}
\text{expns} & ::= \quad \text{expn} \mid \text{expn} , \text{expns}
\end{align*}
\]

\[
\begin{align*}
\text{bv} & ::= \quad \text{id} \quad \text{globally or locally declared} \\
| & \quad \text{id : type [kind]} \\
\end{align*}
\]

\[
\begin{align*}
\text{index} & ::= \quad \text{int-const} \mid \text{int-const .. int-const} \quad \text{indexing variable}
\end{align*}
\]

\[
\begin{align*}
\text{infix} & ::= \quad / \mid \backslash \mid \lor \mid \land \mid \rightarrow \mid \leftarrow \mid \leftarrow \rightarrow \mid \leftarrow \leftarrow \mid + \mid * \mid - \mid / \mid \% \mid \div \mid \mod \mid \geq \mid \leq \mid \neq \mid = \mid < \mid > \mid >= \mid <= \mid \downarrow \mid \uparrow \quad \text{binary boolean operators} \\
| & \quad \text{Until} \mid \text{Awaits} \mid \text{Since} \mid \text{Backto} \quad \text{binary temporal operators} \\
| & \quad \implies \mid \iff \quad \text{entailment and congruence} \\
| & \quad + \mid * \mid - \mid / \mid \% \mid \div \mid \mod \mid \geq \mid \leq \mid \neq \mid = \mid < \mid > \mid >= \mid <= \mid \downarrow \mid \uparrow \quad \text{arithmetical operators} \\
| & \quad \neg \mid \text{future operators} \\
| & \quad \text{past operators} \\
| & \quad \text{channel operations} \\
| & \quad \text{array update}
\end{align*}
\]

\[
\begin{align*}
\text{prefix} & ::= \quad ! \mid ~ \quad \text{negation} \\
| & \quad - \quad \text{minus} \\
| & \quad () \mid [ ] \mid <> \quad \text{future operators} \\
| & \quad (-) \mid [-] \mid <= \quad \text{past operators} \\
| & \quad \text{append} \mid \text{head} \mid \text{tail} \mid \text{length} \\
| & \quad \text{assign}
\end{align*}
\]

\[
\begin{align*}
\text{bind} & ::= \quad \text{Forall} \mid \text{Exists} \mid \text{Exists!} \mid \text{Sum} \mid \text{Prod} \quad \text{binding operators}
\end{align*}
\]

\[
\begin{align*}
\text{kind} & ::= \quad \text{Rigid} \mid \text{Flexible} \quad \text{flexibility of variable}
\end{align*}
\]

\[
\begin{align*}
\text{bool-const} & ::= \quad \text{true} \mid \text{false}
\end{align*}
\]

\[
\begin{align*}
\text{int-const} & ::= \quad \text{digit \{ digit \}}
\end{align*}
\]
id := alpha { alpha | digit | _ }  \\
alpha := upper or lower case letter  \\
digit := number between 0 and 9

**location disjunction:** As an abbreviated notation for the disjunction of consecutive locations, the specification may contain expressions of the form

\[ l\{1,5,7,9,11\}\{i\} \]

which is shorthand for

\[ l\{i\} \lor l\{i\} \lor l\{i\} \lor l\{i\} \lor l\{i\} \lor l\{i\} \lor l\{i\} \lor l\{i\} \lor l\{i\} \lor l\{i\} \]

**channel operations** are described separately in Section 2.4.5.

**array assignment:** Arrays are multidimensional tables. Individual table entries can be updated by the built-in function `assign`.

**Example:** The expression entered below:

```plaintext
in N, M : int where N > 0 \ M > 0 
local A : array [1..N, 1..M] of int
    assign(A,5,1,1)
```

is identical to the array \( A \) except for position \( [1,1] \) where it has been updated to the value 5.

The simplifier uses the following simplification rule automatically (so it really does not have to be declared):

```plaintext
variable a,x : [1..N] 
variable b,y : [1..M] 
variable e : int 
SIMPLIFY : (assign(A,e,a,b))[x,y] ---
    if a = x \ b = y then e else A[x,y]
```

See section 2.4.4 for how to declare your own simplification rules.

**tuple projection:** The \( i \)'th element of a tuple \( t \) can be accessed as:

\[ \#_i t \]

Tuple indices are counted from 1, i.e., the first element in a tuple has index 1.
### 2.3 Specification Files

A specification file usually contains three parts: declarations, axioms, and properties.

1. The *declarations* define new types, values, macros, rewrite rules, and auxiliary variables.
2. The axioms define properties that may be assumed by step in a verification session. Each axiom becomes a background property and may be activated or deactivated from the Top-level Prover.

3. The properties are those formulas you want to prove true for a program or system. They are state-valid in general. These become top-level goals in the Top-level Prover.

The format of the specification file is described by the following grammar:

\[
\text{specification} ::= \{ \text{declaration} \} \text{ expn} \\
| \quad \text{SPEC} \{ \text{declaration} | \text{axiom} | \text{property} \}
\]

\[
\text{axiom} ::= \text{AXIOM desc : expn} \quad \text{named axiom}
\]

\[
\text{property} ::= \text{PROPERTY desc : expn} \quad \text{named property specification}
\]

The nonterminal desc is an informal description of the corresponding axiom or property and can be any sequence of characters except colon (:).

Note that step does not guarantee the consistency of the given set of axioms so the specification of axioms should be done with caution.

A specification file may contain a single formula which by default becomes the property to be proven. It may contain multiple axioms and properties. In addition, the specification file may contain declarations of macros, values, and auxiliary variables. An example of a file with multiple axioms and properties is shown in Figure 2.1.

```
SPEC (* Greatest common divisor spec file *)

value COMMUTATIVE gcd : int*int --> int

AXIOM gcd1: [\forall m,n:int . (m != n --> gcd(m,n) = gcd(m-n,n))]

AXIOM gcd2: [\forall m:int . (m > 0 --> gcd(m,m) = m)]

AXIOM gcd3: [\forall m:int . (m < 0 --> gcd(m,m) = -m)]

PROPERTY aux1: [\forall (y1 > 0)]
PROPERTY aux2: [\forall (y2 > 0)]
PROPERTY aux3: [\forall (gcd(y1,y2) = gcd(a,b))]

PROPERTY partial correctness: 18 ==> g = gcd(a,b)
```

Figure 2.1: Specification file with multiple axioms and properties

2.4 Declarations

Declarations can be in one of the following forms which we describe below:
2.4. DECLARATIONS

declaration ::= type-decl | datatype-decl declaration of types
    | value-decl | macro-decl declaration of constructs
    | aux-var-decl | system-var-decl declaration of variables
    | simplify | rewrite | order rewrite rules

datatype-decl ::= type id = = type
    | type id = { ids }
    | type id underspecified type identifier
type-decl ::= type id = = type
    | type id = { ids }
    | type id enumeration type

2.4.1 Type Declarations

Type declarations are used to define abbreviations for compound types, new underspecified types, and new datatypes.

Type abbreviations are declared as type id = = type where type is a type expression (see Section 2.1). Omitting = and the type expression declares id as a new underspecified type:

type-decl ::= type id = type
    | type id = { ids }
    | type id underspecified type identifier

datatype-decl ::= type id = = constructors
    { and id = = constructors }

The enumeration type declaration

type id = = { id1, ..., idn }

is shorthand for the datatype declaration

type id = = id1 | ... | idn.

Example: A tree datatype can be defined recursively with a forest datatype as follows:

type ident

  type tree = = Node : : id : ident, children: forest

  and forest = = Nil
    | Cons : : first: tree, next: forest

The type ident is defined as a new underspecified type. A forest is a set of trees represented as a list—either Nil or the Cons of a tree and a forest. A tree itself is a Node containing an id and a set of children represented as a forest.

The above datatype definition implicitly declares the following function symbols:
value Node: ident * forest --&gt; tree
value id: tree -&gt; ident
value children: tree -&gt; forest
value Nil: forest
value Cons: tree * forest --&gt; forest
value first: forest -&gt; tree
value next: forest --&gt; forest

The constructors (Node Nil Cons) are used to create new elements of the corresponding type while the deconstructors (id children first next) are used to access the different components of the datatypes.

For the deconstructors the following simplification rules explained in Section 2.4.4 are automatically inferred and do not have to be declared:

variable i: ident
variable ch,n: forest
variable t: tree

 SIMPLIFY : id(Node(i,ch)) ----&gt; i
 SIMPLIFY : children(Node(i,ch)) ----&gt; ch
 SIMPLIFY : first(Cons(t,n)) ----&gt; t
 SIMPLIFY : next(Cons(t,n)) ----&gt; n

These rules are used automatically by the simplification procedures. Note that no simplification rules are given for first(Nil) and next(Nil) although these expressions are well typed. Their interpretation is left underspecified but they are not erroneous.

For the constructors the axioms asserting injectivity are also asserted automatically and used by the simplifier so they don’t have to be declared:

Forall t:tree . Forall f:forest . Nil != Cons(t,f)
Forall i,j:ident . Forall c1,c2:forest .
Node(i,c1) = Node(j,c2) --&gt; i = j \ c1 = c2
Forall t1,t2:tree . Forall sib1,sib2:forest .
Cons(t1,sib1) = Cons(t2,sib2) --&gt; t1 = t2 \ sib1 = sib2

The complete axiomatization of datatypes also includes well-founded induction schemas (see [Manna and Waldinger 1993]). These are not automatically provided in the current release of StepI so the user may have to provide suitable instances.

2.4.2 Value and Macro Declarations

Program and specification files may contain value and macro declarations. Their syntax is given by the following grammar:

value-decl ::= value [ac] ids : [type --&gt;] type
macro-decl ::= macro id : [type --&gt;] type
where id [( ids )] = expn

value declaration
macro declaration
Uninterpreted function symbols are declared as values. Binary function symbols can have additional attributes used by the Simplifier: the keyword \texttt{AC} states that the function is both associative and commutative; \texttt{COMMUTATIVE} and \texttt{ASSOCIATIVE} state that the function is only commutative or associative, respectively. For example, in Figure 2.1, the \texttt{gcd} function was defined to be commutative. Thus the simplifier assumes that \texttt{Forall a,b: int . gcd(a,b) = gcd(b,a)} besides the other axioms.

Macros which may take optional arguments are declared with the keyword \texttt{macro}. A \texttt{where} clause defines the expression corresponding to the macro. Each macro takes a fixed number of arguments and is expanded by the parser through purely syntactic substitution. A macro definition can use other previously defined macros, which will be expanded in turn.

Example: Figure 2.2 shows an example of value and macro declarations as they may appear in a program or a specification file. The value declarations define the integer constant \texttt{c} and function symbol \texttt{f} which takes a pair of arguments of boolean type and returns an integer. The macro declarations indicate that the symbol \texttt{m} should be expanded to the expression \texttt{c < c + 1} and the symbol \texttt{g} with integer arguments \texttt{(i,j)} should be expanded into \texttt{i + i + j + 5}.

\begin{verbatim}
value c:int
value f:bool * bool -- > int
macro m:bool where m = c < c+1
macro g:int * int -- > int where g(i,j) = i+i+j+5
\end{verbatim}

Example: Figure 2.3 shows a specification file to prove 1-bounded overtaking for program \texttt{mux-pet1} (Figure 3.4). The proof by rule G-WAIT requires the input of four auxiliary assertions. Having defined these assertions in the specification file allows the user to enter them by name \texttt{phi0\phi1\phi2} or \texttt{phi3} in the dialog window, reducing the chance of typing errors and saving time if the proof needs to be repeated multiple times.

### 2.4.3 Variable Declarations

\begin{verbatim}
aux-var-decl := variable ids : type [kind]
kind := Rigid | Flexible
system-var-decl := mode ids : type [where expn]
  | clock ids [where expn]
mode := in | out | local

ids := id { , id }
\end{verbatim}
% 1-bounded overtaking for mux-pet1
% for proof by rule G-WAIT use the following intermediate assertions:
% phi0: 14
% phi1: 13 \ ((m0\ /m1\ /m2\ /m5) \ \ (m3 \ \ !(s=1)))
% phi2: 13 \ m4
% phi3: 13 \ m3 \ s=1
SPEC
macro phi0: bool where phi0 /= l/4
macro phi1: bool where phi1 /= l/3 /\ (/m/0/./m/1/./m/2/./m/5/./m/3/./!(s=1))
macro phi2: bool where phi2 /= l/3 /\ m/4
macro phi3: bool where phi3 /= l/3 /\ m/3 /\ s=1
PROPERTY 1-bounded overtaking:
  13 == (\m/4\ Awaits (\m/4\ Awaits (!m/4\ Awaits 14)))

Figure 2.3: Specification file for proving mux-pet1 1-bounded overtaking

Auxiliary variables are declared with the keyword variable. Auxiliary variables can only be declared in the specification file for in a directly entered goal and are not part of the system being verified. Each auxiliary variable can be optionally declared to be Rigid (its value does not change over time) or Flexible (its value may change from one point in time to another). By default, auxiliary variables are rigid. Auxiliary variables are often used as part of macros and rewrite rules.

For convenience, a specification may also include declarations for system variables; these are further described in Section 2.5.

Example: The declarations

  variable x,y,z : int
  variable a,b,c : bool Flexible
  variable i : [1..N]

declare the rigid integer-type variables \(x, y\) and \(z\) the flexible boolean variables \(a, b, c\) and the rigid variable \(i\) of type range \([1..N]\).

Previously declared auxiliary variables can be used in quantification so the type of the quantified variable type does not have to be declared within the quantifier (see Section 2.2).

2.4.4 Rewrite and Simplification Rules *

Specifications can also include simplification and rewrite rules to efficiently build in equational axioms as well as ordering relations to control automatic rewriting.
User-defined rewrite rules can be specified in one of two ways: rewrite rules declared with the \texttt{REWRITE} keyword are applied \textit{under the user's control} step-by-step in the Interactive Prover. \textit{Simplification rules} declared with the \texttt{SIMPLIFY} keyword are applied automatically and exhaustively whenever the Simplifier is invoked from either the Top-level Prover or the Interactive Prover. Thus the set of user-defined simplification rules is expected to terminate while the set of rewrite rules under user control does not have to be terminating.

An ordering relation given by the \texttt{ORDER} keyword specifies a linear order over uninterpreted function symbols as a list. This order is used when converting equalities into rewrite rules during simplification; roughly speaking if \( t_1 = t_2 \) or \( t_2 = t_1 \) is an equality encountered in contextual simplification \( t_1 \) will be rewritten to \( t_2 \) if \( t_1 \) is built from a function symbol that appears earlier in the order.\footnote{The Simplifier uses the \textit{recursive path ordering on terms} to orient equalities, see [Dershowitz and Jouannaud, 1990].}

The syntax for simplification and rewrite rules is given by the following grammar:

\[
\begin{align*}
\text{rewrite} &::= \text{REWRITE } \text{desc} : \text{expn} \rightarrow\rightarrow \text{expn} & \text{rewrite rule} \\
\text{simplify} &::= \text{SIMPLIFY } \text{desc} : \text{expn} \rightarrow\rightarrow \text{expn} & \text{simplification rule} \\
\text{order} &::= \text{ORDER } \text{ids} \\
\end{align*}
\]

The nonterminal \texttt{desc} can be any sequence of characters except colon (:) and is used as an informal description of the simplification or rewrite rule. The auxiliary variables free in the expression to the left of \( \rightarrow\rightarrow \) should be a superset of the free variables of the right hand side expression. In connection with rewrite and simplification rules system variables are treated as constants (constructors).

\textbf{Example:}

Figure 2.4 shows an example of two rewrite rules. The rule \texttt{Peano 1} rewrites \( 6 + 0 \) to \( 6 \) and the rule \texttt{ad hoc} rewrites \( f(4+2) \) to \( \text{if } y \text{ then } x \text{ else } -(4+2) \).

\begin{verbatim}
variable x:int
local y:bool
value f:int --> int
REWITE Peano 1: x + 0 --> x
REWITE ad hoc: f(x) --> if y then x else -x
\end{verbatim}

Figure 2.4: Rewrite rules in a specification file

As with axiomatizations the user is responsible for the consistency of the set of rewrite and simplification rules. The reader interested in learning more about rewrite rules and their applications should consult [Dershowitz and Jouannaud, 1990]. An example of simplification rules appears in Section 2.4.1.
2.4.5 Channel Operations *

Asynchronous channels are modeled as queues, i.e., new elements are appended to the tail of the queue and elements are extracted from the head.

The basic operations on channels, `head`, `tail`, `append` and `length` are best described in the specification file that users should provide when they want to establish properties of channels. That is, step does not include the partial axiomatization given below; instead, specifications should provide separate axiomatizations for each type of asynchronous channel used.

```plaintext
% Sample channel specification
SPEC
  type t
  type chan : channel [!..] of t
  local ch : chan
  value pick : chan * int --> t
  variable x : t

AXIOM : [] Forall i : int .
  pick(append(ch,x),i) =
      if i-1 = length(ch) then x else pick(ch,i)
AXIOM : [] Forall i : int . pick(tail(ch),i) = pick(ch,i+1)
SIMPLIFY : head(ch) --> pick(ch,1)
SIMPLIFY : length(append(ch,x)) --> i + length(ch)
SIMPLIFY : length(tail(ch)) -->
      if length(ch) = 0 then 0 else (length(ch)) - 1
```

Hence the operation `head` extracts the first element from the buffer; `tail` returns the buffer excluding its first element; `append` inserts a new element to the end of the buffer; and `length` measures the number of elements residing in the buffer. The auxiliary function `pick`, axiomatized in the specification, provides us with a formal definition of the `head`, `tail`, `append` operations. We notice that taking `head` of an empty buffer only simplifies to some expression involving `pick` and its value is therefore underspecified.

2.5 SPL Programs

The syntax of SPL programs follows that of traditional imperative languages such as Pascal. In addition to the basic constructs found in these languages, SPL supports nondeterminism by means of the `selection` statement or and parallel composition by means of the `cooperation` statement ||. Parallel processes can interact through shared variables such as semaphores as well as by synchronous and asynchronous channels. Execution of parallel processes is assumed to proceed by interleaving (see Appendix A for a complete account of SPL semantics).
2.5. SPL PROGRAMS

Program variables

As in most imperative languages, program variables must be declared prior to their use in the appropriate scope. SPL provides an additional mode associated with the declaration of program variables. A variable declared as in is an input to the program; it may not be modified by any of the program statements and is assumed to have a fixed value throughout the program execution. Variables declared as local or out may be changed by the program.

Variables may be initialized or constrained with an optional where clause. The given condition is assumed to hold only at the start of the program, not each time the program enters the block where the variable is declared.

Parameterized programs

SPL allows you to write parameterized (N-process) programs in which an arbitrary number of identical processes are declared. Both parameterized composition and parameterized selection are supported.

Example:

Figure 2.5 shows a deadlock-free solution to the N-process Dining Philosophers problem. The program contains N instances of the infinite loop 10-18 each with its unique value of i.

Grouped statements

A grouped statement is a set of statements enclosed by << and >>. A grouped statement is translated into a single transition that is an atomically executable unit. Grouped

```
in N : int where N >= 2
local f : array [1..N] of int where Forall i:[1..N] . (f[i] = 1)
local r : int where r = N - 1

|| P[i:[1..N]] ::
  10:  loop forever do [           
    11:  noncritical;
    12:  request r;
    13:  request f[i];
    14:  request f[(i mod N) + 1];
    15:  critical;
    16:  release f[i];
    17:  release f[(i mod N) + 1];
    18:  release r
  ]
```

Figure 2.5: Program dine (dining philosophers)
statements may only contain basic statements and may only be combined into composite statements by selection and concatenation. A grouped statement may contain at most one send or receive statement on the same channel.

Example: To request resources \( r \) and \( s \) simultaneously one can enter

\[
\langle \langle \text{request } r; \text{ request } s \rangle \rangle
\]

which can also be encoded as

\[
\text{guard } r > 0 \land s > 0 \text{ do } (r,s) := (r-1,s-1)
\]

Program statements

The following grammar provides the syntax of SPL programs:

\[
\begin{align*}
\text{program} & ::= \{ \text{decl} \} \text{composite stmt} \ [; \text{label}] \\
\text{composite stmt} & ::= \text{composite stmt} \ ; \text{composite stmt} \quad \text{concatenation} \\
& | \text{composite stmt} \ | \text{composite stmt} \quad \text{cooperation statement} \\
& | \text{composite stmt} \text{or composite stmt} \quad \text{selection statement} \\
& | \text{[label]} \text{ stmt} \\
\text{stmt} & ::= \text{basic group stmt} \\
& | \text{request variable} \quad \text{request statement} \\
& | \text{release variable} \quad \text{release statement} \\
& | \text{noncritical} \quad \text{noncritical statement} \\
& | \text{critical} \quad \text{critical statement} \\
& | \text{choose variable} \quad \text{choose} \\
& | \text{produce variable} \quad \text{produce statement} \\
& | \text{consume variable} \quad \text{consume statement} \\
& | \text{guard } p_{-} \text{expn} \text{ do assignment} \quad \text{guarded assignment} \\
& | \text{if } p_{-} \text{expn} \text{ then composite stmt} \quad \text{1-way conditional} \\
& | \text{if } p_{-} \text{expn} \text{ then composite stmt} \quad \text{2-way conditional} \\
& | \text{else composite stmt} \\
& | \text{when } p_{-} \text{expn} \text{ do composite stmt} \quad \text{when statement} \\
& | \text{while } p_{-} \text{expn} \text{ do composite stmt} \quad \text{while statement} \\
& | \text{repeat composite stmt until } p_{-} \text{expn} \quad \text{repeat statement} \\
& | \text{loop forever do composite stmt} \quad \text{infinite loop} \\
& | \text{[ id :: ] } [ \text{ program }] \quad \text{block statement} \\
& | \langle \langle \text{comp group stmt} \rangle \rangle \quad \text{grouped statement} \\
& | \text{or binding composite stmt} \quad \text{parameterized selection} \\
& | \text{binding composite stmt} \quad \text{parameterized cooperation} \\
\text{comp group stmt} & ::= \text{basic group stmt} \\
& | \text{if } p_{-} \text{expn} \text{ then comp group stmt} \quad \text{1-way conditional}
\end{align*}
\]
| if \( p \expn \) then \textit{comp-group-stmt} \hspace{1em} 2-way conditional  
| \hspace{1em} else \textit{comp-group-stmt}\n| when \( p \expn \) do \textit{comp-group-stmt} \hspace{1em} when statement  
| \textit{comp-group-stmt} or \textit{comp-group-stmt} \hspace{1em} selection statement  
| \textit{comp-group-stmt} ; \textit{comp-group-stmt} \hspace{1em} concatenation  

\textit{basic-group-stmt} ::= \textit{skip} \hspace{1em} skip statement  
| \textit{assignment} \hspace{1em} assign statement  
| \textit{await} \( p \expn \) \hspace{1em} await statement  
| \textit{variable} \( <= \) \( p \expn \) \hspace{1em} send statement  
| \textit{variable} \( => \) \textit{variable} \hspace{1em} receive statement  

\textit{assignment} ::= \textit{left-val} \( := \) \( p \expn \) \hspace{1em} assignable value  
\textit{left-val} ::= \( p \expn \) \hspace{1em} variable  
\textit{variable} ::= \textit{id} \hspace{1em} array variable  
| \textit{id} \[ \textit{p \expns} \]  
\textit{binding} ::= \textit{id} \[ \textit{params} \] \::\hspace{1em} named binding  
| \textit{params} \hspace{1em} unnamed binding  
\textit{params} ::= \textit{param} \{ , \textit{param} \} \hspace{1em} list of parameters  
\textit{param} ::= \textit{id} \[ \textit{range} \] \hspace{1em} parameter  
\textit{label} ::= \textit{id} \hspace{1em} label  
\textit{ids} ::= \textit{id} \{ , \textit{id} \} \hspace{1em} list of identifiers  

\textit{decl} ::= \textit{type-decl} \mid \textit{datatype-decl} \hspace{1em} SPL declaration  
| \textit{value-decl} \mid \textit{macro-decl} \hspace{1em} SPL declaration  
| \textit{system-var-decl} \hspace{1em} SPL declaration  

\textit{system-var-decl} ::= \textit{mode} \textit{ids} \[ \textit{type} \{ \textit{where} \textit{expn} \} \hspace{1em} basic variable declaration  
\textit{mode} ::= \textit{in} \mid \textit{out} \mid \textit{local} \hspace{1em} mode of variable  

Assignable expressions are built from system variables declared as \texttt{local} or \texttt{out} and array references to \texttt{local} or \texttt{out} arrays. Composite assignable expressions are obtained by collecting simpler assignable expressions into tuples.

**Example:** An example of a composite assignment swaps the values of \texttt{x} and \texttt{y} as follows:

\[(x, y) := (y, x)\]
Expressions

The expressions allowed in SPL programs are a subset of the expressions allowed in specifications. In particular, the following are not allowed in SPL programs:

- pruned variables
- Prod and Sum
- temporal operators
- location expressions

Furthermore, when binding variables by quantifiers inside an SPL program, their types must always be declared locally and the optional kind is not allowed (they are always Rigid).

The syntax of SPL expressions is then as follows:

\[
p_{expn} ::= \begin{align*}
& \text{bool-const} & \text{boolean value} \text{ true or false} \\
| & \text{int-const} & \text{integer value} \\
| & \text{id} & \text{identifier} \\
| & \text{id}(p_{expns}) & \text{function application} \\
| & \text{expr}[p_{expns}] & \text{array reference} \\
| & (p_{expn}) & \text{parenthesized expression} \\
| & (p_{expn},p_{expns}) & \text{a tuple of expressions} \\
| & \# \text{int-const } p_{expn} & \text{int-const}'\text{th projection of tuple} \\
| & p_{expn} \text{ infix } p_{expn} & \text{infix operator} \\
| & \text{prefix } p_{expn} & \text{unary prefix operator} \\
| & \text{if } p_{expn} \text{ then } p_{expn} \text{ else } p_{expn} & \text{if then else} \\
| & \text{bind } \text{id} : \text{type} . \ p_{expn} & \text{binding }
\end{align*}
\]

\[
p_{expns} ::= p_{expn} \mid p_{expn} , p_{expns}
\]

\[
\text{infix} ::= \begin{align*}
& \land \mid \lor \mid \rightarrow \mid 
\leftarrow \mid \text{and} \mid \text{or} \mid \text{impl} \\
| & \text{+} \mid \text{*} \mid 
\text{-} \mid / \mid \text{mod} \mid \text{div} \\
| & = \mid \text{!=} \mid < \mid \text{>} \mid \text{>=} \mid \text{<=} \mid \text{<>}
\end{align*}
\text{binary boolean operators}
\]

\[
\text{prefix} ::= \begin{align*}
& \neg \mid \text{negation} \\
| & \text{append} \mid \text{head} \mid \text{tail} \mid \text{length} \\
| & \text{assign} \\
\end{align*}
\text{channel operations}
\]

\[
\text{bind} ::= \begin{align*}
& \text{Forall} \mid \text{Exists} \mid \text{Exists}! \\
\end{align*}
\text{binding operators}
\]

\[
\text{bool-const} ::= \begin{align*}
& \text{true} \mid \text{false}
\end{align*}
\]

\[
\text{int-const} ::= \begin{align*}
& \text{digit} \{ \text{digit} \}
\end{align*}
\]

\[
\text{id} ::= \begin{align*}
& \text{alpha} \{ \text{alpha} \mid \text{digit} \mid _ { } \}
\end{align*}
\]
2.6 Transition Systems

Transition systems are the basic system representation in step. SPL programs are parsed into fair transition systems; an alternative to SPL is to describe transition systems directly. Appendix A includes the basic definitions of transition systems. For a more leisurely introduction, see [Manna and Pnueli 1991].

2.6.1 Fair Transition Systems

A transition system specification contains:

- Declarations of types and free variables.
- An optional initial condition which is assumed to hold at the initial state of every computation.
- A set of transitions.

Transitions are expressed in terms of and are usually equivalent to (see below) transition relations. A transition relation describes how the system can change from one state to the next. Strictly speaking, a transition relation can be expressed by a formula over the set of primed and unprimed system variables; however, for added convenience, transition relations in step can be specified by a combination of four different fields. Each field is optional, and step inserts default values for absent ones. The four fields are the following:

- The modvar field: contains a set of variables that are changed when the transition is taken. To indicate that the transition relation allows a variable to change value nondeterministically, one can include it in the modvar field and not mention it elsewhere. By default, the only variables that are changed by the transition relation are the ones primed in the modrel field and the ones to the left-hand side of assignments in the assign field described below. The modvar field does not need to contain those variables.

- The enable field: the enabling condition, a formula that may not contain any primed variables. The transition can only be taken if the enabling condition is satisfied. By default, the enabling condition is true.

- The modrel field: contains the modifying relation, an arbitrary relation over primed and unprimed variables that is assumed to hold when the transition is taken. Its default value is true.

- The assign field: lists the assignments for those variables whose next-state values can be described as a function of the current-state (unprimed) variables.

An assignment \( x := t \) (in the assign field) can be equivalently expressed as a conjunct \( x' = t \) in the modifying relation but assignment relations need not also appear in the modrel field. In fact, it is preferable to only include them in the assign field. For example, rather than having \( x' = x + 1 \) as a conjunct in the modrel field, you should include \( x := x + 1 \) in the assign field.
In general, variables that appear on the left-hand side of an assignment statement should not appear primed in the modifying relation (there is the risk of unwillingly specifying an unsatisfiable transition relation). Still there is the possibility that the user will include a conjunct of the form \( x' = x + 1 \) in the model field or will mention a primed variable that is also assigned to. The parser will automatically discover these cases and normalize the transition relation while printing a warning to the console.

A transition consists of one or more transition relations. This partition allows a more convenient representation of certain transition relations, e.g., those for the while and if-then-else SPL constructs. A transition is considered enabled if at least one of its constituent transition relations is enabled. It is considered taken when one of its transition relations is taken. A transition can be taken only if it is enabled.

Transitions that only differ with respect to some parameters can be collected in one parameterized transition. The optional parameters are listed after the transition's name and declare one transition for each value assignment to the parameters.

A transition contains an optional fairness requirement which can be JustICompassionate or NoFairness. The default fairness requirement is NoFairness.

Example: The transition system in Figure 2.6 implements Euclid's algorithm for computing the greatest common divisor of two positive integers \( x \) and \( y \).

```
Transition System % Euclid's algorithm
in a,b: int where a > 0 \( \land \) b > 0
local x,y: int
out gcd: int
Initially x = a \( \land \) y = b
Transition t1 Just:
  enable x > y
  assign x := x - y
Transition t2 Just:
  enable x < y
  assign y := y - x
Transition t3 Just:
  enable x = y
  assign gcd := x
```

Figure 2.6: Euclid's algorithm for greatest common divisor

The system illustrates that no control locations need to be associated with the transitions when it is entered directly as a transition system. This contrasts with the transitions generated from SPL programs. An SPL program corresponding to the above

---

The verification rules generate one verification condition for each transition relation, which may be simpler to manipulate than the verification condition for the entire transition. See Appendix A for the semantics of while and conditional statements. See [Manna and Pnueli, 1995] for an example where transition relations are used to simplify verification conditions.
transition system is shown in Figure 2.7. It generates the transition system shown in Figure 2.8.

\[
\text{in } a,b: \text{ int where } a > 0, b > 0 \\
\text{local } x,y: \text{ int where } x = a, y = b \\
\text{out } \text{gcd}: \text{ int} \\
10: \text{ [loop forever do} \\
\text{[} \\
\text{t1: guard } x > y \text{ do } x := x - y \\
\text{or} \\
\text{t2: guard } x < y \text{ do } y := y - x \\
\text{or} \\
\text{t3: guard } x = y \text{ do } \text{gcd} := x \\
\text{]} \\
\text{]} \\
\]

Figure 2.7: SPL program for Euclid’s algorithm

Transition System
in a,b: int
local x,y: int
out gcd: int
control pi0 : [0..0]
Initially a > 0 \& b > 0 \& x = a \& y = b \& pi0 = 0
Transition t1 Just:
  enable pi0 = 0 \& x > y
  assign x := x - y
Transition t2 Just:
  enable pi0 = 0 \& x < y
  assign y := y - x
Transition t3 Just:
  enable pi0 = 0 \& x = y
  assign gcd := x

Figure 2.8: Transition system obtained from the Euclid SPL program (Figure 2.7)

### 2.6.2 Transition System Syntax

The syntax of transition systems is given by the following grammar:

\[
\text{transition-system} ::= \text{ Transition System}
\]

```
 CHAPTER 2. SYSTEMS AND SPECIFICATIONS

{ decl } { initial } { transition }+

| Clocked Transition System
  { decl } { initial } { progress } { transition }+

decl ::= type-decl | datatype-decl
    | value-decl | macro-decl
    | system-var-decl

system-var-decl ::= mode ids : type [where expn]
    | clock ids [where expn]

mode ::= in | out | local

initial ::= Initially expn

progress ::= Progress expn

transition ::= Transition name [ [ params ] ] [fairness] : relation { ; relation }

relation ::= [ modvar : ids ]
    | [ enable : expn ]
    | [ modrel : expn ]
    | [ assign : assignment { , assignment }]

name ::= id

params ::= id : type { , id : type }

fairness ::= Justice | Compassionate | NoFairness

assignment ::= left-val := expn

left-val ::= expn

Assignable expressions are built from system variables declared as local/out or clockC as well as array references to local or out arrays.

2.6.3 Clocked Transition Systems **

Clocked transition systems [Kesten et al.] can be used to model discrete or continuous real-time systems and are partially supported in the educational release of step. Clocked transition systems differ from fair transition systems in the following ways:

- There are designated clock variables T (global time) and tick (time progress measure) which may not be changed by the standard transitions. Only T may be used in the standard transitions. Initially T is set to 0. These variables are declared automatically when a clocked transition system is specified.

- Fairness conditions on transitions are irrelevant and therefore only the condition NoFairness is allowed.

- The user can specify a special progress condition. It restricts the amount that time is allowed to progress between the standard transitions.

The progress condition is written as:
Progress \( \text{expn} \)

where \( \text{expn} \) is assumed to be a first-order formula \( F(\text{clock}_1, \ldots, \text{clock}_n, T) \) with the auxiliary clocks \( \text{clock}_1, \ldots, \text{clock}_n \) and the global clock \( T \) as free variables. The progress condition corresponds to a transition of the form:

**Transition tick:**

\[
\begin{align*}
\text{modvar} & \quad \text{tick} \\
\text{enable} & \quad \text{tick} > 0 \\
\& \quad \forall t : \text{rat} . \quad (0 <= t \land t <= \text{tick} \rightarrow F(\text{clock}_1*t, \ldots, \text{clock}_n*t, T+t)) \\
\text{assign} & \quad (\text{clock}_1, \ldots, \text{clock}_n, T) := (\text{clock}_1+\text{tick}, \ldots, \text{clock}_n+\text{tick}, T+\text{tick})
\end{align*}
\]

which is generated automatically by the parser. It is helpful to think about this transition as one that allows time to progress from \( T \) to \( T + \text{tick} \) whenever the progress condition \( F \) is satisfied for all intermediary values between \( T \) and \( T + \text{tick} \).

Clock variables are always of type \( \text{rat} \).

### 2.7 When Parsing Fails

It is common for programs and specifications to fail to parse on the first try. The error messages generated by the parser typically indicate the line number and character range where the error was detected, separated by a “:” and provide some extra information about the offending part.

Unfortunately, the error messages are not always as illuminating as we would like. Here are some hints for when things don’t work as expected:

- The different kinds of arrows can be confusing:
  - \( \leftrightarrow \) is the temporal operator \( \Diamond \) (“once”);
  - \( \leftrightarrow \rightarrow \) is double implication
  - \( \Leftarrow \) is “less than or equals”
  - \( \rightarrow \) is normal implication
  - \( \rightarrow \) is also used in function types
  - \( \Rightarrow \) is the temporal “entails” relation (\( p \Rightarrow q \) stands for \( \Diamond (p \Rightarrow q) \))
  - \( \Rightarrow \) is also used for channel operations in SPL
  - \( \Leftarrow \) is a built-in macro expanded to \( \Box (p \Leftarrow q) \)
  - \( \Leftarrow \) is used to specify rewrite rules

- A period (“.”) is expected after quantifiers.

- Variables declared in the specification file and quantified variables are rigid by default. To specify a flexible variable in quantification, just write

\[
\forall x : \text{int} \ \text{Flexible.} \ \text{expn}.
\]

Even after parsing a file, watch out for the following:
• Arithmetization:
  \[ 1 \leq i \leq n \] is parsed as \((1 \leq i) \leq n\) hence as “the truth value of 1 \leq i is less than or equal to N” and not as \(1 \leq i \land i \leq n\).

• Rigid vs. flexible variables (see above).

• Scope of quantifiers and declarations:
  Binding operators have maximal scope. Hence

  \[
  \forall x : \text{int} . \ p(x) \rightarrow q
  \]

  is parsed as

  \[
  \forall x : \text{int} . \ (p(x) \rightarrow q)
  \]

  and not as

  \[
  (\forall x : \text{int} . \ p(x)) \rightarrow q
  \]

  although \(x\) is not free in \(q\).

• When in doubt, add parentheses!
Chapter 3
The Top-Level Interface

Starting step brings up the Top-level Prover window (Figure 3.1) a point-and-click X-windows interface. The Top-level Prover window has the following main components:

1. At the top of the main step window is the main menu used to load systems and specifications, save proof searches, view systems, select properties to be used in proofs, exit the system, and online help.

2. Under the main menu is the status line. It displays the name of the current system if any, the number of unfinished proof trees, the number of open goals in the current proof tree, the current simplifier settings, and whether automatic simplification is on or off.

3. Below the status line is the current goal window where the current unproven goal or subgoal is displayed. This window has a scroll bar at the left. Verification rules are applied to the formula that appears in this window.

4. Below the current goal window is the output window which displays the results of applying rules and generating invariants. It will also state when a proof is complete and generally leave a record of the system’s activities. This window can be reset with the Clear text window item under the Settings pull-down menu.

5. Under the output window at the bottom of the top-level prover is step’s message/help line. In this window step indicates what action is being done, which dialog window is waiting for input, or if an action has terminated successfully or not. It will also display a brief description of any feature that is being selected. For example, if Undoselected, this window will display “Backtrack to parent”.

6. At the right side of the top-level window is the action region a double column of buttons that invoke verification rules and decision procedures and apply them to the current goal. Besides the (temporal) verification rules, buttons in this column invoke the automatic Simplifier or start up the Interactive Prover. The [Previous] and [Next] buttons cycle through the open goals in the current search.
Figure 3.1: Top-level Prover window
7. In the lower right-hand corner of the window is a button with the step logo. This button functions as the interrupt button and can be used to halt most of the automatic operations performed by step.

3.1 Menu Options

The File Menu

The File pull-down menu (Figure 3.2) is used to load system description and specification files and to save finished and unfinished proofs.

![File Menu](image)

**Figure 3.2: Top-Level Prover File Menu**

Several options in this menu activate the File Browser utility that allows you to move around in the directory hierarchy and select files.

Activating the File Browser brings up a window with several panels as shown in Figure 3.3. The name of the window reflects the function that called it (in the figure Load program). The window under File name shows the current file selected and you can enter the filename here directly. The two buttons below allow you to display all files or only those with the default extension for the current function (in this case, the default extension for programs is .sp). The left panel below these buttons shows the files in the selected directory and the right panel shows the directory hierarchy. To move to another directory, click on the directory name in the right panel. To select a file, click on the filename in the left panel. To load or save a file, click OK after selecting the file or entering its name.
CHAPTER 3. THE TOP-LEVEL INTERFACE

Figure 3.3: File Browser

If a file does not parse correctly a Read Error window will appear with an indication of the first parsing error encountered. The file can be corrected and read again by clicking Try same file again or the operation canceled with Cancel.

**Load program** Loads an SPL program from a file. The default extension for a program file is .spl. The program will be parsed into a fair transition system. The program text is displayed in a separate window (setting the STEP_SHOW_LOADED environment variable to OFF disables this—see Appendix E). The program text window may be deleted at any time by clicking Quit and redisplayed with the View program text option below.

Figure 3.4 shows the program window for program *mux-pet1*. The program window also displays the correspondence between the program locations and the value of the internal control location variable. Control locations are explained in detail in Appendix C.2.

If Load program is selected while another verification session is still in progress a confirmation window will first appear. The entire verification session environment is reset when a new program is loaded i.e. all background properties and unproven goals are deleted.

**Load transitions** The Load transitions option works like Load program but expects a transition system file. The default extension for transition system files is .trans. The format for transition systems was given in Section 2.6.

As with programs the transition system is displayed in a separate window (unless the STEP_SHOW_LOADED environment variable is OFF). Loading a new transition system
Figure 3.4: Program Window for program `mux-pet1`
terminates the current verification session.

**Load specification**  Loads a specification (a list of axioms and properties to be proved) from a file. The default extension of a specification file is `.spec`. The input format of specification files was given in Section 2.3. Each property becomes the root of a separate proof tree and the first one appears in the current goal window. If a specification file is loaded during another goal session, the current proof search is suspended and the first property in the specification file will become the new current goal as the root of a new proof search. The old proof search however remains in the system and may be returned to later (e.g. with the select search option).

**Reset Searches**  Selecting *Reset Searches* deletes all pending proof trees and goals as well as all background properties. However, the current program or transition system is retained. Thus it resets a verification session.

**Reset All**  Terminates a system-verification or theorem-proving session: it removes all current proof trees, background properties, and the current system description if any.

**Save search**  Saves the current proof tree to a file in ASCII format for off-line inspection. Note that the current proof tree is necessarily unfinished i.e. it still has open subgoals. Once all subgoals are closed the proof tree is removed from the list and the next proof tree if any becomes the current one. To save the last complete proof use *Save last proof* described below.

**Note:** the current version of step does not support reloading this file to resume the proof. We plan to add this capability in future releases.

**Save goal**  Saves the current goal to a file. The file will contain declarations for all free variables.

**Save properties**  Saves all background properties to a file. The background properties include all properties proven and all invariants generated during the current program verification session as well as all axioms asserted by the user.

**Save transitions**  Saves the current transition system to a file; this is useful to see how SPL programs are converted to transition systems.

**Save last proof**  Saves the finished proof of the most recently established goal.

**View transitions**  Displays a separate window containing the current transition system. It can also be selected when the system description is loaded as an SPL program. The transition window can be deleted at any time by clicking on the *Quit* button of this window.

**View program text**  Displays a separate window with the text of the current SPL program and is similar to *View transitions* above.

**Quit**  Terminates the step session after confirmation from the user.
The Properties Menu

The Properties pull-down menu (Figure 3.5) is used to manage the background properties, move between different proof trees, generate invariants, and enter new goals and axioms directly.

![Properties Menu](image)

Figure 3.5: Properties menu

Activate/Deactivate This option displays a window with all the background properties, i.e., all axioms entered, invariants generated, and properties proven during the current system verification session. (See for example Figure 3.6 which displays the axioms, some invariants and a proven property aux1 of the program gcd.) By default all properties are active, which means that they are used by the Simplifier. This window allows you to deactivate properties or delete them altogether. A property is deactivated by clicking on the switch and removed by clicking on the skull icon. The buttons Select All and Deselect All select or deselect all the listed properties.

No window will appear when there are no background properties.

Select goal This menu option allows you to move to another open goal in the current proof tree. When selected, it displays a window listing all open goals in the current proof tree (see Figure 3.7). Each goal has a select button. By clicking on Select, the corresponding goal is made the current one. The Cancel button returns to the current goal.

Next search Moves to the next proof tree with open subgoals. The previous proof tree is kept in its current state and may be returned to later. This option is useful if you reach a dead-end in a proof and realize you need to prove another property first.
Figure 3.6: Activate/Deactivate Window
Figure 3.7: Goal selection window
Select search Generalizes Next search above to move to any other proof tree. A window listing the root nodes of all proof trees is displayed. See Figure 3.8 for an example. By clicking on the Select button, the corresponding proof tree is made the current proof tree.

![Proof tree selection window](image)

Figure 3.8: Proof tree selection window

No window will appear if there is nothing to select.

Abandon search Deletes the current proof tree and makes the next proof tree in the list if any the current proof tree. A confirmation window appears before this action is taken.

The following options generate program invariants. When the invariant generation is done, the invariants are displayed in the output window and added to the list of background properties. By default they are active and thus used in the simplification of goals. They may be deactivated with the Activate/Deactivate menu option (above). When the invariant generation is finished a message will appear in the message window.

Newly generated invariants are simplified using the basic simplification and decision procedures independently of previous background properties. Invariant generation and simplification can be halted by clicking on the interrupt button (the step logo).

Get local invariants Activates the local invariant generator; requires an SPL program.

Get global linear invariants Activates the linear invariant generator; requires an SPL program or fair transition system.
3.1. MENU OPTIONS

Get global polyhedral invariants: Activates the polyhedral invariant generator after prompting the user for a number of parameters that control the generation method. See Chapter 7 for details. Polyhedral invariants are generated for SPL programs and fair transition systems.

Enter axiom: Used to enter a property that the system may assume to be true during the current program verification session. If it is added to the list of background properties. By default, the new axiom is active and will thus be used by the Simplifier.

Note: Step does not guarantee the consistency of the set of background axioms nor does it require you to prove any of them. This feature should be used with caution.

Enter new goal: Used to enter a new goal to be proven. If the formula parses correctly it will become the root of a new proof tree which is made the current proof tree while the previous one is put on hold.

Check runtime system: Generates a set of goals that, if proved valid, ensure that the transition system will not generate any runtime errors such as division by 0 or array references out of range.

Weakest precondition: This option allows you to calculate weakest precondition for an arbitrary formula and transition in the current system (see Section 4.3). The dialog window shown in Figure 3.9 (upper window) will appear; after entering the transition name and formula, clicking on the OK button will display the weakest precondition in a separate window (shown in Figure 3.9's lower window). The formula may be saved to a file with the Save button.

This function has no effect on the proof tree or on the background properties; it serves a purely informative role. Weakest precondition is also provided as an inference rule useful for strengthening invariants and is described in Section 4.3.

The Tactics Menu

The Tactics pull-down menu is used to run tactics loaded from a file or entered directly. Tactics can be run in two modes: batch mode and interactive. Batchmode tactics are applied without any user interaction; they may not always terminate but can be interrupted by clicking on the step icon (the interrupt button).

For interactive tactics, a window appears with the first atomic step of the tactic. The Apply button applies the displayed step and the next one will appear. In this way you can apply tactic steps until the proof is finished or at any point interrupt the tactic by clicking on the Quit button.

Tactics and their syntax are presented in Section 6.3.

Load batch tactic: Reads a tactic from a file using the File Browser and applies it in batch mode.

Load interactive tactic: Reads a tactic from a file and runs it in interactive mode.
CHAPTER 3. THE TOP-LEVEL INTERFACE

Enter batch tactic  Prompts the user to enter a tactic which is then parsed and run in batch mode. Error messages from parsing the tactic are displayed in the bottom of the input window.

Enter interactive tactic  Prompts the user to enter a tactic which is then parsed and run in interactive mode.

The Diagrams Menu

The Diagrams pull-down menu is used to start up the Verification Diagram Editor and use verification diagrams in the current verification session. The Verification Diagram Editor may be used in parallel with the Top-level Prover and is described in more detail in Chapter 5.

Edit diagram  Starts the Verification Diagram Editor in a separate window.

Verification Diagram rule  Uses the current verification diagram as a verification rule for the current goal or subgoal. It generates subgoals corresponding to all the verification conditions implicit in the diagram. If the diagram is applicable, the verification diagram can be seen as a specialized verification rule.

A verification diagram is chosen to be the current one by the Current-to-verify menu option in the Verification Diagram Editor.
The Logical rules Menu

The Logical rules pull-down menu (Figure 3.10) provides inference rules that simplify temporal properties. Thus, they can be used to derive new properties from previously proven ones. Unlike verification rules, these rules are independent of the system being verified. Section 4.4 presents a full description of these rules.

![Logical Rules menu](image)

Figure 3.10: Logical Rules menu

The Settings Menu

The Settings pull-down menu (Figure 3.11) controls some of the global step environment variables, such as automatic simplification of subgoals and the strength of the Simplifier.

![Settings menu](image)

Figure 3.11: Settings menu

**Automatic Simplification ON/OFF** Sets and resets automatic simplification; see Section 4.1.

**Simplification Flags** Brings up a window used to set the default strength of the Simplifier. See Section 6.1 for more details.
Reset BDDs  Resets the internal BDD package; if the package is used often, this option can be used sporadically to save memory. If some (usually small) short-term efficiency loss is incurred (see Section 6.2). This option should also be used if the number of BDD nodes reaches the maximum given by the `STEP_BDD_NODES` environment variable (see Section 5).

Clear text window  Clears the output window but has no other effect on the system.

System information  Prints out miscellaneous statistics (in the output window) for the current verification session, mostly related to the BDD package (see Section 6).

The Help menu

On-line help is available via your favorite WWW browser when clicking on the Help menu near the right corner of the top-level interface. The hyper-text help pages give an alternative and often quicker introduction to step's facilities.

3.2 Action Buttons

The right-hand column of the Top-level Prover contains rules to manipulate and construct the main proof search. The first four buttons manipulate the structure of the proof tree:

- **Undo**: This button makes the parent of the current subgoal into the current goal. The previous goal is remembered to enable a **Redo** until another verification rule is applied to the new goal.
- **Redo**: Redoes a previous Undo, that is, restores the part of the proof tree eliminated by **Undo**.
- **Previous**: Postpones the current goal and turns the previous subgoal in the subgoal queue into the current subgoal.
- **Next**: Postpones the current subgoal and moves to the next subgoal in the subgoal queue.

The next set of buttons invoke verification rules as described in [Manna and Pnueli 1995]. These rules are not applicable if there is no current program or transition system.

Verification rules:

- **B-INV**
- **B-WAIT**
- **B-CAUS**
- **B-BACKTO**
- **G-INV**
- **G-WAIT**
- **G-CAUS**
- **G-BACKTO**

See Section 4.2 for a description of the main verification rules in their basic and general versions. Section 4.2 also describes the **WPC** and **Strengthen** proof rules.

The remaining buttons in the action region invoke the Interactive Prover and the Simplifier as proof rules applied to the current goal:
3.2. ACTION BUTTONS

Interactive In vokes the Interactive Prover on the current goal. The Interactive Prover window will appear with the current goal as the single consequent in the sequent. Once the Interactive Prover is active the Top-level Prover is suspended to be reactivated when the Interactive Prover session is finished.

Upon exiting the Interactive Prover you have the option to return the result to the Top-level Prover. If the goal was proved then the current goal is closed and the next goal in the proof tree if any becomes the current goal. If the goal was not proved a formula equivalent to the remaining subgoals is returned to the Top-level Prover as a new subgoal.

The Interactive Prover is described in Chapter 8.

Modelcheck In vokes the Model Checker on the current goal. Before the Model Checker starts you can enter additional assumptions to limit the search space. If the goal is proved without any additional assumptions the current goal is closed and the next goal in the proof tree if any becomes the current goal. If the goal is not proved it remains unchanged.

If the goal is proved under additional assumptions it is not added to the set of background properties.

The Model Checker results are reported in a logfile. See Section 4.5 for a more detailed description of the modelchecker.

BDD-split, Fast-simplify, Boolean, Simplify These buttons invoke various versions of the automatic Simplifier; while BDD-split can generate a set of subgoals the others can only reduce the formula to a simpler one. See Section 6.1 for details.

step logo The step logo serves as the interrupt button and can be used to exit from each of the following time-consuming operations:

- Simplification of verification conditions
- Generation and simplification of invariants
- Model checking
Chapter 4

Verification and Logical Rules

The main function of the Top-level Prover is to apply verification rules to temporal properties, reducing them to simpler verification conditions. These verification conditions become subgoals in the top-level proof tree. The verification rules depend on the system being verified. In contrast, a set of logical rules reduces temporal properties independent of the system being verified.

This chapter describes the general structure of the proof search and the rules used to construct it in the Top-level Prover.

4.1 The Proof Search

The structure of the proof search is similar in the Top-level Prover and the Interactive Prover: the proof is a tree with the original goal as its root node. Nodes are labeled with subgoals called verification conditions in the Top-level Prover and sequents in the Interactive Prover and the application of different rules generates new subgoals (or children nodes) for the current node.

Since more than one proof rule can be applied at each node, the process is a search. The proof is finished when there are no subgoals left; the number of subgoals is reduced whenever a simplification step reduces a subgoal to true, which is the only formula that requires no proof.

In the Top-level Prover, the rules applied are temporal verification rules, logical rules, or simplification steps. In addition, rules that can strengthen a verification condition are available, such as weakest precondition. Finally, goals in the Top-level Prover can also be closed by sessions of the Interactive Prover.

The Background Properties:

Background properties include axioms, automatically generated invariants, and previously established properties. When a top-level proof is completed, the property is added to the list of background properties.

In the Top-level Prover, simplification is carried out relative to the properties and axioms that are active at the time (except for the $\text{Fast-simplify}$ option, discussed in Section 6.1).
In the Interactive Prover these background properties have to be explicitly added by the user to the current sequent before the simplification rule is called.

Navigating the Proof Tree:
The options `Postpone` (in the Interactive Prover) and `Previous` (in the Top-level Prover) and `Select goal` (in both provers) can be used to jump from one open node to another. The `Undo` and `Redo` buttons alternate between a given node and its children.

In the Top-level Prover any number of proof searches can be conducted simultaneously sharing the same set of background properties. The `Next search` button can be used to jump from one proof search to the next.

Automatic Simplification
A verification rule can generate a large number of verification conditions depending on the size of the underlying transition system. After having applied a verification rule a message in the message window will display how many subgoals were generated. If automatic simplification is `OFF` the first subgoal will be displayed in the specification window. All the generated subgoals can be viewed by selecting the `Select goal` option of the `Properties` pull-down menu.

If automatic simplification is `ON` the Simplifier will be applied to each subgoal. Only those subgoals that fail to simplify to `true` will remain. Simplifications steps which fail to simplify to `true` will be undone to enable subsequent application of rules such as `Strengthen` and `WPC`; these are only applicable to unsimplified verification conditions since they require knowing the particular transition that generates the verification condition.

When automatic simplification is `OFF` a verification rule only generates subgoals but does not simplify them. These then have to be manually simplified by successive clicks on `Simplify` for with the Interactive Prover. If you unintentionally invoke a verification rule with automatic simplification `OFF` you can click on `Undo` to turn automatic simplification `OFF` and apply the rule again.

4.2 Verification Rules
This section describes the verification rules available in step. In the following description `Θ` refers to the initial condition of the current program or transition system and `T` refers to its set of transitions (see Appendix A). For a formula `pΓp0` is the initial version of `pΓ` that is the first-order formula equivalent to `p` at the initial state of the computation.

As usual the notation `p{q}τ{q}Γ` stands for the verification condition `(p ∧ rτ) ⇒ q′`. The inverse verification condition `Γ{p}τ−1{q}Γ` is equivalent to `−q τ −p Γ` giving an alternate presentation of verification conditions. Below we try to use the most convenient one in each case. The notation `p{T}q` stands for the set of verification conditions `p τ q` for each transition `τ` in the system; similarly `p{T}−1{q}Γ` is the set `p τ−1 q` for each `τ`.

Verification conditions of the form `α ⇒ β` are replaced by `α → β` when `α` and `β` are assertions that is state-formulas with no temporal operators. In this case the `P-validity`
of $\alpha \rightarrow \beta$ implies that of $\alpha \Rightarrow \beta$. If this $P$-validity cannot be established the invariant $\alpha \Rightarrow \beta$ may be proved separately for an incremental proof.

- **B-INV** Applies the basic invariance rule to the current subgoal $\Gamma$ which has to be of the form $\square p \Gamma$ for a past formula $p$. This rule generates the following subgoals:
  
  \begin{align*}
  & p \Rightarrow p \quad \text{Monotonicity} \\
  & \Theta \rightarrow p_0 \quad \text{Initial condition} \\
  & \{p\} \tau\{p\}, \quad \text{for each } \tau \in \mathcal{T}
  \end{align*}

  The monotonicity subgoal is generated because this rule is based on rule G-INV below; this goal always trivially simplifies to $true$.

- **G-INV** Applies the general invariance rule to the current subgoal. The current goal has to be of the form $\square q \Gamma$ with $q$ a past formula. After clicking a dialog window will appear that allows you to enter a strengthened version $p$ of $q$. The default is $q$. After entering the auxiliary assertion $p \Gamma$ the following subgoals are generated:
  
  \begin{align*}
  & p \Rightarrow q \quad \text{Monotonicity} \\
  & \Theta \rightarrow p_0 \quad \text{Initial condition} \\
  & \{p\} \tau\{p\}, \quad \text{for each } \tau \in \mathcal{T}
  \end{align*}

- **B-WAIT** Applies the basic nested wait-for rule. This rule is applicable only if the current goal is a nested wait-for formula $\Gamma$ that is $\Gamma$ is of the form
  
  \[ p \Rightarrow q_n \mathcal{W} q_{n-1} \mathcal{W} q_{n-2} \ldots q_1 \mathcal{W} q_0, \]

  where $p, q_n, \ldots, q_0$ are past formulas. Informally $\Gamma$ the nested wait-for formula asserts that whenever $p$ becomes true when there is an possibly empty interval where $q_n$ holds followed by a $q_{n-1}$ interval up to $q_0$ becomes true unless one of the previous $q$'s remains true infinitely after. After clicking on this button the following subgoals are generated:
  
  \begin{align*}
  & p \Rightarrow \bigvee_{i=0}^n q_i \quad \text{Premise N1} \\
  & q_i \Rightarrow \hat{q}_i \quad \text{Monotonicity $\forall i$ for each $i = 0 \ldots n$} \\
  & \{q_i\} \tau\{q_i\} \quad \text{for each $i = 1 \ldots n$}
  \end{align*}

  The monotonicity conditions are generated because the implementation of B-WAIT is based on that of G-WAIT below. They always trivially simplify to $true$.

- **G-WAIT** This button invokes the general nested wait-for rule. This rule is applicable only if the current goal is a nested wait-for formula $\Gamma$ that is $\Gamma$ is of the form
  
  \[ p \Rightarrow q_n \mathcal{W} q_{n-1} \mathcal{W} q_{n-2} \ldots q_1 \mathcal{W} q_0, \]

  where $p, q_n, \ldots, q_0$ are past formulas. A dialog window will appear which allows you to enter the auxiliary assertions $\varphi_n, \ldots, \varphi_0$. Their defaults are $q_n, \ldots, q_0 \Gamma$ respectively. After entering these assertions the following subgoals are generated:
  
  \begin{align*}
  & p \Rightarrow \bigvee_{i=0}^n \varphi_i \quad \text{Premise N1} \\
  & \varphi_i \Rightarrow \hat{q}_i \quad \text{Monotonicity $\forall i$ for each $i = 0 \ldots n$} \\
  & \{\varphi_i\} \tau\{\varphi_i\} \quad \text{for each $i = 1 \ldots n$}
  \end{align*}
• **B-CAUS** The basic causality rule is a specialization of the basic invariance rule. It is applicable to goals of the form

\[ p \Rightarrow \Box q, \]

where \( p \) and \( q \) are past formulas. It usually generates simpler verification conditions than its counterpart B-INV which is also applicable to goals of this form by taking into account the special form of the goal.

This rule generates the following subgoals:

- \( p \Rightarrow p \lor q \quad \text{Monotonicity} \)
- \( \Theta \Rightarrow \neg(p)_0 \lor (q)_0 \quad \text{Initial condition} \)
- \( \{p\}T^{-1}\{p \lor q\} \)

• **G-CAUS** The general causality rule specializes the general invariance rule. It is applicable to goals of the form

\[ p \Rightarrow \Box q, \]

where \( p \) and \( q \) are past formulas. It usually generates simpler verification conditions than its counterpart G-INV which is also applicable to goals of this form by taking into account the special form of the formula to be proved.

After clicking on this button a dialog window will appear which allows you to enter an intermediate formula \( \varphi \) (default \( q \)). After \( \varphi \) is entered the following subgoals are generated:

- \( p \Rightarrow \varphi \lor q \quad \text{Monotonicity} \)
- \( \Theta \Rightarrow \neg(\varphi)_0 \lor (q)_0 \quad \text{Initial condition} \)
- \( \{\varphi\}T^{-1}\{\varphi \lor q\} \)

• **B-BACKTO** The basic back-to rule establishes properties of the form

\[ p \Rightarrow q_n B \ldots B q_0 \]

for past formulas \( p, q_n, \ldots, q_0 \). It generates the following subgoals:

- \( p \Rightarrow \bigvee_{i=0}^n q_i \quad \text{Premise N1} \)
- \( q_i \Rightarrow q_i \quad \text{Monotonicity for each } i \text{ in } 1 \ldots n \)
- \( \{q_i\}T^{-1}\{\bigvee_{j=i}^n q_j\} \quad \text{for each } i \text{ in } 1 \ldots n \)

• **G-BACKTO** The general back-to rule generalizes the above by allowing strengthening of the \( q_i \)’s and weakening of \( p \). To prove

\[ p \Rightarrow q_n B \ldots B q_0, \]

given formulas \( \varphi_0, \ldots, \varphi_n \) the following subgoals are generated:

- \( p \Rightarrow \bigvee_{i=0}^n \varphi_i \quad \text{Premise N1} \)
- \( \varphi_i \Rightarrow q_i \quad \text{Monotonicity for each } i \text{ in } 1 \ldots n \)
- \( \{\varphi_i\}T^{-1}\{\bigvee_{j=i}^n \varphi_j\} \quad \text{for each } i \text{ in } 1 \ldots n \)
4.3 WPC and Strengthening

A basic and often successful heuristic for constructing inductive invariance properties is based on weakest preconditions.

- **WPC** The weakest precondition verification rule can be used to prove a verification condition that does not itself simplify to true. The rule is applicable to subgoals of the form

\[
\{ \varphi \} \tau \{ \psi \}
\]

and when applied it generates the subgoal

\[
\Box (\rho \rightarrow \psi')
\]

which implies the original verification condition.

The symbol \( \rho \) refers to the transition relation \( \Gamma \) and is the logical form of the transition \( \tau \). The formula \( \psi' \) is the primed version of \( \psi \) in which each system variable occurring free in \( \psi \) has been replaced by its primed version. Appendix A describes transitions and transition relations in more detail.

- **Strengthen** The strengthening rule can be used to strengthen a property that is not inductive. The rule is applicable to subgoals of the form

\[
\{ \varphi \} \tau \{ \psi \}
\]

and when applied it generates the subgoal

\[
\Box (\varphi \land (\rho \rightarrow \psi')).
\]

while deleting the original verification condition and its siblings from the proof tree.

4.4 Logical Rules

The Logical rules pull-down menu provides inference rules that allow the derivation of new properties from previously proven properties.

If the current formula does not match the kind of formula that the given rule applies to, a message will appear in the bottom help window \( \Gamma \) and the given function will have no effect.

**MON-1** The monotonicity rule for invariances can be used to prove a property of the form \( \Box p \) where \( p \) is a state formula. If the current goal is \( \Box p \) the following subgoal is generated:

\[
\bigwedge_i \varphi_i \rightarrow p
\]

where \( \varphi_i \) ranges over all the currently active in the background properties.
TRN-C The *transitivity of causality rule* proves a causality property from two other causality properties. If the current goal has the form \( p \Rightarrow \Box r \Gamma \) this option displays a dialog window for entering an auxiliary assertion. After a new assertion \( q \) is entered (default is \( r \)) the following two subgoals are generated:

\[
\begin{align*}
p &\Rightarrow \Box q \quad \text{Left Transitivity} \\
q &\Rightarrow \Box r \quad \text{Right Transitivity}
\end{align*}
\]

MON-C The *monotonicity rule for causality* can prove a causality property from a stronger causality property. If the current goal has the form \( p \Rightarrow \Box u \Gamma \) for past formulas \( p \) and \( u \Gamma \) this option generates a dialog window for entering two auxiliary assertions. After entering the two assertions \( q \) and \( r \) (defaults are \( p \) and \( u \)) the following three subgoals are generated:

\[
\begin{align*}
p &\Rightarrow q \quad \text{Monotonicity 1} \\
q &\Rightarrow \Box r \quad \text{Causality} \\
r &\Rightarrow u \quad \text{Monotonicity 2}
\end{align*}
\]

MON-W The *monotonicity rule for wait-for formulas* can be used to prove a wait-for formula from a stronger wait-for formula. If the current goal has the form \( p \Rightarrow q_n W \ldots W q_0 \Gamma \) a dialog window will appear for entering \( n + 2 \) auxiliary assertions. The default assertions are \( p \Gamma q_0, I \Gamma q_0 \). After entering these assertions \( \Gamma p_{new}, q_{n,new}, \ldots q_{0,new} \) and clicking the OK button the following \( n + 3 \) subgoals are generated:

\[
\begin{align*}
p &\Rightarrow p_{new} \quad \text{Monotonicity} \\
q_{n,new} &\Rightarrow q_n \quad \text{Monotonicity } n \\
\vdots \\
q_{0,new} &\Rightarrow q_0 \quad \text{Monotonicity } 0
\end{align*}
\]

and

\[
p_{new} \Rightarrow q_{n,new} W \ldots W q_{0,new}
\]

CONT The *contradiction rule* can be used to prove a general safety property by converting it into a causality formula.

If the current goal is of the form \( \square \neg p \Gamma \) for past formula \( p \Gamma \) the following subgoal will be generated:

\[
p \Rightarrow \Box \text{false} \quad \text{Contradiction}
\]

4.5 The Model Checker

*Model checking* determines if a given temporal formula holds for a given system by a systematic exploration of the state-space of the system. **STEP** includes a model checker for transition systems and formulas of linear-time temporal logic \( \Gamma \) which can be used to close subgoals or produce counterexamples to particular properties.

The **STEP** model checker checks the current goal under an optional additional first-order assumption. If the goal is found to be false a counterexample is reported.
When the `Modelcheck` proof rule is invoked, a dialog window appears requesting the input of an auxiliary first-order assumption by default `true`. You may enter restrictions on parameters or system variables with infinite domains in order to make the program finite-state. For example, for the program `MPX-SEM` you may enter `#=2` indicating that the number of processes you want to consider is two.

The goal is closed only if the Model Checker proves it with assumption `true`.

The Model Checker Preprocessor

The Model Checker first applies a preprocessor to the given system and the goal. If possible, a finite number of problem instances with constant-size arrays and constant-bounded parameters equivalent to the original goal is produced. If this is not possible, the Model Checker terminates, control returns to the Top-level Prover and a message is displayed. Some of the tasks accomplished by the preprocessor are:

- simplification of data types;
- elimination of unnecessary variables;
- instantiation of parameters and other variables;
- introduction of additional transitions corresponding to possible run-time errors;
- general simplification of the transition system and of the property using the Simplifier.

Note that the additional assumption can be used to restrict the range of a variable on which array or parameter bounds depend.

The Core Model Checker

For each problem instance, an external core model checker is invoked. It searches the state-space and stops when

1. a run-time error is found;
2. a counterexample computation is found for;
3. the entire state space has been explored.

The main algorithm used by the core model checker is adapted from [Hojati et al. 1993].

Restrictions

Not all programs and properties accepted by `step` can be model checked. The datatypes handled by the core model checker are: boolean, integer, and channels, and arrays over booleans and integers. The preprocessor can reduce some other datatypes to these. The model checker can only handle basic logic, arithmetic, and channel operations applied to these variables. Formulas given to the Model Checker cannot have temporal operators in the scope of binding operators.
Log file

The model checking results are reported in a log file selected by the user before model checking begins.

After calling the core model checker, the log file will contain: the list of relevant system variables, the preprocessed transition system, some statistical information, and, if the formula proved to be invalid, a counterexample computation.

Environment Variables

The search space may be infinite if there are system variables that range over an infinite domain, and in this case the Model Checker may or may not terminate. Even if the system is finite-state, the state-space may prove too large for the time and memory available. The following settings can be used to limit the Model Checker resources:

- **STEP_MC_SPACE**: Maximum amount of memory in Megabytes.
  Default: unlimited.

- **STEP_MC_TIME**: Maximum amount of user time in minutes.
  Default: unlimited.

- **STEP_MC_CPU**: Maximum amount of CPU time in minutes.
  Default: unlimited.

The Model Checker can be interrupted with step’s interrupt button (the step logo). NOTE: This may leave a UNIX process running in the background; see the UNIX kill man pages for details on how to kill runaway processes.
Chapter 5

The Verification Diagram Editor

Verification diagrams are a visual representation of a proof. A verification diagram describes sets of states given by assertions and possible transitions between them. A well-formed verification diagram represents a set of verification conditions that are sufficient to establish a given temporal formula. Verification diagrams are described in detail in [Manna and Pnueli 1994] and [Manna and Pnueli 1995].

Step currently supports three types of verification diagrams:

- **Invariance diagrams** to represent proofs of invariance formulas: $\Box p$.

- **Wait-for diagrams** to represent proofs of nested wait-for formulas:

$$p \Rightarrow q_n W_{q_{n-1}} \cdots W_{q_0}.$$ 

- **Chain diagrams** to represent proofs of response formulas $p \Rightarrow \Diamond q$ that do not require induction.

Here $p, q_0, \ldots, q_n$ are state assertions.

Hierarchical verification diagrams (Section 5.7) support distributing a large verification diagram over several files.

5.1 Definitions

A *verification diagram* is a directed labeled graph constructed as follows:

- **Nodes** in the graph are labeled by assertions.

- **Edges** in the graph represent transitions between nodes. Each edge connects one node to another and is labeled by the name of a transition in the program. We refer to an edge labeled by $\tau$ as a $\tau$-edge.

- One of the nodes may be designated as a *terminal node*. This node is distinguished by having a boldface boundary. No edges may depart from a terminal node.
Verification diagrams represent sets of verification conditions as follows: For every non-terminal node (labeled by) \( \varphi \) and transition \( \tau \) let \( \varphi_1, \ldots, \varphi_k \) be the nodes to which \( \varphi \) is connected by \( \tau \)-edges. The verification condition associated with \( \varphi \) and \( \tau \) is given by:

\[
\{ \varphi \} \tau \{ \varphi \lor \varphi_1 \lor \cdots \lor \varphi_k \}.
\]

Note that if \( k = 0 \), i.e., no \( \tau \)-edges depart from \( \varphi \), the verification condition is:

\[
\{ \varphi \} \tau \{ \varphi \}.
\]

No verification conditions are associated with terminal nodes. A diagram is called valid over program \( P \) (\( P \)-valid) if all the verification conditions associated with nodes of the diagram are \( P \)-state valid. Thus, a valid diagram provides a succinct representation of the verification conditions of the form \( \{ \varphi \} \tau \{ \psi \} \) for all combinations of assertions \( \varphi \) and \( \psi \) and transitions \( \tau \) that appear in the diagram.

### 5.2 Wait-for Diagrams

A verification diagram is called a wait-for diagram if there is only one terminal node and it is labeled by \( \varphi_0 \) and the diagram is weakly acyclic, i.e., whenever node \( \varphi_i \) is connected by an edge to node \( \varphi_j \) for \( i \geq j \). \( P \)-valid wait-for diagrams can be used to establish the \( P \)-validity of a nested wait-for formula.

A \( P \)-valid wait-for diagram establishes that the formula

\[
\left( \bigvee_{j=0}^{m} \varphi_j \right) \Rightarrow \varphi_m W \varphi_{m-1} \cdots W \varphi_1 W \varphi_0
\]

is \( P \)-valid. To establish the \( P \)-validity of

\[
p \Rightarrow q_m W q_{m-1} \cdots W q_0
\]

the following additional verification conditions (referred to as side verification conditions) have to be established:

\[
p \Rightarrow \bigvee_{j=0}^{m} \varphi_j \quad \text{(W1)}
\]

\[
\varphi_i \Rightarrow q_i \quad \text{for } i = 0, 1, \ldots, m. \quad \text{(W2)}
\]

### 5.3 Invariance Diagrams

A verification diagram is called an invariance diagram if it is exit-free, i.e., it has no terminal node. Unlike wait-for diagrams, invariance diagrams may contain cycles.

A \( P \)-valid invariance diagram with nodes \( \varphi_1, \ldots, \varphi_m \) establishes that the formula

\[
\left( \bigvee_{j=1}^{m} \varphi_j \right) \Rightarrow \square \left( \bigvee_{j=1}^{m} \varphi_j \right)
\]
is $P$-valid. To establish the validity of $\Box q$ the following two side verification conditions have to be established:

\[
\Theta \rightarrow \bigvee_{j=1}^{m} \varphi_j \quad (11)
\]

\[
(\bigvee_{j=1}^{m} \varphi_j) \rightarrow q \quad (12)
\]

### 5.4 Chain Diagrams

Verification diagrams can also represent fairness requirements. The main extension over the previous types of diagrams is the introduction of two additional types of edges: double and solid. A double edge corresponds to a helpful transition that is just and a solid edge corresponds to a helpful transition that is compassionate. Thus, edges connecting nodes in a diagram can be single, double, or solid.

A verification diagram is said to be a chain diagram if its nodes are labeled by assertions $\varphi_0, \ldots, \varphi_m$ with $\varphi_0$ being the terminal node and if it satisfies the following requirements:

- If a single edge connects node $\varphi_i$ to node $\varphi_j$, then $i \geq j$.
- If a double or solid edge connects $\varphi_i$ to $\varphi_j$, then $i > j$.
- Every node $\varphi_i$, $i > 0$, has at least one departing edge which is either double or solid. The transition labeling this edge is identified as helpful for assertion $\varphi_i$.
- Every transition can label at most one type of edge (single, double, or solid) departing from the same node.

The first two requirements ensure that the diagram is weakly acyclic in the same sense as defined for the wait-for diagram. The stronger second requirement ensures that the subgraph based on the double and solid edges is acyclic, forbidding self-connections by double or solid edges. The third requirement demands that every non-terminal node has at least one helpful transition associated with it.

Chain diagrams allow proving progress properties that do not require an infinite size well-founded domain. Later releases of step will allow the user to enter well-founded domains as part of the chain diagram.

#### Verification Conditions

Let $\varphi$ be a non-terminal node and $\varphi_1, \ldots, \varphi_k$, $k \geq 0$, be the $\tau$-successors of $\varphi$.

- If $\tau$ labels a single edge, we associate with $\varphi$ and $\tau$ the usual verification condition
  \[
  \{\varphi\} \tau \{\varphi \lor \varphi_1 \lor \ldots \lor \varphi_k\}.
  \]

- If $\tau$ labels a double or solid edge, we associate with $\varphi$ and $\tau$ the verification condition
  \[
  \{\varphi\} \tau \{\varphi_1 \lor \ldots \lor \varphi_k\}.
  \]
• If $\tau$ labels a double edge departing from $\varphi$ we associate with $\varphi$ and $\tau$ the implication

$$\varphi \rightarrow E\!n(\tau).$$

i.e. when $\varphi$ holds $\tau$ is enabled.

• If $\tau$ labels a solid edge departing from $\varphi$ we associate with $\varphi$ and $\tau$ the compassion requirement in the form of the response formula

$$\varphi \Rightarrow \Box(\neg \varphi \lor E\!n(\tau)).$$

Valid Chain Diagrams

A chain diagram is $P$-valid if the first three types of verification conditions are $P$-state valid and the compassion requirements associated with solid edges are $P$-valid.

A $P$-valid chain diagram establishes that the response formula

$$\left(\bigvee_{j=0}^{m} \varphi_j\right) \Rightarrow \Box \varphi_0$$

is $P$-valid. To establish the $P$-validity of

$$p \Rightarrow \Box q$$

the following two additional side verification conditions have to be established:

$$p \Rightarrow \bigvee_{j=0}^{m} \varphi_j \quad (FC1)$$

$$\varphi_0 \Rightarrow q \quad (FC2).$$

5.5 Compound Nodes

To make verification diagrams more readable, Step supports encapsulation conventions similar to those of Statecharts [Harel 1987]. The basic encapsulation construct is the compound node containing internal nodes. We refer to the contained nodes as the descendants of the compound node. Nodes that are not compound are called basic nodes.

Departing edges An edge departing from a compound node is interpreted as though it departed from each of its descendants.

Arriving edges An edge arriving at a compound node is interpreted as though it arrived at each of its descendants. An assertion $\varphi$ labeling a compound node is interpreted as though it were a conjunct added to each of its descendants.
Figure 5.1: The Verification Diagram Editor
5.6 Verification Diagram Editor: Interface

The Verification Diagram Editor allows you to load and manipulate verification diagrams. The interface is shown in Figure 5.1. The two top panels are used to edit the label and the assertions on nodes and edges. The diagram itself is drawn in the large panel using the drawing tools displayed in the column to its left.

The verification diagram does not have to be related to the current system description and you can use the Verification Diagram Editor independently of the step verification session in progress. By selecting the Current-to-Verify option under the File menu, you make the current diagram the one used by the verification session. In this way, a verification diagram can be applied as a verification rule to generate verification conditions that establish a given property.

The Verification Diagram Editor does not parse the expressions labeling the nodes for check the existence of the transitions labeling the edges. These checks are performed when the verification diagram rule is invoked. If an assertion does not parse or there is any other problem with the diagram, a message indicating the error will appear in the message window of the Top-level Prover. The verification diagram can then be corrected within the Verification Diagram Editor made current again, and the verification diagram rule applied again.\(^1\)

Drawing Tools

The leftmost column contains the tools for drawing basic and compound nodes and edges corresponding to the different justice requirements.

This tool sets the pointer to selection mode. It enables you to select nodes and edges by clicking on them. When an edge is selected, all the nodes to which it is attached are selected too. To select a single edge (e.g., to convert it into a different type of edge) you first have to move the edge away from the nodes. You can do this by pressing the left mouse button while pointing at the edge and then moving it quickly while keeping the button pressed. To select multiple edges and nodes, click on some location outside these nodes and while keeping the left mouse button pressed, drag the mouse until the nodes and edges are enclosed in the region indicated by the dashed rectangle.

\(^1\)An inquisitive user may wonder why the Verification Diagram Editor does not check for parsing errors. The reason is that to check parsing errors, one requires knowing the system for which a diagram is being applied to. However, step allows the Verification Diagram Editor to run independent to any system. This feature is useful when the same (or similar) diagrams are being applied to different systems.
This tool is used to draw non-terminal nodes. After selecting it, move the mouse to the location where you want the upper left corner of the new node and click on the left button; while keeping it pressed, move the mouse down and to the right until the displayed shape has the proper dimensions. Releasing the button will then create the node. The node will be initially selected and the two editable fields above the diagram will contain the default name and label for this node. You can edit the name field and enter a state assertion for this node.

To resize an existing node, select it with the left button (after first selecting the pointer tool) and move the mouse to one of its corners; and resize the node while keeping the button pressed on the black square in the chosen corner.

Works exactly as the tool for non-terminal nodes, but the generated node is a terminal node.

This tool draws an edge without an associated fairness requirement. An edge is drawn using the left mouse button: press the button at the location where you want the edge to start, keep the button pressed while moving the pointer and release the button at the desired endpoint. To draw an edge with corners, release the button momentarily at the location of the turn and move the mouse in the new direction. Several consecutive turns can be made in this way.

The label of the edge will appear in the top panel and can be edited there. The label can correspond to any transition of the system being analyzed.

Works exactly as the tool for drawing a single-lined edge, but draws a double-lined edge that has an associated Justice requirement. The label of such a Just edge must be the name of a transition declared as Just or Compassionate.

Works exactly as the tool for drawing a single-lined edge, but draws a solid-lined edge that has an associated Compassion requirement. The label of a Compassionate edge must be the name of a transition declared as Compassionate.

**Menu Options**

**The File Menu**

The File menu is used to load and save verification diagrams as well as to make verification diagrams accessible to the Top-level Prover.

**New** Resets the verification diagram editor deleting the currently edited diagram.

**Load** Loads a verification diagram from a file.

**Save** Saves the verification diagram currently being edited to a file.
Current-to-verify Selects the currently edited verification diagram as the one used by the Top-Level Prover.

Exit Quits the Verification Diagram Editor after confirmation by the user.

The Edit Menu

The Edit menu is used to cut and paste parts of the diagram.

Cut Deletes one or more selected items from the editing window. The deleted item(s) (node or edge) are put on a scratchpad and can later be recovered by Paste.

Copy Copies the selected item from the editing window to the scratchpad.

Paste Takes the item most recently put on the scratchpad (either by Cut or Copy) and puts it in the editing window in the same location it was before.

Duplicate Duplicates one or more selected items. This is equivalent to doing Copy followed by Paste.

The Node Menu

The Node menu is used to group nodes into compound nodes and change terminal into nonterminal nodes and vice versa.

Group/Compound By selecting a large node and the nodes enclosed by the large node, Group turns the large node into a compound node containing the enclosed and selected sub-nodes (see Figure 5.1).

Subnodes of a compound node cannot be edited. They have to be released with Ungroup before you can change their labels or move them.

The label and assertion of the compound (enclosing) node is displayed in the upper left corner of the node.

Ungroup/Uncompound Converts a selected compound node into a regular node and releases the subnodes such that they can be edited and moved.

Terminal Converts the selected node (only one node should be selected) into a terminal node. The borders of the node are drawn with thick lines and the node is declared as terminal.

Nonterminal Converts a selected terminal node into a nonterminal node.

The Edge Menu

The Edge menu lets you change the fairness requirements associated with a selected edge and check that all edges are properly connected to the nodes.

Cut Head Cuts the head hop of a selected edge. The head hop is the hop with an arrow where an edge ends.
Cut Tail Cuts the tail hop of a selected edge. The tail hop is the hop where an edge starts.

Merge Merges a selected edge with the edges chained to it. Two edges are considered chained if one's ending point is close to the other's starting point.

Unjust Converts a selected edge into an Unjust edge. Unjust edges are drawn with thin black lines.

Just Converts a selected edge into a Just edge. Just edges are drawn with double lines.

Compassionate Converts a selected edge into a Compassionate edge. Compassionate edges are drawn with thick black lines.

Connectivity Highlights those edges whose ends are not connected to nodes. This is a convenient way to check whether all edges are connected properly in the verification diagram.

Global Hookup Checks and hooks up all loose edges. An edge will be hooked up to a node if its end is close enough to or inside the node.

Help
Activation of the on-line help pages using Mosaic or your favorite html (WWW) browser. The Top-level Prover help pages are accessible from the Verification Diagram Editor help pages and vice-versa so a single help session should generally suffice. The executable used for on-line help can be changed by setting the STEP_BROWSER environment variable.

5.7 Hierarchical Verification Diagrams *
In order to allow decomposition of verification diagrams into smaller, more manageable parts, step supports hierarchical verification diagrams. With hierarchical diagrams, several nodes in a large diagram can be represented by a single node, making large diagrams more readable and easier to edit.

Import Nodes
A hierarchical verification diagram has one or more import nodes. An import node imports a verification diagram from a file. The imported diagram may itself have import nodes that import other files. The edges going out from the import node are matched up with outgoing edges from the imported diagram and similarly for incoming edges.

An import node is created by giving it the label #import and giving it as assertion the filename (with full or relative path name) within angle brackets (<>) for example <mux-pet2/mux-h1.diagram> of the file that contains the diagram to be imported. The assertion field may also contain parameters (see next paragraph) enclosed within “%”. Any text in the assertion field of an import node that is not enclosed by angle brackets or percent signs is ignored.
Parameter Passing

Hierarchical verification diagrams allow macro-like text replacement when importing a verification diagram. Parameters (in the node to be imported) are of the form "@1", "@2", etc. and "@1" is replaced by the first argument passed by the importing node. "@2" is replaced by the second argument etc.

When the top-level diagram is processed (i.e., when it is used as a verification rule by the Top-level Prover), it checks whether the number of parameters each imported diagram requires is equal to the number of arguments passed. The arguments passed by the import node are included in the assertion field; each argument is enclosed within percent signs ("%"). An example of the assertion field of an import node that passes parameters is

\[<\text{mux-pet2/mux-h1.diagram}> \%m3\% \%l3\% \%s=1\%\]

The arguments do not have to be on the same line and white space is ignored.

Edges

The connection of edges between the import node and the imported diagram is done by matching unconnected outgoing edges in the imported diagram (also called dangling edges) with outgoing edges of the import nodes that have the same label and similarly for incoming edges. If multiple edges of the imported diagram match multiple edges of the import node, then edges are created for all possible combinations (taking the product over all edges). Thus, one incoming edge in an imported diagram may translate into multiple edges with the same label entering that same node in the imported diagram.

If multiple edges with the same label entering or leaving an import node need to be distinguished (e.g., because they enter different nodes in the imported diagram), the labels may be provided with a secondary label. The secondary label follows the regular transition name separated by a colon. Thus, if 11 is the regular transition name and 11:1 labels an edge for transition 11 with secondary label 1. Edges connected to the import node and dangling edges in the imported diagram only match when both the primary and the secondary label are identical. White space is ignored in secondary labels. Secondary labels default to 0 (zero) if left blank but this value is not displayed in the diagram.

Edges can also be connected to import nodes as if the import node was a compound node (i.e., an incoming edge in the import node is connected to all nodes of the imported diagram and similarly for an outgoing edge) by adding the secondary label All to the label of the edge entering the import node. Thus, an edge labeled with All in the importing diagram is connected to all nodes of the imported diagram without any name matching. It is not meaningful to add the secondary label All to dangling edges in an imported diagram and this is therefore ignored.

If import nodes are within a compound node, then edges entering or leaving the compound node are interpreted as usual to represent edges entering or leaving each enclosed node. Therefore these edges are considered to be connected to the import nodes and interpreted as usual.
Chapter 6

Automatic Theorem-Proving

A computer-aided verification system should try to make the task of proving verification conditions as easy as possible while still ensuring soundness.

To this effect, the system provides powerful, usually fast, automatic simplification procedures. These are supplemented by the interactive Gentzen-style theorem prover discussed in Chapter 8 used to establish properties that the automatic simplifier cannot.

6.1 The Automatic Simplifier

There is a basic trade-off between the strength and the speed of the automatic Simplifier. The more powerful the settings are, the slower the Simplifier is. The default settings are a good compromise: they are able to simplify most commonly occurring verification conditions in a reasonable time.

The impatient user may want to use the interrupt button to halt lengthy simplification operations and try alternative proof techniques.

Selecting the Simplification Flags menu option brings up the window shown in Figure 6.1. It contains the following settings:

![Figure 6.1: Simplifier settings](image-url)
- **Equality and Inequality**
  If ON, the Simplifier will use properties of equalities and inequalities. For example, it will simplify
  \[ y > 1 \rightarrow y > 0 \]
to *true* when this switch is ON but not when it is OFF.
  These decision procedures are quite efficient and useful in general, so they are ON by default.

- **Linear Arithmetic**
  If ON, the Simplifier will use decision procedures for linear arithmetic which use properties of \( >, +, \text{and} = \). For example, it will simplify
  \[ y \geq 0 \land y' = y + 1 \rightarrow y' > 0 \]
to *true* when the switch is ON but will not be able to simplify it when the switch is OFF. The default is OFF.
  The performance of these procedures can vary widely from one formula to the next; the *interrupt button* can be used to halt the computation.

- **Associative and Commutative functions**
  If ON, step will take into account the associativity and commutativity of addition, multiplication, and boolean connectives as well as those user-defined binary function symbols that have been declared as ASSOCIATIVE or COMMUTATIVE (see Section 2.3).
  Note that since step rewrites all formulas into a normal form, it implicitly uses many instances of the associative and commutative laws even when this flag is OFF. The default is OFF.

- **BDD simplification**
  If ON, the (propositionally complete) BDD simplification method will be used as part of the simplifier before other decision procedures are used (see Section 6.2). The default is OFF.

- **Conditional Rewrite Rules**
  If ON, the Simplifier will use a number of built-in conditional rewrite rules. For example, it will simplify
  \[ (y > 0 \land x \cdot y > 0) \rightarrow x > 0 \]
to *true* when the switch is ON but will not be able to simplify it when the switch is OFF.
  Conditional rewrite rules may slow down the Simplifier considerably, so they are switched OFF by default.

Commonly used Simplifier settings are also available as separate buttons in the action region of the Top-level Prover:
**Fast-simplify** Applies only the most basic simplification procedures, disregarding the background properties. This is useful to rewrite formulas into a more standard form after which they can be compared more easily with each other.

**Bool-simplify** Applies a propositional satisfiability check using the active background properties. If the given formula is a propositional tautology with respect to the active background properties, it will be simplified to true. This procedure is complete for propositional logic. The basic simplification procedures (see Fast-simplify above) are called before the propositional test.

This procedure uses step's BDD package (see Section 6.2).

**Simplify** Invokes the simplifier and decision procedures as given by the current simplification settings.

For material related to the decision procedures and indexing used in step see [Bledsoe 1975], [Shostak 1979], [Nelson and Oppen 1980], [Boyer and Moore 1988], [Stickel 1989], [McCune 1992], [Zhang 1993].

### 6.2 BDD Simplification

**step** includes a package for the creation and manipulation of propositional (i.e. boolean) formulas using Binary Decision Diagrams (BDDs). This package is used in the proof rule as well as in boolean simplification steps.

Since the amount of memory used by the BDD package can grow indefinitely (in the form of accumulated BDD nodes), the package can be reset at any time with the Reset BDDs option under the Settings menu. This only incurs a (usually small) short-term efficiency loss and does not affect the soundness of the prover.

This option should also be used if the number of BDD nodes reaches the maximum given by the **STEP_BDD_NODES** environment variable (see Appendix E). The number of existing BDD nodes can be obtained through the **System Information** option.

A number of environment variables control the amount of memory that BDDs use:

- **STEP_BDD_NODES** The maximum number of BDD nodes kept around at one time.
- **STEP_BDD_CACHE** The size of the BDD result cache.
- **STEP_BDD_SPLIT** The maximum number of subgoals that can be generated by a BDD split operation in both the Top-level and Interactive Provers (see Section 6.2.1 below).

For more information on BDDs see [Bryant 1986], [Bryant 1992].

#### 6.2.1 BDD split *

The proof rule uses BDDs to convert the current goal into a list of subgoals, each a disjunction of atomic formulas. The subgoals correspond to the negation of the propositional models that can falsify the formula. If all the subgoals are valid, the original goal is valid as well.
In the worst case exponentially many children are produced using this expansion but in most applications there is a good chance that it will work well. The acceptable maximum number of subgoals is given by the \texttt{STEP\_BDD\_SPLIT} environment variable.

In the Top-level Prover the active background formulas are automatically used to reduce the number of subgoals generated; in the Interactive Prover the background formulas must be added to the sequent before invoking \texttt{BDD-split}. This is analogous to the behavior of the Simplifier in the two provers.

### 6.3 Tactics *

Tactics are a method to automate parts of the high-level proof search by encoding long or repetitive sequences of proof commands. An individual proof command is an \textit{atomic tactic}.

The Top-level Prover and the Interactive Prover share the same tactic language parameterized by their atomic tactics (since each has a different set of proof commands). The general syntax of a tactic is as follows:

\begin{verbatim}
tactic ::= predefined             Predefined tactic
| composite               Composite tactic
| atomic                 Atomic tactic
predefined ::= skip           Identity tactic
| fail                   Failure tactic

composite ::= try ( tactic , tactic , tactic )
| then [form] tactics
| sequence tactics
| else tactics
| repeat [form] tactic
| spread tactic tactics
| branch tactic tactics [ otherwise tactic ]
| tactic ; tactic
| ( tactic )
tactics ::= [ [ tactic , tactic ]]  

form ::= depth first
| breadth first  

atomic ::= \{ atomic-tactic \}
\end{verbatim}

The grammar for \textit{atomic-tactic} for the Top-level Prover is given in Section 6.3.2; the grammar for the Interactive Prover atomic tactics is given in Section 6.3.3.

The curly brackets can be omitted if the atomic tactic can be written without spaces. In other cases curly brackets have to enclose each atomic tactic.
6.3. **TACTICS**

6.3.1 Composition Options

When a verification rule is applied to a goal to which it does not syntactically make sense (for example \(+\-INV\) applied to a response formula) it **fails** otherwise it **succeeds**. Tactics which are composed from verification rules can also fail and succeed as described below.

\[
\text{try}(tac_1,tac_2,tac_3)
\]

Try tactic \(tac_1\) if it succeeds apply \(tac_2\) to each subgoal generated. If \(tac_1\) fails apply tactic \(tac_3\) to the original goal. The entire tactic fails if \(tac_2\) fails on one of the generated subgoals of \(tac_1\) or \(tac_3\) fails.

Figure 6.2 gives an idea of how the proof-search looks like after \(tac_1\) and \(tac_2\) have been applied successfully. For \(tac_1\) failed and \(tac_3\) has been applied. The lower-most dot represents the original goal, the first left-most layer of dots represent the subgoals generated from \(tac_1\) the second left-most layer the subgoals generated from \(tac_2\). To the right is a schematic representation of the subgoals generated by \(tac_3\).

\[
\begin{center}
\text{tac}_2 \\
\text{tac}_1  \\
\text{original goal}
\end{center}
\]

\[
\begin{center}
\text{tac}_3 \\
\text{original goal}
\end{center}
\]

**Figure 6.2: try(\text{tac}_1,\text{tac}_2,\text{tac}_3)**

then \([\text{form}]\) tactics

then \(\square\) has the same effect as skip - the original subgoal is reduced to itself. For non-empty lists we define inductively:

then breadth first \([tac_1,tac_2,\ldots,tac_n]\) applies tactic \(tac_1\) first. If it succeeds it applies the tactic then breadth first \([tac_2,\ldots,tac_n]\) to each of the generated subgoals. If it fails it applies the tactic then breadth first \([tac_2,\ldots,tac_n]\) to the original subgoal.

Figure 6.3 illustrates the levels obtained from successful completion of then breadth first \([tac_1,tac_2,tac_3]\). So for instance the nodes above the original goal in the bottom represent the result of applying \(tac_1\).

\[
\begin{center}
\text{tac}_3 \\
\text{tac}_2 \\
\text{tac}_1  \\
\text{original goal}
\end{center}
\]

**Figure 6.3: then breadth first \([tac_1,tac_2,tac_3]\)**
then depth first \([\text{tac}_1, \text{tac}_2, \ldots, \text{tac}_n]\) applies the tactic \(\text{tac}_1\) first. If it succeeds, it applies then depth first \([\text{tac}_2, \ldots, \text{tac}_n]\) to the first subgoal generated. If it fails, it applies then depth first \([\text{tac}_2, \ldots, \text{tac}_n]\) to the original goal. If any left-most subgoal is closed by a tactic \(\text{tac}_i\) where \(i < n\), then \(\text{tac}_{i+1}, \ldots, \text{tac}_n\) are ignored.

Figure 6.4 illustrates the effect of successful completion of then depth first \([\text{tac}_1, \text{tac}_2, \text{tac}_3]\).

\[
\begin{array}{c}
\text{tac}3 \\
\text{tac}2 \\
\text{tac}1 \\
\text{original goal}
\end{array}
\]

Figure 6.4: then depth first \([\text{tac}_1, \text{tac}_2, \text{tac}_3]\)

If no form is given the default is breadth first.

**sequence tactics**

Applies the tactics in left-to-right order breadth first until the first tactic fails. The constructor sequence can be defined in terms of try; namely sequence \([\text{tac}_1, \text{tac}_2, \text{tac}_3]\) has the same effect as try(\(\text{tac}_1\) try(\(\text{tac}_2, \text{tac}_3\ fail) fail).

\(\text{tac}_1; \text{tac}_2\) is shorthand for then \([\text{tac}_1, \text{tac}_2]\). The semicolon ; associates to the left and has the highest precedence.

**else tactics**

Applies the first tactic that succeeds from the list of tactics supplied.

Figure 6.5 illustrates the result of else \([\text{tac}_1, \text{tac}_2, \ldots, \text{tac}_i, \ldots, \text{tac}_n]\) where \(\text{tac}_i\) is the first tactic that succeeds.

\[
\begin{array}{c}
\text{tac}_i \\
\text{original goal}
\end{array}
\]

Figure 6.5: else \([\text{tac}_1, \text{tac}_2, \ldots, \text{tac}_i, \ldots, \text{tac}_n]\)

**repeat [form] tactic**

repeat breadth first \(\text{tac}\) applies \(\text{tac}\) repeatedly until failure appears on one of the generated subgoals in a layer.

repeat depth first \(\text{tac}\) applies \(\text{tac}\) repeatedly on the left-most subgoal generated.

**spread tactic tactics**
spread tac \([tac_1, tac_2, \ldots, tac_n]\) applies first \(tac_1\) and then applies \(tac_i\) to the \(i\)th subgoal generated. The tactic fails if the number of subgoals generated from \(tac\) is different from \(n\). Also if any of the tactics \(tac\) or \(tac_i\) fail the entire tactic fails.

branch tactic tactics \([\text{otherwise} \ tactic]\)

branch \(tac\ \[tac_1, \ldots, tac_n\] \) works like spread\ except that if the number of subgoals is \(m \geq n\) then it applies tactic \(tac_n\) to the subgoals indexed \(n, n+1, \ldots, m\).

branch \(tac\ \[tac_1, \ldots, tac_n\] \text{otherwise} tac' \) first applies the tactic branch \(tac\ \[tac_1, \ldots, tac_n\]\). If this fails it applies \(tac'\) to the subgoal.

6.3.2 Top-level Prover Tactics

atomic-tactic ::= basic-rule | arg-rule expns
Basic rule not requiring arguments
Rule requiring argument expressions

basic-rule ::= B-INV | B-WAIT | B-BACKTO | B-CAUS | Simplify | Interactive | Undo | Redo | Postpone | BDD

arg-rule ::= G-INV | G-WAIT | G-BACKTO | G-CAUS | Modelcheck

expns ::= \{''expn''\}+

6.3.3 Interactive Prover Tactics

To appreciate this section it is helpful to have read Chapter 8.

atomic-tactic ::= basic-rule | e-rule expns | p-rule positions | pe-rule expn-poss | p-rule | replace | skolemize
Basic rule not requiring arguments
Rule taking expression argument
Rule taking position arguments
Rule taking position and expression arguments
Position rule without position arguments
Replacement of equals for equals
Skolemization
**basic-rule** ::= Undo
  | Redo
  | Postpone
  | Flatten
  | Simplify
  | Make-first-order
  | Next
  | Presburger
  | All-Propositional
  | BDD-split
  | PTL-expansion

**cut** ::= Add
  | Add-Axiom
  | Rewrite
  | Free-Induction

**p-rule** ::= Duplicate
  | Delete
  | Hide
  | Unhide
  | 1-Step-Propositional
  | Induction

**e-rule** ::= Cut
  | Add-Axiom
  | Rewrite
  | Free-Induction

**pe-rule** ::= Instantiate

**replace** ::= Replace at i-val left by right
  | Replace at i-val right by left

**skolemize** ::= Skolemize
  | Skolemize "" expn "" at position
  | Skolemize position

**expns** ::= \{ "" expn "" \}+

**positions** ::= \{ position \}+

**position** ::= i-val
  | [ i-val { , i-val } ]
If a \textit{p-rule} is given without arguments then it is applied exhaustively in a breadth-first manner.

### 6.3.4 Interrupting Tactics

Tactics are not guaranteed to terminate or can otherwise run for a very long time; the \textit{interrupt button} (the step logo) is useful to stop the execution of a batchmode tactic.

Two things can actually happen if the interrupt button is pressed during the execution of a tactic: if the interrupt button is activated during a simplification step the simplification will be canceled and the system will continue with the next one; if the system is interrupted when no simplification is going on the entire tactic will be interrupted.

### 6.3.5 Commonly used Tactics

**B-INV; Simplify**

Applies the rule B-INV followed by simplification of each subgoal. This differs from the effect of setting Automatic Simplification ON by not backtracking to the unsimplified verification condition when simplification fails.

**B-INV; Simplify; Undo**

If simplification does not establish the given subgoal of B-INV the current leaf in the proof search tree is undone getting back to the unsimplified subgoal.

**repeat(B-INV; Simplify; Undo; WPC)**

Repeats application of B-INV to each unsimplified subgoal that has been strengthened by the WPC rule.

**{ G-INV 
\[(15 \land m5) \lor (14 \land m5 \implies s = 2 \land y1)\] }; Simplify**

Applies rule G-INV with a strengthening formula and each subgoal is then simplified. G-INV only requires one strengthening formula; rules such as G-WAIT require more so a list of expressions must be entered following the keyword "G-WAIT".

**Skolemize; Simplify; BDD-split; Simplify**

Called from the Interactive Prover this tactic attempts to establish a subgoal by first skolemizing quantifiers of universal force and then simplifying in the hope that simplification will reduce the boolean complexity of the verification condition. BDD-split is then invoked to split the goal into subgoals of atomic formulas; the simplifier is finally invoked to establish each of these subgoals.

**1-Step-Propositional; Simplify**
CHAPTER 6. AUTOMATIC THEOREM-PROVING

The Interactive Prover [1-Step-Propositional] rule is used without a position. It is therefore used exhaustively until it cannot be applied. Then each generated subgoal is simplified by [Simplify].
Chapter 7

Automatic Generation of Invariants

*step* includes three different methods that automatically generate *bottom-up invariants* based solely on the system to be verified of increasing power and complexity. Since these invariants are, by construction, guaranteed to be true of the system, they are automatically added to the list of active background properties.

Of the three methods, the first, *local invariants*, operates on an SPL program; the other two operate on the transition system directly. A description of the general principles behind *step*’s invariant generation algorithms is [Björner et al., 1995].

To obtain invariants, select the appropriate option under the *Properties* menu; when the invariant generation is done, each invariant is simplified, displayed in the output window, and added to the set of background properties. By default, they are active, and thus used in the simplification of goals. They may be deactivated with the *Activate/Deactivate* menu option.

Each invariant is simplified on its own (i.e., no background properties are used) using the basic decision procedures for equality (see Section 6.1).

When the invariant generator is finished, a message will appear in the message window. Invariant generation and simplification can be halted by clicking on the interrupt button (the *step* logo).

### 7.1 Local Invariants

Local invariant generation produces very simple but useful invariants based on local analysis of the program text. The generated invariants can be classified in the following three groups:

1. *Finite-domain invariants*. Invariants of the form

   \[ l_i \Rightarrow x = c_1 \lor x = c_2 \lor \ldots \lor x = c_n \]

   where \( l_i \) is a program location, \( c_1, \ldots, c_n \) are constants, and \( x \) is a program variable.

2. *Conditional invariants*. Invariants of the form \( l_i \Rightarrow \text{cond} \) where \( l_i \) is a program location and *cond* is a formula arising from the initial condition or a branch statement.
e.g., the program fragment \[ \text{if cond then } l_1: S_1 \text{ else } l_2: S_2 \] generates the conditional invariants \[ l_1 \Rightarrow \text{cond} \Gamma \text{ and } l_2 \Rightarrow \neg \text{cond} \Gamma \] if no variables in \text{cond} are modified by parallel statements.

3. **Range invariants.** Invariants of the form \[ l_i \Rightarrow lb \leq x \land x \leq ub \Gamma \] where \( lb \) and \( ub \) are integer or rational constants (or infinity).

### 7.2 Linear Invariants

Linear invariant generation arises from analyzing the linear relationships between system variables in the course of a program.

The invariants generated are usually linear equalities of the form

\[
a_1 x_1 + a_2 x_2 + \ldots + a_n x_n = b
\]

where \( a_1, \ldots, a_n, b \) are constants. Each \( x_i \) can be (a) a variable \( \Gamma \) (b) the sum \( \sum_{i=\text{[m..M]}} A[i] \Gamma \) of the values of an array (c) a sum \( \sum_{i=\text{[m..M]}} x[i] \Gamma \) where \( x \) is a local variable of a parameterized process \( P[\text{[i: [m..M]]}] \).

### 7.3 Polyhedral Invariants *

The polyhedral invariant generator is the most powerful of the methods provided (currently in "experimental" state). The invariants generated depend on the set of variables under consideration \( \Gamma \) which can be controlled by the user. The more variables that are included \( \Gamma \) the more powerful the invariants that can be generated \( \Gamma \) but the time spent generating invariants will also increase (possibly in an exponential manner).

When the polyhedral invariant generation is selected \( \Gamma \) a dialog window appears where the user can enter the following:

1. **Variables to be ignored:** you may enter a list of program and auxiliary variables \( \Gamma \) separated by spaces or commas. These variables will be ignored by the invariant generator \( \Gamma \) that is if the invariant generation will not try to deduce polyhedral relationships which include them.

2. **Location variables to be considered:** By default location variables are not considered by the polyhedral invariant generator. Here you may enter a list of location variables to be added to the vocabulary of the invariant generator.

The generator uses an external package \( \Gamma \) whose time and space limitations can be controlled by setting the following environment variables:

- `STEP_POLY_SPACE` (Megabytes)
- `STEP_POLY_TIME` (minutes)
- `STEP_POLY_CPU` (minutes)

The invariant generation can be interrupted by clicking on the interrupt button (the `step` logo); this may leave a separate UNIX process running \( \Gamma \) which users may have to terminate independently from a UNIX prompt (see the UNIX `kill` command man pages).
Chapter 8

The Interactive Prover

The Interactive Prover is based on a Gentzen-style proof system specialized to linear-time temporal logic. For more background see e.g. Gallier [1987]. This chapter explains the basic functionality of the various rules provided in the Interactive Prover and gives examples of their use.

8.1 Proof Structure

A goal or subgoal in the Interactive Prover is a sequent of the form

$$\Gamma \Rightarrow \Delta$$

where $\Gamma$ and $\Delta$ are multisets of formulas (i.e., formulas can be repeated). A sequent stands for the formula

$$\bigwedge \gamma_i \rightarrow \bigvee \delta_j$$

where $\gamma_i$ are the elements of $\Gamma$ and $\delta_j$ are the elements of $\Delta$. Free variables are implicitly universally quantified.

The Interactive Prover creates a proof tree similar to that in the Top-level Prover. An inference rule applies to zero or more of the formulas in a sequent $\Gamma \Rightarrow \Delta$ and produces a number of subgoals, the branches of the tree again in the form of sequents. The proof is finished when all subgoals are closed.

A subgoal is closed when one of the following conditions is satisfied:

- The sequent contains \texttt{true} on the right-hand side of the long doubly lined implication.

$$\Gamma \Rightarrow \Delta, \texttt{true}$$

- The sequent contains \texttt{false} on the left-hand side of the long doubly lined implication:

$$\Gamma, \texttt{false} \Rightarrow \Delta$$

- The sequent contains a formula $p$ in both the left and right sides of the long doubly lined implication:

$$p, \Gamma \Rightarrow \Delta, p$$
The sequent contains a formula \((-\)p) on the left side of the long doubly lined implication and no applications of \textit{Next} (see Section 8.3) separate the sequent from the root of the proof tree:

\[ (-\)p, \Gamma \Rightarrow \Delta \]

This axiom reflects the well-foundedness of time. In the first time instance \(\text{the truth-value of } (-\)p is false for any formula p} because there is no past at this instance.

- The sequent \(\Gamma \Rightarrow \Delta\) is \textit{duplicated} along a branch in the proof tree i.e., the following conditions are satisfied:
  
  1. There is a sequent \(\Gamma' \Rightarrow \Delta'\) between the root and \(\Gamma \Rightarrow \Delta\) such that \(\Gamma' \subseteq \Gamma\) and \(\Delta' \subseteq \Delta\).
  2. The two sequents are separated by at least one application of the \textit{Next}-rule.
  3. There is an \textit{unfulfilled} formula \(\text{in the sense of the temporal tableau construction [Manna and Pnueli 1995]}\) for the sequents below applications of \textit{Next} and above \(\Gamma' \Rightarrow \Delta'\).

It helps to think of duplication in the presence of unfulfilled formulas as corresponding to an induction step. The requirement also replaces inference rules formalizing induction on temporal operators.

Fortunately all these conditions are kept track of automatically and you do not have to worry about the detailed conditions so that you can focus on the outline of the proof.

### 8.2 User Interface

Figure 8.1 shows the interface of the Interactive Prover. It has a menu bar on top and action buttons to the right. Most interaction in the Interactive Prover is accomplished by button clicks. If a rule is applicable to multiple formulas in the sequent \(\Gamma\) the candidates are highlighted (shown in reverse video) and you can select the one to which you intend to apply the rule. Some rules require additional information to be entered through dialog windows.

**Menu Options**

- **Refresh** redraws the Interactive Prover windows.
- **Tactics** is equivalent to the Tactics menu in the Top-level Prover (Section 3.1) except now it invokes tactics composed of Interactive Prover primitive commands (see Section 6.3).
- **Settings**
  
  - **Simplification Flags** This menu option is equivalent to the \textit{Simplification Flags} option in the Top-level Prover. It brings up a window used to set the power of the automatic \textit{Simplifier}; see Section 6.1.
Current goal:

\[ \implies 1 : 0 < y2 \land p10 = 3 \implies 0 < y2 - y1 \]
Select boxes This menu option allows the user to control whether boxes i.e., the temporal operator □ known as "henceforth" in front of axioms added to the sequent during the proof should be kept or not. By default boxes are removed from axioms when axioms are added to the sequent. See also the explanation of Add-Axiom below for an example of the usage of Select boxes.

Quit brings up the window shown in Figure 8.2.

Figure 8.2: Exit Window for Interactive Prover

If you choose to use the results obtained any unfinished goals are returned to the Top-level Prover as a new goal with the name Interactive. If there are no goals left the current goal in the Top-level Prover is closed. If you choose to ignore the effect of the Interactive Prover the proof tree in the Top-level Prover stays as it was before invoking the Interactive Prover. 

The button saving verbose versions allows you to save the entire proof in the format seen during verification. Each sequent is printed in its full length and annotated with its position in the proof-search.

The button saving for reruns saves a dense proof-script which can be loaded as a tactic for later automatic recreation of the proof.

8.3 Interactive Prover Rules

The right column of the Interactive Prover contains buttons to manipulate the proof tree and invoke the Gentzen proof rules. The proof search proceeds as usual with logical rules that reduce the current goal to a number of subgoals and simplification steps that close or simplify the current goal.

A number of inference rules require the user to select subformulas in the sequent. The subformulas that may be selected are highlighted when selecting the inference rule and the user clicks on the relevant highlighted formula to invoke the rule on that formula.

Undo Makes the ancestor of the current subgoal into the current goal. The former current goal is retained to enable a Redo unless another proof rule is applied to the new current goal.
Redo  Redoes a previous Undo.

Hide  Temporarily hides a formula from the current sequent. Hidden formulas do not participate in simplification steps and cannot be acted upon by inference rules. Hidden formulas however can be recovered with the Unhide rule below. Hiding formulas can make the sequent more readable and may speed up simplification.

After clicking this button all formulas are highlighted and you can select the formula to hide. To hide several formulas you must use Hide multiple times.

Unhide  Recovers a formula hidden with Hide.

Postpone  The next subgoal in the subgoal queue becomes the current subgoal.

Duplicate  Duplicates a formula in the sequent. This is useful if you need to instantiate a quantified variable more than once using different values. Note that duplicated formulas can be hidden with Hide until they are needed.

Delete  This button implements the *weakening rule*. It allows you to delete formulas that are not needed to establish the goal. Deletion of unnecessary or redundant formulas makes the current goal easier to read and speeds up the simplification procedures.

Flatten  Creates a new sequent in which all conjunctions in the antecedent and all disjunctions in the consequent are split into separate formulas.

Example: For formulas \( a, b, c, d, e, f, g \) Flatten has the following effect:

\[
\begin{align*}
& a, b, b \land c \imp d \land e, f, g \\
& a \lor b, b \land c \imp d \land e, f \lor g
\end{align*}
\]

Simplify  Invokes the Simplifier on the current goal. The strength of the Simplifier is controlled via the Simplification Flags option under the Settings menu.

1-Step-Propositional  Application of propositional inference rules is controlled uniformly with this button.

After clicking this button formulas which contain propositional connectives that can be analyzed are highlighted. Clicking on one of the highlighted formulas applies the appropriate propositional inference rule to the selected formula.

The boolean connectives analyzed are:

\[ \land, \lor, \imp, \iff, \neg \]

which are entered as:

\[ \land \lor, \imp, \iff, \neg \]

The temporal connectives analyzed are:

\[ \lozenge \Box, \Box, \lozenge, \Box \lor, \Box \land, \lozenge \Box, \Box, \lor, \land \]

\[ \Box, \lozenge, \Box, \lozenge, \Box \lor, \Box \land \]
which are entered as:

[] , <->, Until, Awaits, [ ], <->, Since, Backto

Notice that ◯ and ⊕ are not in this group.

Figure 8.3 shows all the rules that can be applied by 1-Step-Propositional.

**Skolemize** Quantifiers of universal force can be eliminated by skolemization which replaces the bound variable by a new variable.

If there are multiple instances of universally quantified variables all of the corresponding formulas are highlighted upon clicking on this button and you can select the one you want to skolemize by clicking on it.

**Skolemize** implements the inference rule:

\[
\frac{\Gamma, f(b)}{\Gamma, \text{Exists } x . f(x) \Rightarrow \Delta}
\]

where \( b \) is a fresh variable not occurring free in \( \Gamma \Delta \) or \( \text{Exists } x . f(x) \).

\[
\frac{\Gamma}{\Gamma, f(b) \Rightarrow \Delta}
\]

\[
\frac{\Gamma}{\Gamma, \text{Forall } x . f(x) \Rightarrow \Delta}
\]

where again \( b \) is not free in either \( \Gamma \Delta \) or \( \text{Forall } x . f(x) \).

Besides these cases the Interactive Prover supports skolemization of quantifiers under any nesting of the boolean connectives \( \land, \lor, \rightarrow, \neg \) and the then and else branches in if-then-else.

The type of the new variable \( b \) is that of \( x \Gamma \) where range types have been replaced by \( \text{int} \). Range constraints are reconstructed in a type condition that is added to the formula \( f \).

**Example:** For formulas \( e, f, g \), Forall \( x : [1..N] . h(x) \Gamma \) and \( y \) of type integer not free in these then

**Skolemize** eliminates the universal force quantifier by:

\[
\begin{align*}
f & \Rightarrow e \rightarrow \neg g \land (1 \leq y \land y \leq N \rightarrow h(y)) \\
& \Rightarrow e \rightarrow g \land \text{Forall } x : [1..N] . h(x)
\end{align*}
\]

**Instantiate** Quantifiers of existential force can be instantiated by terms.

After clicking on this button a data entry window appears for entering the term you want to instantiate the bound variable with.

**Instantiate** implements the inference rules:

\[
\frac{\Gamma, p(t)}{\Gamma \Rightarrow \Delta}
\]

\[
\frac{\Gamma, \text{Forall } x . p(x)}{\Gamma \Rightarrow \Delta}
\]
Propositional Rules

Sequents and formulas in square brackets \([\,]\) are only generated when the sequent below the line is above an application of the rule Next. This guarantees that statements such as \(\square\square(first)\) are not derivable as opposed to \(\square first\).

(first is shorthand for \(\neg \square true\) which only holds at position 0.)

\[
\begin{align*}
\Rightarrow (\neg) & \quad \frac{\Gamma, \varphi \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, \neg \varphi} & \quad (\neg \Rightarrow) & \quad \frac{\Gamma \Rightarrow \Delta}{\Gamma, \neg \varphi \Rightarrow \Delta} \\
\Rightarrow (\lor) & \quad \frac{\Gamma \Rightarrow \Delta, \varphi, \psi}{\Gamma \Rightarrow \Delta, \varphi \lor \psi} & \quad (\lor \Rightarrow) & \quad \frac{\Gamma, \varphi \Rightarrow \Delta \quad \Gamma, \varphi \Rightarrow \Delta}{\Gamma, \varphi \lor \psi \Rightarrow \Delta} \\
\Rightarrow (\rightarrow) & \quad \frac{\Gamma, \varphi \Rightarrow \Delta, \psi}{\Gamma \Rightarrow \Delta, \varphi \rightarrow \psi} & \quad (\rightarrow \Rightarrow) & \quad \frac{\Gamma, \varphi \Rightarrow \Delta \quad \Gamma \Rightarrow \Delta, \varphi}{\Gamma \Rightarrow \Delta, \varphi \rightarrow \psi \Rightarrow \Delta} \\
\Rightarrow (\leftrightarrow) & \quad \frac{\Gamma, \varphi \Rightarrow \Delta, \psi \quad \Gamma, \psi \Rightarrow \Delta, \varphi}{\Gamma \Rightarrow \Delta, \varphi \leftrightarrow \psi} & \quad (\leftrightarrow \Rightarrow) & \quad \frac{\Gamma, \varphi, \psi \Rightarrow \Delta \quad \Gamma \Rightarrow \Delta, \psi, \varphi}{\Gamma, \varphi \leftrightarrow \psi \Rightarrow \Delta} \\
\Rightarrow (\land) & \quad \frac{\Gamma \Rightarrow \Delta, \psi \quad \Gamma \Rightarrow \Delta, \varphi \leftrightarrow \psi}{\Gamma \Rightarrow \Delta, \varphi \land \psi} & \quad (\land \Rightarrow) & \quad \frac{\Gamma, \varphi, \psi \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, \varphi \land \psi} \\
\Rightarrow (\square) & \quad \frac{\Gamma \Rightarrow \Delta, \varphi \quad \Gamma \Rightarrow \Delta, \square \varphi}{\Gamma \Rightarrow \Delta, \square \varphi} & \quad (\square \Rightarrow) & \quad \frac{\Gamma, \varphi, \square \varphi \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, \square \varphi} \\
\Rightarrow (\Diamond) & \quad \frac{\Gamma \Rightarrow \Delta, \varphi, \Diamond \Diamond \varphi}{\Gamma \Rightarrow \Delta, \Diamond \varphi} & \quad (\Diamond \Rightarrow) & \quad \frac{\Gamma, \varphi \Rightarrow \Delta}{\Gamma, \Diamond \varphi \Rightarrow \Delta} \\
\Rightarrow (W) & \quad \frac{\Gamma, \varphi, \Diamond (\varphi W \psi) \Rightarrow \Delta \quad \Gamma, \psi \Rightarrow \Delta}{\Gamma, \varphi W \psi \Rightarrow \Delta} & \quad (W \Rightarrow) & \quad \frac{\Gamma \Rightarrow \Delta, \varphi, \psi, \varphi, \Diamond (\varphi W \psi), \psi}{\Gamma \Rightarrow \Delta, \varphi W \psi} \\
\Rightarrow (U) & \quad \frac{\Gamma, \varphi, \Diamond (\varphi U \psi) \Rightarrow \Delta \quad \Gamma, \psi \Rightarrow \Delta}{\Gamma, \varphi U \psi \Rightarrow \Delta} & \quad (U \Rightarrow) & \quad \frac{\Gamma \Rightarrow \Delta, \varphi, \psi, \varphi, \Diamond (\varphi U \psi), \psi}{\Gamma \Rightarrow \Delta, \varphi U \psi} \end{align*}
\]
where \( t \) is a term and \( \Gamma p(x) \) is a formula containing \( x \). Complementing skolemization \( \Gamma \) instantiation applies to any existential force quantifiers under \( \rightarrow \), \( \vee \Gamma \) \( \rightarrow \Gamma \) if-then-else.

If the user-supplied term \( t \) is not obviously of the same type as \( x \), \( \text{Instantiate} \) generates additional type constraint conditions along with the instantiated formula.

**Example:** For formulas \( (N > 0) \exists x : [0..N-1]. h(x) \) and term \( t \), \( \text{Instantiate} \) \( x \) with \( t \) eliminates the existential quantifier in the following way:

\[
\begin{align*}
N > 0 \Rightarrow 0 <= t \& t <= N - 1 \& h(t) \\
N > 0 \Rightarrow \exists x : [0..N-1]. h(x)
\end{align*}
\]

**Replace** This button allows you to replace equals by equals, that is, any term that is asserted to be equal to another term in the antecedent may be replaced with that other term throughout the sequent.

Upon clicking on this button all terms that are candidates to be replaced are highlighted and you can select the term that you want to replace with.

**Example:** Starting with the sequent

\[
x = 3, y = 4 \Rightarrow x * x + y * y = 25
\]

Clicking on **Replace** highlights: \( x \), \( y \), \( 3 \), and \( 4 \). Clicking on 3 replaces \( x \) with 3 in the consequent, resulting in

\[
x = 3, y = 4 \Rightarrow 3 * 3 + y * y = 25
\]

**Rewrite** Brings up the Select rewrite window which displays all rewrite rules that were declared in the specification during the current verification session via

\[
\text{REWRITE } \text{desc} : \text{expn} \rightarrow \text{expn}
\]

Rules can be selected and de-selected by clicking on the Select buttons. When clicking on the Apply button the current goal is simplified according to the selected rewrite rules.

**Make-first-order** Upon clicking on this button with current goal sequent \( G \), a first-order subgoal \( G' \) is generated such that the first-order validity of \( G' \) implies the first-order temporal validity of \( G \).

**Next** Upon clicking on this button a new subgoal is created in which the operator \( \ominus \) is deleted from (sub)formulas preceded by \( \vee \) and in which \( \ominus \) is appended to (sub)formulas not preceded by \( \ominus \).
**Cut** With the Cut rule you can perform a case split analysis on the current goal. Upon clicking on this button a dialog window will appear where you can enter the formula to split on. Thus upon having entered the cut-formula the Cut implements the inference rule

\[
\Gamma, p \rightarrow \Delta \\
\Gamma \rightarrow \Delta, p \\
\Gamma \rightarrow \Delta
\]

**Induction** Upon clicking on this button a dialog window will appear in which you can enter the variable you want to do induction on. The variable has to be a universally quantified integer variable. The button implements the inference rule:

\[
\Gamma \Rightarrow \Delta, \ p(0) \\
\Gamma \Rightarrow \Delta, \ \text{Forall } x : \text{int} . \ (x \geq 0 \ \text{\lor} \ p(x) \rightarrow \ p(x+1)) \\
\Gamma \Rightarrow \Delta, \ \text{Forall } x : \text{int} . \ (x \leq 0 \ \text{\lor} \ p(x) \rightarrow \ p(x-1)) \\
\Gamma \Rightarrow \Delta, \ \text{Forall } x : \text{int} . \ p(x)
\]

where \( p \) is a formula containing \( x \).

**Free-Induction** supports simple mathematical induction for free variables. Suppose that the integer variable \( x \) occurs free in the sequent \( \Gamma \Rightarrow \Delta \) which we recall corresponds to a formula:

\[
p(x) : \bigwedge_{\gamma(x) \in \Gamma} \gamma(x) \rightarrow \bigvee_{\delta(x) \in \Delta} \delta(x),
\]

Since \( x \) is implicitly universally quantified it makes sense to formulate the induction principle:

\[
\Rightarrow p(0) \\
\Rightarrow \text{Forall } x : \text{int} . \ (x \geq 0 \ \text{\lor} \ p(x) \rightarrow \ p(x+1)) \\
\Rightarrow \text{Forall } x : \text{int} . \ (x \leq 0 \ \text{\lor} \ p(x) \rightarrow \ p(x-1)) \\
\Gamma \Rightarrow \Delta
\]

**Presburger** Decides formulas of Presburger arithmetic. Formulas belonging to Presburger arithmetic may involve integer valued variables, addition, multiplication by constants, boolean connectives, and quantifiers over integer variables.

The decision problem is super-exponential and thus only works on small examples. It is seldom needed in practice as the decision procedure cannot handle uninterpreted function symbols (e.g., arrays).

**Example:** The following are Presburger formulas:

- \( \text{Forall } x : \text{int} . \ \text{Exists } y : \text{int} . \ (x + y > 0) \)
- \( \text{Exists } x : \text{int} . \ \text{Forall } y : \text{int} . \ \text{Exists } z : \text{int} . \ (x > y \ \lor \ x < y \ \lor \ z = y - x \ \lor \ 5z = x) \)

The following are not Presburger formulas:
For all \( x : \text{int} \). Exists \( y : \text{int} \). \((f(x) + y > 0)\)

Exists \( x : \text{int} \). For all \( y : \text{rat} \). \((x > y)\)

since the first contains an uninterpreted function symbol \( f \) while the second includes the unsupported type \( \text{rat} \).

**All-Propositional** Applies **1-Step-Propositional** recursively until there are no new formulas to which it is applicable.

**BDD-split** Simplifies the purely propositional structure of the formulas in the sequent. Thus, a formula whose main connective is a temporal operator \( \Box \), \( < > \), \( \text{Awaits} \), \( \text{Until} \), \( \text{Since} \), \( \text{Backto} \), \([-] \), \( < > \)'s treated as an atomic formula (as opposed to **All-Propositional**). See Section 6.2 for further explanations.

**Add-Axiom** A list of previously established properties or asserted axioms is displayed in a separate window. The user selects the previously proved property or asserted axiom to be inserted in the antecedent of the sequent.

The **Select boxes** menu-option under **Settings** allows the user to automatically remove or retain preceding boxes from the axioms added by **Add-Axiom**. By default boxes are removed from axioms added to the sequent.

**Example:** By default, adding the axiom \( \Box (p(x) \land q(y)) \) to the sequent \( \Gamma \Rightarrow \Delta \)

generates

\[ (p(x) \land q(y)), \Gamma \Rightarrow \Delta \]

Selecting **Select boxes** keeps the boxes. Hence adding the same axiom generates the sequent:

\[ \Box (p(x) \land q(y)), \Gamma \Rightarrow \Delta \]

**PTL-expansion** The structure of the Gentzen sequent calculus resembles the one used in temporal tableau-like decision procedures for propositional linear-time temporal logic. The prover therefore supports an automatic decision procedure for propositional temporal logic.

**Disclaimer:** This implementation of PTL-expansion only applies well to small formulas; its use should be limited to textbook examples. More efficient implementation are available e.g., \[\text{Kesten et al.} \Gamma 1993\].
Chapter 9

Basic Tutorial

This chapter presents detailed verification sessions that can be followed as a tutorial for step and used as templates for other verifications. All input files mentioned in this chapter are available in the examples directory in the step distribution. We assume that you have successfully started step (see Section 1.2).

9.1 Preliminaries

The input to step will be an SPL program $P$ and a temporal-logic formula $\varphi$ that expresses a property of $P$ to be verified. A verification session deals with a single program and one or more properties to be verified for that program. Thus, loading a new program reinitializes the system and starts a new verification session.

Loading a program

To load an SPL program, select the File menu option in the upper left corner (see Figure 3.2) and select Load program. This will start up the File Browser with a Load program window as shown in Figure 3.3. By clicking on the * .spl button, you can choose to see only the files with extension .spl, the standard extension for SPL program files. The right column shows the directory hierarchy, and the left column shows the files in the current directory. Clicking on a directory in the right column makes it the current directory. You can load a file by clicking on its filename in the left column. Alternatively, you may enter the full filename directly in the entry line under File name. After entry, click the OK button, and the selected file will be loaded.

Loading the specification

To load a specification, select the Load specification option from the File menu. Again, this activates the File Browser, which brings up a Load specification window. The default extension for specification files is .spec. If the specification file contains a single formula, this formula is understood to be the top-level goal and is displayed in the top window as the new current goal. If the file contains multiple formulas to be proved, the first one
is displayed as the current top-level goal. The other top-level goals can be accessed by selecting the Next search or Select search option under the Properties menu.

Specification files can also contain declarations, axioms and definitions—see Chapter 2.

Help

At any time you can get online help by selecting the Help option from the menu. This will bring up a WWW browser with step's help pages.

Quitting step

To leave step select the Quit option from the top-level menu or select the Quit option from the File menu option. Both will bring up a confirmation window asking if you really want to exit step.

9.2 MUX-SEM: Mutual Exclusion

We now assume that you have started step and are able to load program and specification files. The program and specifications mentioned in this section are available in the directory examples/mux-sem.

Figure 9.1: Program MUX-SEM (mutual exclusion by semaphores)

We begin with the proof of two simple safety properties for the program MUX-SEM shown in Figure 9.1: the first \( \varphi_1 \) states that \( y \) is always greater than 0, and the second \( \varphi_2 \) expresses mutual exclusion. These properties are expressed by the formulas

\[
\varphi_1 : \square (y \geq 0) \\
\varphi_2 : \square (\neg (\ell_3 \land m_3))
\]

or in step's input format

\[
\varphi_1 : \Box (y \geq 0) \\
\varphi_2 : \Box ! (13 \land m_3).
\]
To illustrate the various methods available for verification we will prove these properties in a number of different ways.

In this example we will load each property from a separate file. Subsequent examples will illustrate specification files with multiple properties as well as entering goals directly.

### 9.2.1 Using B-INV

The first property $\Box (y \geq 0)$ is inductive so we should in principle be able to be verify it using rule B-INV. We can do this in the following steps:

1. Load the program `mux-sem` by loading the file `mux-sem.spl`. This will bring up the Program Text window shown in Figure 9.2. Apart from the program it contains a list of how the program's control locations correspond to step's internal variables. Control locations are explained in detail in Appendix C.2. When an SPL program is loaded a fair transition system is automatically generated which you can view by selecting View Transitions from the File menu.

2. Load the specification file `mux-sem/yge0.spec`. The property appears in the current goal window as shown in Figure 9.3.

3. Enable automatic simplification by selecting the Automatic Simplification ON/OFF option under the Settings menu.

4. Enable the use of decision procedures for linear arithmetic by selecting the Simplification Flags option under the Settings menu. A window with the title “Main Flags” will appear as shown in Figure 6.1. Click on the second switch to include decision procedures for linear arithmetic in the standard simplification and press the OK button.

   (Throughout this tutorial we will assume that the decision procedures for equality are also enabled; this is the default for step).

5. Click on the B-INV button in the action region. One by one the verification conditions for all transitions are displayed in the current goal window and the result of their simplification is displayed in the output window (see Figure 9.4). Since all verification conditions are simplified to true the proof is complete as indicated.

The single original goal has now been proved so there is no current (unproven) goal as indicated in the current goal window. The proved property is saved as a background property and can be used in the proof of subsequent properties. The background properties can be viewed by selecting the Activate/Deactivate option under the Properties menu. This brings up a Property Checklist window shown in Figure 9.5. By default properties are active and will be used in the proof of subsequent properties. However you may deactivate a property by clicking on the switch next to it.

Inactive properties can be reactivated later on; however you may also entirely delete a property by clicking on the skull.

To save the proof to a file (in ASCII format) select the Save last proof option of the File menu. The File Browser brings up a window with the title “Save completed proof”.

Figure 9.2: mux-sem program text window
Figure 9.3: Loaded specification for MUX-SEM

Figure 9.4: Verification of $\Box y \geq 0$ for MUX-SEM

Figure 9.5: Background properties
As when loading a file you can select an existing filename or enter a new one. The default extension for saved proofs is .search. The window also allows you to enter three lines of text that will be added to the top of the file as a comment. Finally click on OK to save your proof. Figure 9.6 shows the format of the saved proof.

% Proof of □(y>=0) for program mux-sem, with decision % procedures for linear arithmetic turned on, using rule B-INV.
Formula abbreviations:
phi0: 0 <= y
Reduced via INV:
□0 <= y
given the strengthening formula:
  0 <= y
1 Established via Simplify: true
2 Established via Simplify: y = 1 \ ( pi0 = 0 \ pi1 = 0 --> 0 <= y
3 Established via Simplify: {phi0} idle {phi0}
4 Established via Simplify: {phi0} 10 {phi0}
5 Established via Simplify: {phi0} 11 {phi0}
6 Established via Simplify: {phi0} 12 {phi0}
7 Established via Simplify: {phi0} 13 {phi0}
8 Established via Simplify: {phi0} 14 {phi0}
9 Established via Simplify: {phi0} m0 {phi0}
10 Established via Simplify: {phi0} m1 {phi0}
11 Established via Simplify: {phi0} m2 {phi0}
12 Established via Simplify: {phi0} m3 {phi0}
13 Established via Simplify: {phi0} m4 {phi0}

Figure 9.6: Transcript for proof of □(y>=0) over MUX-SEM

9.2.2 Using G-INV

The next property we will prove \( \varphi_2 \) expresses mutual exclusion. This property is not inductive even relative to the simple property \( \varphi_1 \) proved before. However we can prove it by strengthening the invariant being proved as follows:

1. Load the specification file mux-sem/mux.spec. The property \( \varphi_2 \) will appear in the current goal window.

2. Click on G-INV. This will bring up a window with the title “Input auxiliary assertions”. By default the auxiliary assertion is the goal to be proven so when the default is used G-INV behaves exactly as B-INV. Now enter the following formula after deleting the default one:

\[
(13+14+m3+m4+y) = 1
\]
and click on [OK]. If there is a syntax error in the entered formula a message will be displayed at the bottom of the entry window and you will be asked to try again.

Using boolean terms such as 13 and m3 as integers is a convenient feature for writing succinct specifications and is called *arithmetization*. The boolean value true is also interpreted as 1 and false as 0 (see Chapter 2).

After the strengthened assertion is entered all verification conditions are simplified to true and the proof is complete.

**Note:** The strengthened property only implies mutual exclusion based on the invariant \( \varphi_1 : \Box(y \geq 0) \) that we proved before. If you do not prove this property before attempting the proof of mutual exclusion the Monotonicity verification condition will fail to simplify to true.

The basic but useful property \( y \geq 0 \) can also be generated automatically using step’s invariant generation tools. To see how this can be used reset the searches with the Reset searches option under the File menu and repeat the steps using G-INV above. The Monotonicity property will not be simplified to true. Now we can do the following:

3. Select the Get local invariants option from the Properties menu. Five invariants appear in the output window. The first invariant in Figure 9.7 is \( \varphi_1 \) the property we need.

![Figure 9.7: Local invariants for MUX-SEM](image)

4. Select the Simplify button. Now the Monotonicity condition should simplify to true and the proof is complete.

Note also that these simplification steps succeed only if the linear arithmetic decision procedures are enabled. (This is done with the Simplification Flags option under the Settings menu). If they are not enabled the Monotonicity premise will still fail to be simplified to true.

### 9.2.3 Using the Model Checker

Since program MUX-SEM is a finite-state program we may use the Model Checker to prove mutual exclusion as follows:
1. Load again the mutual-exclusion property from the `mux-sem/mux.spec` file. (The model checker does not use background properties, so you don’t have to reset the current background properties at this point.)

2. Click on the `Modelcheck` button. This will bring up a window titled “Input auxiliary assertions”. This window allows the user to enter additional assumptions under which model checking is to be done. Since we don’t need any, we can just click the `OK` button. A File Browser window titled “Modelcheck log file” will appear. Here, you have to specify a file in which you want to have model checker results recorded. The default extension of the model checker log file is `.mclog`.

3. After the log file is specified, the model checker will run. We will then see in the output window:

   Modelcheck succeeded. The proof is COMPLETE.

The model checker log file provides more detailed information such as the number of states explored. In general, however, the log file is interesting only when the model checker fails. In this case, the file will include a counterexample computation that is a computation of the system which violates the original property.

9.2.4 Using Verification Diagrams

In the discussion of `MUX-SEM` [Manna and Pnueli, 1995] presents a state-space partition graph. This graph can easily be converted into the invariance verification diagram shown in Figure 9.8. This diagram may be used to prove mutual exclusion for `MUX-SEM` as follows:

1. Reset the background properties and current searches with the `Reset searches` option under the `File` menu.

2. Load again the mutual-exclusion property from the `mux-sem/mux.spec` file.

3. Select the `Edit diagram` option under the `Diagrams` menu. This brings up the Verification Diagram Editor.

4. Select the `Load` option under the `File` menu in the Verification Diagram Editor window. This brings up the File Browser.

5. Select the file `mux-sem/state-partition.diagram`. This loads the verification diagram which is displayed in the editor window as shown in Figure 9.8.

6. Select the `Current-to-Verify` option of the `File` menu of the Verification Diagram Editor. This will make the verification diagram accessible to the Top-level Prover.

7. Select the `Verification diagram rule` option of the `Diagrams` option. This generates all the verification conditions corresponding to the given verification diagram as subgoals. If automatic simplification is enabled, the simplifier will proceed to simplify each subgoal.
Figure 9.8: Invariance verification diagram for MUX-SEM
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The first two subgoals are side verification conditions. They state that the label on each of the nodes implies the invariant to be proven and that the initial condition implies the label of one of the nodes. Then the verification conditions corresponding to each of the edges in the diagram are simplified (including those for the implicit self-loops). In this case, all verification conditions simplify to true (even without the linear arithmetic decision procedures) and the proof is complete.

9.2.5 Using MON-I and Linear Invariants

Finally, we show how we can prove \( \varphi_2 \) based on \( \varphi_1 \) and automatically generated linear invariants:

1. Load the examples/mux-sem.spl program and prove \( \varphi_1 : \Box(y \geq 0) \) using B-INVT as described in Section 9.2.1.

2. Load the goal \( \varphi_2 \) from examples/mux.spec.

3. Select the Get global linear invariants option under the Properties menu. This will automatically generate a number of linear invariants which are added to the set of background properties including

   \[ \Box(l_0 + l_1 + l_2 = m_3 + m_4 + y). \]

   Your set of background properties should now contain this formula as well as \( \varphi_1 \).

4. Make sure that linear decision procedures are enabled (with the Simplification Flags option under the Settings menu).

5. Select the MON-I rule under the Logical rules menu. This will generate a single verification condition as the subgoal stating that the conjunction of all the background invariants implies the original goal.

6. Click on the Simplify button; the verification condition will be simplified to true and the proof is complete. (If automatic simplification is ON, this simplification will be done automatically after MON-I is invoked.)

9.3 MUX-PET1: Invariance Strengthening

In this section, we illustrate the construction of the proof tree with the program MUX-PET1 shown in Figure 9.9. The program and specification used in this section can be found in the files mux-pet1/mux-pet1.spl and mux-pet1/mux.spec respectively.

9.3.1 Using WPC

Mutual exclusion for MUX-PET1 is not an inductive property. Instead of finding a strengthened assertion and using G-INVT as we did for program MUX-SEM, we use step to do the strengthening for us:
1. Load the program `mux-pet1.spl` and the mutual exclusion property `mux.spec` from the `mux-pet1` directory.

2. Turn automatic simplification on by selecting the Automatic Simplification ON/OFF option under the Settings menu.

3. Generate local invariants by selecting the Get local invariants from the Properties menu.

4. Invoke rule B-INF. As may be expected, the verification conditions for \( \ell_3 \) and \( m_3 \) do not simplify to `true` since they are not valid. The first unproven verification condition is now displayed in the current goal window (in unsimplified form). The other open subgoal The verification condition for \( \ell_3 \) can be viewed by selecting the Next button. Again selecting `Next` goes back to the previous goal.

**The Proof Tree**

We interrupt our proof for a moment to look at the construction of the proof tree. We started with the property we wanted to prove as the single top-level goal. This goal is the root of the proof tree. When rule B-INF was applied it generated subgoals were generated in this case: these are the verification conditions for monotonicity (always trivial for B-INF) initial condition the idle transition and the 12 program transitions. Since automatic simplification was on all but two of these subgoals were immediately simplified to `true` and closed which left us with two open goals.

The buttons `Previous` and `Next` allow you to cycle through the entire list of open goals in the tree. Two other buttons that manipulate the proof tree are `Undo` and `Redo`. Applying `Undo` to a subgoal returns to its parent in the proof tree and the subgoal and all its siblings are removed from the tree. However, they may be recovered by clicking on `Redo` which restores the proof tree in the reverse order that steps were undone.

When multiple properties are being proved at the same time, multiple proof trees are maintained each with its own set of open subgoals. The `Next search` and `Select search` options under the `Properties` menu are used to move from one proof search to another.
Finishing the Proof

We now continue with the proof:

5. Click on the [WPC] button which stands for Weakest Pre-Condition. This generates the subgoal \( \square wpc(\neg(\ell_4 \land m_4), m_3) \). In this case

\[
(\neg y_1 \lor s = 1) \land pi_1 = 3 \Rightarrow \neg(pi_0 = 4),
\]

where \( pi_1 = 3 \) stands for \( at_m m_3 \) and \( pi_0 = 4 \) stands for \( at_f f_4 \) (you may check this by looking at the control correspondence information provided in the Program Text window).

6. The subgoal generated by WPC is again an invariance property. Thus we can try to prove it using B-INV again. Click on [B-INV] to prove this subgoal: all the verification conditions generated by B-INV are simplified to true, closing this branch of the proof tree.

7. We can do the same for the unproved verification condition for \( f_3 \) which is now the only open goal left. Click again on the [WPC] button and then on the [B-INV]. Again all verification conditions are simplified to true, thus closing this second branch. This leaves us without any open goals in the tree so the proof of our original top-level goal is complete.

9.3.2 Using Tactics *

In the previous section we manually applied [WPC] to the two verification conditions that were not valid. Applying weakest precondition to invalid verification condition is a common technique that often succeeds so we may want the application to be automatic. This can be done with the help of tactics. step provides a simple tactic language in which sequences of rules can be specified. The tactic language is described in detail in Chapter 6.3 and more examples of tactics are presented in Section 6.3.5.

To prove mutual exclusion for mux-pet1 using tactics we do the following:

1. Load program mux-pet1.spl and the specification file mux.spec.

2. Select Get local invariants from the Properties menu.

3. Select Enter batchmode tactic from the Tactics menu. This will bring up a data entry window with the title “Enter tactic”.

4. Enter the following in this window:

   \[
   \text{repeat}(\text{B-INV}; \text{Simplify}; \text{Undo}; \text{WPC})
   \]

   and click on [OK]. This will apply B-INV to the goal. All resulting subgoals will be simplified. For those subgoals that do not simplify to true the simplification will be undone going back to the unsimplified verification condition to which WPC is then applied. Then the process is repeated for all open subgoals. In this case they are
invariance formula since WPC always generates an invariance formula. Application of the tactic is repeated in this way until there are no open subgoals left. This involves two iterations in this case and the proof is complete.

Thus, mutual exclusion for mux-pet1 can be proved automatically by this tactic. Note that the batchmode application of a tactic such as this one is risky since it may not terminate. However, tactics can be interrupted by clicking on the interrupt button (the step logo) in the bottom-right corner of the main interface.

When applying a tactic for the first time it is recommended to apply it in interactive mode. In this way, the behavior of the tactic can be followed at each step and the tactic can be easily interrupted at any time.

9.3.3 Using verification diagrams

The chain diagram given in Figure 5.1 is a high-level proof-outline for process $P_1$’s accessibility to its critical section. It establishes that whenever $P_1$ requests access to its critical section it will eventually get there. When specified in step this reads

$$12 \implies <>14.$$ 

To use the chain-diagram in proving this response property assume we have already loaded the program, selected automatic simplication, and generated local invariants as described in Section 9.3.1. We now apply the following steps:

1. Select Enter new goal under the Properties menu. Enter the goal $12 \implies <>14$.

2. Invoke the diagram editor by selecting Edit verification diagram in the Diagrams menu.

3. From the verification diagram editor select Load in the File menu to load the verification diagram mux-pet1/response.dia. The diagram in Figure 5.1 should now appear in the verification diagram window.

4. Enable step’s top-level interface to use the verification diagram by selecting Current-to-verify under the File menu in the verification diagram editor.

5. Back in step’s top-level interface use the verification diagram as a rule by selecting Verification diagram rule from the Diagrams menu. The 88 verification conditions generated from the verification diagram will now all simplify to true automatically.
Appendix A

Computational Model

A.1 Fair Transition Systems

An SPL program is compiled into a fair transition system. A fair transition system \( \langle V, \Theta, T, J, C \rangle \) is given by the following components:

- \( V \): system variables including both data variables and control variables.
- \( \Theta \): initial condition which is a satisfiable assertion characterizing all the initial states of a computation.
- \( T \): set of transitions. Each transition \( \tau \in T \) is a function
  \[
  \tau : \Sigma \mapsto 2^\Sigma,
  \]
  where \( \Sigma \) is the set of all states and a state is a type-consistent interpretation of \( V \). Each state in \( \tau(s) \) is called a \( \tau \)-successor of \( s \). A transition \( \tau \) is said to be enabled on \( S \) if \( \tau(s) \neq \emptyset \). Otherwise it is said to be disabled.
- \( J \subseteq T \): the set of just transitions (also called weakly fair transitions).
- \( C \subseteq T \): the set of compassionate transitions (also called strongly fair transitions).

Each transition \( \tau \in T \) is represented by a first-order formula \( \rho_\tau(V, V') \) called the transition relation which may refer to both unprimed and primed versions of the system variables. The transition relation expresses the relation holding between a state \( s \) and any of its \( \tau \)-successors \( s' \in \tau(s) \). The unprimed version of a variable refers to its value in \( s \) and the primed version refers to its value in \( s' \).

Thus the state \( s' \) is a \( \tau \)-successor of the state \( s \) if the formula \( \rho_\tau(V, V') \) evaluates to true when we interpret each \( x \in V \) as \( s[x] \) and its primed version \( x' \) as \( s'[x] \).

If not explicitly specified, \( \text{step} \) adds to each fair transition system the idling transition \( \tau_I \) (also called the stuttering transition) whose transition relation is \( \rho_I : (V = V') \).
A.2 Computations

An infinite sequence of states
\[ \sigma : s_0, s_1, s_2, \ldots \]
is defined to be a computation of a fair transition system \( P \) if it satisfies the following requirements:

- **Initiality**: \( s_0 \) is initial if it satisfies the initial condition.
- **Consecution**: For each \( j = 0, 1, \ldots \) the state \( s_{j+1} \) is a \( \tau \)-successor of the state \( s_j \).
- **Justice**: For each transition \( \tau \in \ junction \) it is not the case that \( \tau \) is continually enabled beyond some position \( j \) in \( \sigma \) but not taken.
- **Compassion**: For each transition \( \tau \in \lumpset \) it is not the case that \( \tau \) is enabled at infinitely many positions in \( \sigma \) but taken at only finitely many positions.

A.3 SPL semantics

System Variables

The system variables \( V \) consist of the program variables declared in the program \( \Gamma \) and a set of control variables (approximately one for each spawned subprocess) \( \Gamma \) and a set of location variables (one for each label in the program). A control variable ranges over a subset of the integers where each value corresponds to a location in a process (see Section C.2). A location variable is a boolean variable with the same name as a program label. Its value is true if control currently resides at the location of the label \( \Gamma \), i.e., iff one of the control variables equals the value of the corresponding location.

Initial Condition

The initial condition \( \Theta \) for program \( P \) with \( n \) top-level processes is defined as
\[
\Theta : \; \pi_0 = 0 \land \pi_1 = 0 \land \ldots \land \pi_n = 0 \land \varphi,
\]
where \( \varphi \) is the conjunction of all the assertions that appear in the where clause of the declaration of \( P \). For a parameterized program \( \Gamma \) with top-level processes \( P[i] \Gamma \leq i \leq NT \) the initial condition is given by
\[
\Theta : \; \forall i : [1 \ldots N].\pi_0[i] = 0 \land \varphi.
\]

Transitions

A transition relation is defined for each of the statements in the language.

To make the definitions more succinct we define the abbreviation
\[
move_S(\ell, \hat{\ell}) : \; \pi_S = val(\ell) \land \pi'_S = val(\hat{\ell}) \land loc(\ell) \land loc(\hat{\ell}),
\]
where $\pi_S$ refers to the control variable of the innermost subprocess in which the statement $S$ appears, and $val(\ell)$ refers to the integer value associated with the location of $\ell$. $loc(\ell)$ refers to the location variable associated with the location of $\ell$.

Since every transition usually modifies only a few variables, we define

\[ \text{pres}(U) : \bigwedge_{u \in U} (u' = u), \]

where $U \subseteq V$. It states that all variables in $U$ preserve their value at the current step.

In the following we represent a statement $S$ with label $\ell$ and post-label $\hat{\ell}$ in the form $[\ell : S; \hat{\ell}]$. For a statement $[\ell : S; \hat{\ell}]$ we denote by $Y_S \cap V = \{\pi_S, loc(\ell), loc(\hat{\ell})\}$. Unless otherwise specified, all transitions are Just.

**Basic Statements**

- **Skip**
  Statement: $\ell : \text{skip}; \hat{\ell}$:
  Transition relation:
  \[ \rho_{\ell} : \text{move}_S(\ell, \hat{\ell}) \land \text{pres}(Y_S) \]

- **Assignment**
  Statement: $\ell : \bar{\pi} := \bar{\tau}; \hat{\ell}$:
  Transition relation:
  \[ \rho_{\ell} : \text{move}_S(\ell, \hat{\ell}) \land \bar{\pi}' = \bar{\tau} \land \text{pres}(Y_S - \{\bar{\pi}\}) \]
  The assignment is always enabled (if control is at the assignment statement) even if $\bar{\tau}$ is not in the range of $\bar{\pi}$, which may happen with e.g. range types. A proof obligation to check for this type of runtime error is generated by the **Check Runtime System** menu option of the Top-level Prover.

- **Await**
  Statement: $\ell : \text{await} \; e; \hat{\ell}$:
  Transition relation:
  \[ \rho_{\ell} : \text{move}_S(\ell, \hat{\ell}) \land e \land \text{pres}(Y_S) \]

- **Asynchronous Send**
  Statement: $\ell : a \Leftarrow e; \hat{\ell}$: where $a$ is an asynchronous channel.

**Unbounded channel** Transition relation:

\[ \rho_{\ell} : \text{move}_S(\ell, \hat{\ell}) \land a' = \text{append}(a, e) \land \text{pres}(Y_S - \{a\}) \]

**Bounded channel** For a channel $a$ with bound $k$ the transition relation is:

\[ \rho_{\ell} : \text{move}_S(\ell, \hat{\ell}) \land \text{length}(a) < k \land a' = \text{append}(a, e) \land \text{pres}(Y_S - \{a\}) \]

To ensure that a process cannot be excluded from sending to a bounded channel if it is infinitely often non-full, we associate the Compassionate justice requirement with this transition.
• **Asynchronous Receive**  
  Statement: \( \ell : \alpha \Rightarrow u; \hat{\ell} : \)  
  Transition relation:  
  \[
  \rho_{\ell} : \ move(\ell, \hat{\ell}) \land length(\alpha) > 0 \land u' = head(\alpha) \land \alpha' = tail(\alpha) \land \text{pres}(Y_S - \{u, \alpha\}).
  \]
  The asynchronous receive statement is enabled only if the channel is currently nonempty. To ensure that a process must receive something if the channel is infinitely often nonempty, the **Compassionate** fairness requirement is associated with this transition.

• **Synchronous Send-Receive**  
  With each pair of matching send and receive statements:  
  \( \ell : \alpha \leftarrow e; \hat{\ell} : m : \alpha \Rightarrow u; \hat{m} : \)  
  where two parallel statements are considered matching if they form an \( \alpha \leftarrow e\alpha \Rightarrow u \) pair for some \( e \) and \( u \) for the same synchronous channel \( \alpha \).  
  Transition relation:  
  \[
  \rho_{(\ell, m)} : \begin{align*}
  \pi_{\text{send}} &= val(\ell) \land \pi'_{\text{send}} = val(\hat{\ell}) \land \\
  \pi_{\text{receive}} &= val(m) \land \pi'_{\text{receive}} = val(\hat{m}) \land \text{pres}(Y_{\text{sendreceive}})
  \end{align*}
  \]
  Thus, transition \( \tau_{(\ell, m)} \) is enabled if control is at \( \ell \) and \( m \) simultaneously. When executed, the transition causes joint progress in the two processes containing the send and receive statement.
  
  The fairness requirement associated with the generated transitions is **Compassionate** to ensure that a process cannot wait to transmit or receive a message indefinitely if infinitely many messages are sent on the channel it is waiting on.

• **Request**  
  With the statement \( \ell : \text{request}(r); \hat{\ell} : \) we associate a transition \( \tau_{\ell} \) with the transition relation  
  \[
  \rho_{\ell} : \ move(\ell, \hat{\ell}) \land r > 0 \land r' = r - 1 \land \text{pres}(Y_S - \{r\}).
  \]
  Thus, this statement is enabled when control is at \( \ell \) and \( r \) is positive. When executed, it decrements \( r \) by 1.
  
  To model fair scheduling of semaphores (in the form of for instance single or multi-level feedback queues), the transition associated with request is **Compassionate**.

• **Release**  
  With the statement \( \ell : \text{release}(r); \hat{\ell} : \) we associate a transition \( \tau_{\ell} \) with the transition relation  
  \[
  \rho_{\ell} : \ move(\ell, \hat{\ell}) \land r' = r + 1 \land \text{pres}(Y_S - \{r\}).
  \]
  This statement increments \( r \) by 1.
A.3. SPL SEMANTICS

- **Noncritical**
  With the statement $\ell$: noncritical; $\hat{\ell}$ we associate a transition $\tau_\ell \Gamma$ with transition relation
  \[
  \rho_\ell : \text{move}(\ell, \hat{\ell}) \land \text{pres}(Y_S).
  \]
  Thus the only observable action of this statement is to terminate. The situation that execution of the noncritical section does not terminate is modeled by a computation that does not take transition $\tau_\ell$. This is allowed by excluding $\tau_\ell$ from the justice set.

- **Critical**
  With the statement $\ell$: critical; $\hat{\ell}$ we associate a transition $\tau_\ell \Gamma$ with transition relation
  \[
  \rho_\ell : \text{move}(\ell, \hat{\ell}) \land \text{pres}(Y_S).
  \]
  The only observable action of the critical statement is to terminate.

- **Produce**
  With the statement $\ell$: produce $x$; $\hat{\ell}$ we associate a transition $\tau_\ell \Gamma$ with transition relation
  \[
  \rho_\ell : \text{move}(\ell, \hat{\ell}) \land x' \neq 0 \land \text{pres}(Y_S - \{x\}).
  \]
  The observable action of the produce statement is to assign a nonzero value to variable $x$. Note that this transition is nondeterministic; that is, a state $s$ may have more than one $\tau_\ell$-successors. In fact it may have infinitely many successors—one for each possible value of $x'$.

- **Consume**
  With the statement $\ell$: consume $y$; $\hat{\ell}$ we associate a transition $\tau_\ell \Gamma$ with transition relation
  \[
  \rho_\ell : \text{move}(\ell, \hat{\ell}) \land y' = 0 \land \text{pres}(Y_S - \{y\}).
  \]
  The observable action of the consume statement is to set variable $y$ to zero.

- **Guarded Assignment**
  Statement: $\ell$: guard $c$ do $\bar{u} := \bar{v}; \hat{\ell}$.
  Transition relation:
  \[
  \rho_\ell : c \land \text{move}_S(\ell, \hat{\ell}) \land \bar{u}' = \bar{v} \land \text{pres}(Y_S - \{\bar{u}\}).
  \]
  The assignment is enabled if control is at the assignment statement and $c$ evaluates to true. Run-time type errors are not considered in the enabling condition of the transition. As with standard assignment the possibility of run-time errors can be checked with the Check Runtime System option under the Properties menu.

**Composite Statements**

Standard control constructs and primitives for concurrency compose simpler program statements into more elaborate statements. A composite statement is in general of the form $\ell[S_1,...,S_n] \Gamma$ where $S_1,...,S_n$ are the immediate substatements. The transitions associated with a composite statement comprise of the union of the transitions associated with
\(S_1, \ldots, S_n\) and the transitions associated with the construct \(C\). Below we describe the transitions associated with each construct. Here \(Y_S\) refers to all system variables except the control and location variables changed by each construct.

- **Conditional**
  - **One-way** With the statement \(k: [\text{if } c \text{ then } \tilde{e}:S]\); \(\tilde{e}:\Gamma\) we associate a transition \(\tau_1\Gamma\) with the transition relation
    \[
    \rho_{\tilde{e}}: \left( c \land \text{move}(\ell, \tilde{e}) \right) \lor \left( \neg c \land \text{move}(\ell, \tilde{e}) \right) \land \text{pres}(Y_S).
    \]
    According to \(\rho_{\tilde{e}}\Gamma\) when \(c\) evaluates to \text{true} control moves from \(\ell\) to \(\tilde{e}\) and when \(c\) evaluates to \text{false} control moves from \(\ell\) to \(\tilde{e}\).
  - **Two-way** With the statement \(k: [\text{if } c \text{ then } \tilde{e}_1:S_1 \text{ else } \tilde{e}_2:S_2]\); \(\tilde{e}:\Gamma\) we associate a transition \(\tau_1\Gamma\) with the transition relation
    \[
    \rho_{\tilde{e}}: \left( c \land \text{move}(\ell, \tilde{e}_1) \right) \lor \left( \neg c \land \text{move}(\ell, \tilde{e}_2) \right) \land \text{pres}(Y_S).
    \]
    According to \(\rho_{\tilde{e}}\Gamma\) when \(c\) evaluates to \text{true} control moves from \(\ell\) to \(\tilde{e}_1\) and when \(c\) evaluates to \text{false} control moves from \(\ell\) to \(\tilde{e}_2\).

- **While**
  With the statement \(k: [\text{while } c \text{ do } \tilde{e}:S]\); \(\tilde{e}:\Gamma\) we associate a transition \(\tau_1\Gamma\) with the transition relation
    \[
    \rho_{\tilde{e}}: \left( c \land \text{move}(\ell, \tilde{e}) \right) \lor \left( \neg c \land \text{move}(\ell, \tilde{e}) \right) \land \text{pres}(Y_S).
    \]
    According to \(\rho_{\tilde{e}}\Gamma\) when \(c\) evaluates to \text{true} control moves from \(\ell\) to \(\tilde{e}\) and when \(c\) evaluates to \text{false} control moves from \(\ell\) to \(\tilde{e}\). Note that in this context the post-location of \(\tilde{S}\) is \(\ell\). Note also that the enabling condition of \(\tau_1\) is \(c\Gamma\) which does not depend on the value of \(c\).

- **Loop forever**
  With the statement \(k: [\text{loop forever do } \tilde{e}:S]\); \(\tilde{e}:\Gamma\) we do not associate any transitions. However the locations \(\ell\Gamma\) and post-location of \(\tilde{S}\) are equivalent.

- **Repeat until**
  With the statement \(k: [\text{repeat } \tilde{e}_1:S \text{ until } c]; \tilde{e}_2\): we associate a transition \(\tau_1\Gamma\) with the transition relation
    \[
    \rho_{\tilde{e}_1}: \left( c \land \text{move}(\tilde{e}_1, \tilde{e}_2) \right) \lor \left( \neg c \land \text{move}(\tilde{e}_1, \ell) \right) \land \text{pres}(Y_S).
    \]
According to $\rho(c, \Gamma)$ when $c$ evaluates to true control moves from $\hat{l}_1$ to $\hat{l}_2 \Gamma$ and when $c$ evaluates to false control moves from $\hat{l}_1$ to $\ell$. The enabling condition of $\tau_{\hat{l}_1}$ is at $\hat{l}_1 \Gamma$ which does not depend on the value of $c$. Note that the locations $\ell$ and $\hat{\ell}$ are equivalent.

- **When**
  
  With the statement $\ell$: [when $c$ do [\ell; S]]; $\hat{\ell}$: we associate a transition $\tau_{\ell} \Gamma$ with the transition relation
  \[
  \rho_{\ell} : c \land \text{move}(\ell, \hat{\ell}) \land \text{pres}(Y_S).
  \]
  According to $\rho_{\ell} \Gamma$ when $c$ evaluates to true control moves from $\ell$ to $\hat{\ell}$.

- **Or**
  
  No special transition is associated with the statement $S_1$ or $S_2$. When control resides at this statement $\Gamma$ it is interpreted to reside simultaneously at the starting points of $S_1$ and $S_2$.

- **Cooperation**
  
  With the statement $\ell$: [$\ell_1; S_1; \hat{l}_1][\ell_2; S_2; \hat{l}_2][\ldots][\ell_n; S_n; \hat{l}_n]$ $\hat{\ell}$: we associate two transitions $\tau_{\ell_{\text{Entr}}} \Gamma$ and $\tau_{\ell_{\text{Exit}}} \Gamma$ with transition relations:
  \[
  \rho_{\ell_{\text{Entr}}} : \text{move}(\ell, \ell_1, \ell_2, \ldots, \ell_n) \land \text{pres}(Y_S).
  \]
  \[
  \rho_{\ell_{\text{Exit}}} : \text{move}(\ell_1, \ell_2, \ldots, \ell_n, \hat{\ell}) \land \text{pres}(Y_S).
  \]
  All they do is to move control from the guarding location $\ell$ into the prelocation of each parallel statement $\Gamma$ and from the post-location of each parallel statement to the post-location $\hat{\ell}$ of the entire statement.

  *Note:* makes a special case for parallel composition at the top-level of the program. They do not get a designated pre- and post-location $\Gamma$ but the initial state is instead what corresponds to $\ell_1, \ell_2, \ldots, \ell_n$ in the schema.

- **Concatenation**
  
  No special transition is associated with sequential composition of statements:
  \[
  S_1; S_2
  \]
  Since the post-location of $S_1$ is the pre-location of $S_2$ the transitions generated from each substatement $S_1$ and $S_2$ will move control appropriately from $S_1$ to $S_2$.

- **Parameterized Cooperation**
  
  With the statement $\ell$: $i : [1..N], [\ell[i]; S[i]; \hat{l}[i]]; \hat{\ell}$: we associate two transitions $\Gamma$ $\tau_{\ell_{\text{Entr}}} \Gamma$ and $\tau_{\ell_{\text{Exit}}} \Gamma$ with transition relations:
  \[
  \rho_{\ell_{\text{Entr}}} : \text{move}(\ell, \ell[i] \mid i = 1, \ldots, N) \land \text{pres}(Y_S).
  \]
  \[
  \rho_{\ell_{\text{Exit}}} : \text{move}(\ell[i] \mid i = 1, \ldots, N, \hat{\ell}) \land \text{pres}(Y_S).
  \]
  They are the parameterized versions of the transitions associated with non-parameterized cooperation.
• **Parameterized Or**
  No special transition is associated with parameterized selection. This reflects that the construct is the parameterized version of standard or.

• **Blocks**
  Blocks are used to encapsulate variable declarations and don’t generate transitions on their own.

**Grouped Statements**

To compose several simple statements into one atomic entity, SPL provides the grouping construct $\langle\langle S_1; S_2 \rangle\rangle$. For instance, the transition associated with a grouped statement of the form $\langle\langle S_1; S_2 \rangle\rangle$ is the relational composition of their transition relations.
Appendix B

Linear-Time Temporal Logic

As a requirement specification language for reactive systems we take linear-time temporal logic. There is an underlying first-order assertion language \( L \) over interpreted symbols for expressing functions and relations over concrete domains such as the integers, arrays, and lists of integers (see Section 2.1).

We refer to a formula in the assertion language \( L \) as a state formula or simply as an assertion. A temporal formula is constructed out of state formulas by applying the boolean operators \( \neg \) and \( \lor \) (the other boolean operators can be defined from these) and temporal operators. There are two classes of temporal operators, future and past. The future and past temporal operators used in state are presented in Tables B.1 and B.2 respectively.

<table>
<thead>
<tr>
<th>Operator</th>
<th>Name</th>
<th>Step representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \square p )</td>
<td>Henceforth ( p )</td>
<td>( \square )</td>
</tr>
<tr>
<td>( \diamond p )</td>
<td>Eventually ( p )</td>
<td>( &lt;&gt; )</td>
</tr>
<tr>
<td>( p U q )</td>
<td>( p ) Until ( q )</td>
<td>Until</td>
</tr>
<tr>
<td>( p W q )</td>
<td>( p ) Waiting-for (Unless) ( q )</td>
<td>Awaits</td>
</tr>
<tr>
<td>( \circ p )</td>
<td>Next ( p )</td>
<td>( )</td>
</tr>
</tbody>
</table>

Table B.1: Future Temporal Operators

<table>
<thead>
<tr>
<th>Operator</th>
<th>Name</th>
<th>Step representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \square p )</td>
<td>So-far ( p )</td>
<td>([-] )</td>
</tr>
<tr>
<td>( \diamond p )</td>
<td>Once ( p )</td>
<td>( &lt;&gt; )</td>
</tr>
<tr>
<td>( p S q )</td>
<td>( p ) Since ( q )</td>
<td>Since</td>
</tr>
<tr>
<td>( p B q )</td>
<td>( p ) Back-to ( q )</td>
<td>Backto</td>
</tr>
<tr>
<td>( \diamond q )</td>
<td>Previously ( p )</td>
<td>(( - ))</td>
</tr>
</tbody>
</table>

Table B.2: Past Temporal Operators

A model for a temporal formula \( p \) is an infinite sequence of states \( \sigma : s_0, s_1, \ldots \), where each state \( s_j \) provides an interpretation for the vocabulary of \( p \) i.e. the variables occurring in \( p \). For a given state \( s_j \) and state formula \( q \) we write \( s_j \models q \) when \( q \) is true when evaluated over \( s_j \). Given a model \( \sigma \) we now present an inductive definition for the notion of a temporal formula \( p \) holding at a position \( j \geq 0 \) in \( \sigma \) written as \((\sigma, j) \models p\):
For a state formula $p\Gamma$

$(\sigma, j) \models p$ $\iff$ $s_j \models p$

That is we evaluate $p$ locally using the interpretation provided by $s_j$.

- $(\sigma, j) \models \neg p$ $\iff$ $(\sigma, j) \not\models p$
- $(\sigma, j) \models p \lor q$ $\iff$ $(\sigma, j) \models p$ or $(\sigma, j) \models q$
- $(\sigma, j) \models \Box p$ $\iff$ for all $k \geq j\Gamma(\sigma, k) \models p$
- $(\sigma, j) \models \Diamond p$ $\iff$ for some $k \geq j\Gamma(\sigma, k) \models p$
- $(\sigma, j) \models p U q$ $\iff$ for some $k \geq j\Gamma(\sigma, k) \models q\Gamma$
  and $(\sigma, i) \models p$ for every $i$ such that $j \leq i < k$
- $(\sigma, j) \models p W q$ $\iff$ $(\sigma, j) \models p U q$ or $(\sigma, j) \models \Box p$
- $(\sigma, j) \models \Box p$ $\iff$ $(\sigma, j + 1) \models p$

For past operators we have:

- $(\sigma, j) \models \Box p$ $\iff$ for all $k\Gamma 0 \leq k \leq j\Gamma(\sigma, k) \models p$
- $(\sigma, j) \models \Diamond p$ $\iff$ for some $k\Gamma 0 \leq k \leq j\Gamma(\sigma, k) \models p$
- $(\sigma, j) \models p S q$ $\iff$ for some $k\Gamma 0 \leq k \leq j\Gamma(\sigma, k) \models p$
  and for every $i$ such that $k < i \leq j\Gamma(\sigma, i) \models p$
- $(\sigma, j) \models p B q$ $\iff$ $(\sigma, j) \models p S q$ or $(\sigma, j) \models \Box p$
- $(\sigma, j) \models \Diamond p$ $\iff$ $j > 0$ and $(\sigma, j - 1) \models p$

Another useful derived operator is the entailment operator defined by: $p \Rightarrow q \iff \Box(p \Rightarrow q)$.

For a state formula $p$ and a state $s$ such that $p$ holds on $s\Gamma$ we say that $s$ is a $p$-state. A state formula that holds on all states is called state-valid.

For a temporal formula $p$ and a position $j \geq 0$ such that $(\sigma, j) \models p\Gamma$ we say that $j$ is a $p$-position (in $\sigma$). If $(\sigma, 0) \models p\Gamma$ we say that $p$ holds on $\sigma\Gamma$ and write it as $\sigma \models p$. A formula $p$ is called satisfiable if it holds on some model. A formula is called (temporally) valid if it holds on all models.

Two formulas $p$ and $q$ are defined to be equivalent written as $p \sim q$ if the formula $p \iff q$ is valid $\Gamma$ i.e. $\Gamma \sigma \models p \iff \sigma \models q \Gamma$ for all models $\sigma$. We adopt the convention by which a formula $p$ that is claimed to be valid is state-valid if $p$ is an assertion and is temporally valid if $p$ contains at least one temporal operator.

The formulas $p$ and $q$ are defined to be congruent written as $p \cong q$ if the formula $\Box(p \cong q)$ is valid $\Gamma$ i.e. $\Gamma(\sigma, j) \models p \iff (\sigma, j) \models q \Gamma$ for all models $\sigma$ and all positions $j \geq 0$. If $p \cong q$ then $p$ can be replaced by $q$ in any context $\Gamma$ i.e. $\Gamma \varphi(p) \cong \varphi(q)$ for any formula $\varphi(p)$ containing occurrences of $p$.

The notion of (temporal) validity requires that the formula holds over all models. Given a program $P\Gamma$ we can restrict our attention to the set of models which correspond to computations of $P\Gamma$ i.e. $\Gamma Comp(P)$. This leads to the notion of $P$-validity $\Gamma$ by which a temporal formula $p$ is $P$-valid (valid over program $P$) if it holds over all the computations of $P$. Obviously any formula that is temporally valid is also $P$-valid for any program $P$. In a similar way we obtain the notions of $P$-satisfiability and $P$-equivalence.

A state $s$ that appears in some computation of $P$ is called a $P$-accessible state. A state formula is called $P$-state valid if it holds over all $P$-accessible states. Obviously any state formula that is state-valid is also $P$-state valid for any program $P$.

Again we refer to a $P$-state valid formula simply as $P$-valid.
Appendix C

Differences with the Book

step was implemented based on [Manna and Pnueli1995]. However due to implementation considerations and limitations there are some differences between step and [Manna and Pnueli1995] hereafter referred to as MP95. For the convenience of those using step together with MP95 this appendix highlights most of the differences. These are deviations from the syntax of programs as well as differences in specification style.

C.1 SPL Programs

step closely follows the SPL syntax described in MP95. However some constructs had to be translated into an ASCII equivalent. To ease transcribing programs from the book Table C.1 presents each statement as it appears in MP95 and its representation in step. In the table $u$ refers to an arbitrary variable $\bar{u}$ refers to a list of variables (i.e. $\langle u_1, \ldots, u_n \rangle$) $\bar{e}$ refers to an arbitrary expression $\bar{z}$ refers to a list of expressions $z$ refers to an arbitrary channel and $S$ refers to an arbitrary statement.

Table C.2 presents the various types of expressions as they appear in MP95 and their representation in step. Again $\bar{e}$ is an arbitrary expression $\bar{u}$ is an arbitrary variable $\bar{z}$ refers to a list of expressions.

The various types allowed in step are presented in Table C.3. In this table $\bar{e}$ is an arbitrary expression $\bar{u}$ is an arbitrary variable $\bar{b}$ is an arbitrary base type $T$ is an arbitrary base or simple type and $T$ is an arbitrary type (i.e. $\langle \text{base} \langle \text{simple or complex} \rangle \rangle$). A range in step is written $e_1..e_2$.

C.2 Control Locations in step

As in MP95 step uses control variables to indicate where the current program control resides. However MP95 uses a single control variable $\pi$ whose value is the set of locations where control currently resides. step on the other hand uses a set of control counters where each control counter has its own subscript. We illustrate the differences between MP95 and step with an example.

Consider the program schema presented in Figure C.1 which lists the value of the control variable $\pi$ (in italics) as it is used in MP95. In the figure $S_i$ refers to any simple
<table>
<thead>
<tr>
<th>Class</th>
<th>In MP95</th>
<th>In step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td>skip</td>
<td>skip</td>
<td>skip</td>
</tr>
<tr>
<td></td>
<td>$a := e$</td>
<td>$a := e$</td>
<td>single assignment</td>
</tr>
<tr>
<td></td>
<td>$(\pi) := (\tau)$</td>
<td>$(\pi) := (\tau)$</td>
<td>multiple assignment</td>
</tr>
<tr>
<td></td>
<td><code>await e</code></td>
<td><code>await e</code></td>
<td><code>await e</code></td>
</tr>
<tr>
<td></td>
<td>$z \Rightarrow e$</td>
<td>$z \Leftarrow e$</td>
<td>send</td>
</tr>
<tr>
<td></td>
<td>$z \Rightarrow a$</td>
<td>$z \Leftarrow a$</td>
<td>receive</td>
</tr>
<tr>
<td></td>
<td><code>request r</code></td>
<td><code>request r</code></td>
<td>request</td>
</tr>
<tr>
<td></td>
<td><code>release r</code></td>
<td><code>release r</code></td>
<td>release</td>
</tr>
<tr>
<td>Schematic</td>
<td>noncritical</td>
<td>noncritical</td>
<td>noncritical</td>
</tr>
<tr>
<td></td>
<td>critical</td>
<td>critical</td>
<td>critical</td>
</tr>
<tr>
<td></td>
<td>produce $a$</td>
<td>produce $a$</td>
<td>produce</td>
</tr>
<tr>
<td></td>
<td>consume $a$</td>
<td>consume $a$</td>
<td>consume</td>
</tr>
<tr>
<td></td>
<td>choose $a$</td>
<td>choose $a$</td>
<td>choose</td>
</tr>
<tr>
<td>Compound</td>
<td>if $e$ then $S_1$</td>
<td>if $e$ then $S_1$</td>
<td>1-branch cond.</td>
</tr>
<tr>
<td></td>
<td>if $e$ then $S_1$ else $S_2$</td>
<td>if $e$ then $S_1$ else $S_2$</td>
<td>2-branch cond.</td>
</tr>
<tr>
<td></td>
<td>$S_1; S_2$</td>
<td>$S_1; S_2$</td>
<td>concatenation</td>
</tr>
<tr>
<td></td>
<td>when $e$ do $S$</td>
<td>when $e$ do $S$</td>
<td>when</td>
</tr>
<tr>
<td></td>
<td>$S_1$ or $S_2$</td>
<td>$S_1$ or $S_2$</td>
<td>selection</td>
</tr>
<tr>
<td></td>
<td>while $e$ do $S$</td>
<td>while $e$ do $S$</td>
<td>while loop</td>
</tr>
<tr>
<td></td>
<td>repeat $S$ until $e$</td>
<td>repeat $S$ until $e$</td>
<td>repeat</td>
</tr>
<tr>
<td></td>
<td>loop forever do $S$</td>
<td>loop forever do $S$</td>
<td>loop forever</td>
</tr>
<tr>
<td></td>
<td>$S_1</td>
<td></td>
<td>S_2$</td>
</tr>
<tr>
<td></td>
<td>[local decl; $S$]</td>
<td>[local decl; $S$]</td>
<td>block</td>
</tr>
<tr>
<td></td>
<td>$\bigoplus_{j=1}^M P[j] :: S[j]$</td>
<td>$\bigoplus_{j=1}^M P[j] :: S[j]$</td>
<td>param. selection</td>
</tr>
<tr>
<td></td>
<td>$\bigoplus_{j=1}^M S[j]$</td>
<td>$\bigoplus_{j=1}^M S[j]$</td>
<td>param. selection</td>
</tr>
<tr>
<td></td>
<td>$\bigoplus_{j=1}^M P[j] :: S[j]$</td>
<td>$\bigoplus_{j=1}^M P[j] :: S[j]$</td>
<td>param. cooperation</td>
</tr>
<tr>
<td></td>
<td>$\bigoplus_{j=1}^M S[j]$</td>
<td>$\bigoplus_{j=1}^M S[j]$</td>
<td>param. cooperation</td>
</tr>
<tr>
<td>Grouped</td>
<td>$(S)$</td>
<td>$\ll S \gg$</td>
<td>group</td>
</tr>
</tbody>
</table>

Table C.1: The statements of spl.
<table>
<thead>
<tr>
<th>In MP95</th>
<th>In step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>T</strong></td>
<td><code>true</code></td>
<td><code>true</code></td>
</tr>
<tr>
<td><strong>F</strong></td>
<td><code>false</code></td>
<td><code>false</code></td>
</tr>
<tr>
<td><code>integer</code></td>
<td><code>integer</code></td>
<td><code>integer value</code></td>
</tr>
<tr>
<td><code>u</code></td>
<td><code>u</code></td>
<td><code>variable</code></td>
</tr>
<tr>
<td><code>u[e]</code></td>
<td><code>u[e]</code></td>
<td><code>array access</code></td>
</tr>
<tr>
<td><code>(e)</code></td>
<td><code>(e)</code></td>
<td><code>parenthesized expression</code></td>
</tr>
<tr>
<td><code>(r)</code></td>
<td><code>(r)</code></td>
<td><code>tuple of expressions</code></td>
</tr>
<tr>
<td><code>e_1 = e_2</code></td>
<td><code>e_1 = e_2</code></td>
<td><code>equal</code></td>
</tr>
<tr>
<td><code>e_1 ≠ e_2</code></td>
<td><code>e_1 != e_2</code></td>
<td><code>not equal</code></td>
</tr>
<tr>
<td><code>e_1 &lt; e_2</code></td>
<td><code>e_1 &lt; e_2</code></td>
<td><code>less than</code></td>
</tr>
<tr>
<td><code>e_1 &gt; e_2</code></td>
<td><code>e_1 &gt; e_2</code></td>
<td><code>greater than</code></td>
</tr>
<tr>
<td><code>e_1 ≤ e_2</code></td>
<td><code>e_1 &lt;= e_2</code></td>
<td><code>less than or equal to</code></td>
</tr>
<tr>
<td><code>e_1 ≥ e_2</code></td>
<td><code>e_1 &gt;= e_2</code></td>
<td><code>greater than or equal to</code></td>
</tr>
<tr>
<td><code>-e</code></td>
<td><code>-e</code></td>
<td><code>unary minus</code></td>
</tr>
<tr>
<td><code>e_1 mod e_2</code></td>
<td><code>e_1 mod e_2</code></td>
<td><code>modulo</code></td>
</tr>
<tr>
<td><code>e_1 div e_2</code></td>
<td><code>e_1 div e_2</code></td>
<td><code>integer division</code></td>
</tr>
<tr>
<td><code>e_1 + e_2</code></td>
<td><code>e_1 + e_2</code></td>
<td><code>addition</code></td>
</tr>
<tr>
<td><code>e_1 - e_2</code></td>
<td><code>e_1 - e_2</code></td>
<td><code>subtraction</code></td>
</tr>
<tr>
<td><code>e_1 * e_2</code></td>
<td><code>e_1 * e_2</code></td>
<td><code>multiplication</code></td>
</tr>
<tr>
<td><code>e_1 / e_2</code></td>
<td><code>e_1 / e_2</code></td>
<td><code>rational division</code></td>
</tr>
<tr>
<td>[ \prod_{i=e_1}^{e_2} e ]</td>
<td><code>Prod i:[e_1..e_2].e</code></td>
<td><code>product</code></td>
</tr>
<tr>
<td>[ \sum_{i=e_1}^{e_2} e ]</td>
<td><code>Sum i:[e_1..e_2].e</code></td>
<td><code>summation</code></td>
</tr>
<tr>
<td><code>¬e</code></td>
<td><code>¬e</code></td>
<td><code>negation</code></td>
</tr>
<tr>
<td><code>¬e</code></td>
<td><code>!e</code></td>
<td><code>negation</code></td>
</tr>
<tr>
<td><code>e_1 \land e_2</code></td>
<td><code>e_1 \land e_2</code></td>
<td><code>conjunction</code></td>
</tr>
<tr>
<td><code>e_1 \lor e_2</code></td>
<td><code>e_1 \lor e_2</code></td>
<td><code>disjunction</code></td>
</tr>
<tr>
<td><code>e_1 \rightarrow e_2</code></td>
<td><code>e_1 \rightarrow e_2</code></td>
<td><code>implication</code></td>
</tr>
<tr>
<td><code>e_1 \leftrightarrow e_2</code></td>
<td><code>e_1 \leftrightarrow e_2</code></td>
<td><code>equivalence</code></td>
</tr>
<tr>
<td><code>∀ u. e</code></td>
<td><code>Forall u. e</code></td>
<td><code>universal quantification</code></td>
</tr>
<tr>
<td><code>∃ u. e</code></td>
<td><code>Exists u. e</code></td>
<td><code>existential quantification</code></td>
</tr>
<tr>
<td><code>∃! u. e</code></td>
<td><code>Exists! u. e</code></td>
<td><code>unique existential quantification</code></td>
</tr>
</tbody>
</table>

Table C.2: spl expressions
<table>
<thead>
<tr>
<th>Class</th>
<th>In MP95</th>
<th>In step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base type</td>
<td>integer boolean rational</td>
<td>int boolean rational</td>
<td></td>
</tr>
<tr>
<td>Simple type</td>
<td>[range] ( t_1 \times \cdots \times t_k ) ( { \text{values} } )</td>
<td>[range] ( t_1 \ast \cdots \ast t_k ) ( { \text{values} } )</td>
<td>range type tuple type enumeration type</td>
</tr>
<tr>
<td>Complex type</td>
<td>array [ranges] of ( T ) channel of b ( \text{channel} [1..] ) of b ( \text{channel} [range] ) of b ( (T) )</td>
<td>array [ranges] of ( T ) channel of b ( \text{channel} [1..] ) of b ( \text{channel} [range] ) of b ( (T) )</td>
<td>array type synchronous unbounded async. bounded async. parenthesized type</td>
</tr>
</tbody>
</table>

Table C.3: SPL types in step

or schematic statement (i.e. any statement that has no substatements). The step version of the same program schema is presented in Figure C.2 which lists all the control counters used (in italics) and their value.

The following highlights the similarities and differences between step’s interpretation of control counters and the MP95 use of the control variable \( \pi \):

1. **step:** The ordering of the labels does not affect the value of the control counters. For example if we renamed label 12 to be 125 the control counters would still have the same values. In general the program counter \( \pi-j \) associated with the process \( P_j \) starts at 0 and increases by 1 as control proceeds sequentially through the process.

MP95: The ordering of the labels does affect the value of the control variable \( \pi \). So if in the above example if we rename \( \ell_2 \) to \( \ell_{25} \) then \( \pi = \{\ell_2\} \) would change to \( \pi = \{\ell_{25}\} \).

2. **step:** A cooperation statement among \( N \) processes employs \( N \) control counters. If the cooperation is a top-level cooperation statement (i.e. no entry and exit transitions) then the program has only \( N \) control counters. This is not the case in Figure C.2 where the cooperation statement has entry and exit transitions. In this case we have two control counters \( \pi1 \) and \( \pi2 \) in addition to \( \pi0 \) whose value within the cooperation statement is constant.

MP95: There is always only one control variable called \( \pi \).
Figure C.1: Program schema illustrating MP95's use of the control variable $\pi$
P1:: [ 10: (pi0=0) S_0; 
12: (pi0=1) S_1; 
13: [(pi0=2) 
  P2:: [ m0: (pi0=3, pi1=0) S_2; 
m1: (pi0=3, pi1=1) S_3; 
m2: (pi0=3, pi1=2) ] 
] 
] 
P3:: [ k0: (pi0=3, pi2=0) S_4; 
k1: (pi0=3, pi2=1) ] 
]; 
14: [(pi0=4) 
  [ m4: (pi0=4) S_5; 
m5: (pi0=5) S_6; 
m6: (pi0=6) S_7; 
m7: (pi0=7) S_8; 
m8: (pi0=10) ] 
or 
  [ k4: (pi0=4) S_9; 
k5: (pi0=8) S_{10}; 
k6: (pi0=9) S_{11}; 
k7: (pi0=10) ] 
or 
  [ j4: (pi0=4) S_{12}; 
j5: (pi0=10) ] 
] ]; 
15: (pi0=10) S_{13}; 
16: (pi0=11) 

Figure C.2: Program schema illustrating step's use of control counters.
3. In both step and MP95 the same rules govern the notion of label equivalence.

   step: This is achieved by assuming label-equivalent locations the same control-counter values.

   MP95: This is achieved by using the notational convenience that a label $\ell_i$ really refers to its (label) equivalence class $[\ell_i]$. Consequently $\ell_4 \sim m_4 \sim k_4 \sim j_4$ and $\ell_5 \sim m_8 \sim k_7 \sim j_5$ (see page 13-14 in MP95).

**Location equivalences**

In step a **loop forever do** statement does not generate a transition so the pre-location and post-location of this statement are equivalent. However, both locations are entered as location variables in the symbol table and can thus be referred to in the specification.

### C.3 Type Declarations

In several programs MP95 uses multiple lines of declarations as follows:

```plaintext
in   x, y, z: integer
     a, b, c: bool
local d, e, f: integer
     g, h, i: bool
```

In step each line of declarations must be preceded by the mode of the variable so the above has to be entered as:

```plaintext
in   x, y, z: integer
in   a, b, c: bool
local d, e, f: integer
local g, h, i: bool
```

### C.4 Initialization

#### Array Initialization

In several programs MP95 uses the following construct to initialize an array:

```plaintext
in   M: integer where $M \geq 1$
y: array[1..M] of integer where $y = T$
```

In step this causes an error due to the incompatible types of $y$ and $T$. In step this has to be written as:

```plaintext
in   M: int where $M \geq 1$
in   x: array[1..M] of int where Forall i:[1..M] . y[i] = true
```
APPENDIX C. DIFFERENCES WITH THE BOOK

Channel Initialization

In MP95 a channel may be initialized as the empty channel with

\[ send = \lambda \]

In step this is accomplished with

\[ \text{length}(send) = 0 \]

In MP95 a channel may be initialized with values as follows:

\[ \text{ack} = [1 \ldots 1] \]

\[ \text{length}(send) /= 0 \]

\[ \text{length}(ack) = N \]

Step does not provide a construct to initialize a channel in the where clause. If the initial values matter, the channel has to be initialized explicitly within the program with \( \text{example} \) a while statement. If the initial values do not matter, which is the case for program \( \text{prod-cons-c} \) of Figure 1.18 in MP95, the channel may be initialized with the following where clause:

\[
\begin{align*}
\text{in} & \quad N : \text{int where } N > 0 \\
\text{local} & \quad \text{send, ack : channel } [1..] \text{ of int where } \text{length}(\text{ack}) = N \land \\
& \quad \text{length}(\text{send}) = 0
\end{align*}
\]

C.5 Parameterized Programs

In MP95 (page 174) terms such as \( N_3 \) are often used to denote the number of processes residing at a particular location. This abbreviation is not available in step, so there are two ways to express such formulas: In a formula of the form

\[ N_3 \leq 1 \]

(i.e., the number of processes at location 3 is less than 1) the recommended step representation is the following:

\[
\text{Forall } i : [1..N] \quad \text{Forall } j : [1..N] \quad (13[i] \land 13[j] \rightarrow i = j)
\]

This may be generalized to small numbers greater than 1 using disjunction in the consequent. When this representation is not possible, e.g., in a formula of the form

\[ N_3 + y = z \]

you can use \( \text{Sum} \) as follows:

\[
(\text{Sum } i : [1..N] \quad 13[i]) + y = z
\]

We discourage the use of \( \text{Sum} \) for the simpler cases because reasoning with \( \text{Sum} \) is more difficult than reasoning with quantifiers only.
Appendix D

Distribution and Installation

Installation

STEP is available for platforms that are supported by Standard ML of New Jersey version 1.08I including:

- Sun SPARC workstations (SunOS and Solaris)
- DEC Alpha workstations (OSF1)

You should have copied the distribution file specific to your machine. The only difference is in the binaries included with each.

Files associated with running STEP are located in the bin directory. The executable files are found in the .bin and .heap directories and accessed through the STEPpolyserv and model_checker scripts. These scripts determine the machine type and operating system using the .arch-n-opsys command and then invoke the appropriate binary file (from the .run or .bin directory) on the given arguments. Thus none of these files should be moved or renamed.

You can edit the STEP script to set the STEP_DIR environment variable once and for all.

Note: These scripts use the /bin/sh shell; if this shell is not found you should edit them and change the first line to indicate the appropriate location (actually ksh is preferred).

STEP Distribution

When the distribution has been un-tarred it will have created three directories:

- The bin directory containing binaries step, model_checker, and polyserv for executing STEP and from within it the model checker and utilities for generating polyhedral invariants.

- The doc directory. It contains:

  1. The step manual (postscript).
APPENDIX D. DISTRIBUTION AND INSTALLATION


3. The list of known problems and bugs known-bugs.txt.


5. The text of the licence agreement license-agreement.txt.

6. The WWW subdirectory which contains the help pages.

- The examples directory containing a selection of simple example programs and specifications that can be verified using step.

New releases will include updated help pages; please let us know how those pages and this manual can be improved.

D.1 Example Programs

The examples directory adheres to the default filename extensions used by the File Browser. These conventions are shown in Table D.1.

<table>
<thead>
<tr>
<th>Filename extension</th>
<th>contents of file</th>
</tr>
</thead>
<tbody>
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<td>.spl</td>
<td>SPL program</td>
</tr>
<tr>
<td>.trans</td>
<td>transition system</td>
</tr>
<tr>
<td>.cts</td>
<td>clocked transition system</td>
</tr>
<tr>
<td>.spec</td>
<td>specification</td>
</tr>
<tr>
<td>.search</td>
<td>proof search</td>
</tr>
<tr>
<td>.diagram</td>
<td>verification diagram</td>
</tr>
<tr>
<td>.tactic</td>
<td>tactic</td>
</tr>
<tr>
<td>.mclog</td>
<td>modelchecker logfile</td>
</tr>
</tbody>
</table>

Table D.1: Filename conventions

The example programs and specification files listed in Tables D.2 and D.3 are included in the distribution. They correspond to simple examples and exercises from [Manna and Pnueli 1995]. In most cases the user will have to complete the specifications for exercises in order to succeed deductive verification. Thus possible answers to the exercises are not included in the distribution. The directory mux-fisher contains a simple real-time protocol implemented with Clocked Transition Systems.
### Table D.2: Example programs and specifications

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<th>program file</th>
<th>specification files</th>
<th>other files</th>
</tr>
</thead>
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<td>ADD-TWO</td>
<td>add-two</td>
<td>yge0.spec</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>l1x=2.spec</td>
<td></td>
</tr>
<tr>
<td>ANY-NAT</td>
<td>any-nat</td>
<td>any-nat.spl</td>
<td></td>
</tr>
<tr>
<td>BINOM</td>
<td>binom</td>
<td>binom.spl</td>
<td>part-corr.spec</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>str-part-corr.spec</td>
</tr>
<tr>
<td>BINOM-C</td>
<td>binom</td>
<td>binom-c.spl</td>
<td>part-corr.spec</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>str-part-corr.spec</td>
</tr>
<tr>
<td>CUBE</td>
<td>cube</td>
<td>cube.spl</td>
<td>part-corr.spec</td>
</tr>
<tr>
<td>DIFF-INC</td>
<td>diff-inc</td>
<td>diff-inc.spl</td>
<td>psi.spec</td>
</tr>
<tr>
<td>DINE</td>
<td>dine</td>
<td>dine.spl</td>
<td></td>
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<td>dine-contr</td>
<td>dine-contr.spl</td>
<td></td>
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<td>fact.spl</td>
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<tr>
<td>INC</td>
<td>inc</td>
<td>inc.spl</td>
<td>psi.spec</td>
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<td>loop+.trans</td>
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<td>specification files</td>
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<td>--------------</td>
<td>---------------------</td>
<td>-------------</td>
</tr>
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<td>max-array.spl</td>
<td>part-corr.spec phi3.search phi4.search phi5.search</td>
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<td>mpx-sem.spl</td>
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<td>SQRT-C</td>
<td>sqrt</td>
<td>sqrt-c.spl</td>
<td></td>
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</tbody>
</table>

Table D.3: More example programs and specifications.
Appendix E

STeP Environment Variables

A number of UNIX environment variables can be set before running step. The only mandatory variables are STEP_DIR which indicates the directory where step resides and the DISPLAY environment variable which should be set to the IP-address of your screen. If these variables are not set you will not be able to run step.

To set a UNIX environment variable enter

```
setenv variable-name new-value
```
from the UNIX prompt. You can also add these commands to your .cshrc file to make them the defaults when you log in.

General variables

- **STEP_DIR**: Local step installation directory.
  Default: /local/step.

- **STEP_BROWSER**: The location of the WWW/http browser executable used for on-line help.
  Default /usr/local/bin/mosaic.

- **STEP_AUTO_SIMPLIFY**: If this flag is set to anything other than “OFF” this will cause step to start with the automatic simplification flag (see Section 4.1) ON.

- **STEP_SHOW_LOADED**: Set this to “OFF” if you do not want loaded programs and transition systems to be automatically displayed.

Verification Diagram settings

The following values can be increased when very large diagrams need to be drawn.

- **STEP_DIAGRAM_WIDTH**: Maximum width in pixels of the verification diagram canvas.
  Default: 1200.

- **STEP_DIAGRAM_HEIGHT**: Maximum height in pixels of the verification diagram canvas.
  Default: 1200.
APPENDIX E. STEP ENVIRONMENT VARIABLES

BDD-package settings
These variables control the amount of memory used by the BDD’s.

- **STEP_BDD_NODES**: Maximum number of BDD nodes that are kept around by the BDD package. After this maximum is reached, the BDD packages have to be reset for BDD-based operations to succeed.
  Default: 100000.

- **STEP_BDD_SPLIT**: Maximum number of subgoals that are generated by the BDD-split rule in the Top-level Prover and Interactive Prover. If this maximum is reached, the BDD-split rule fails. (BDD split can generate exponentially many subgoals in the worst case.)
  Default: 1000.

- **STEP_BDD_CACHE**: Size of the BDD result cache used for efficient BDD operations.
  Default: 32000.

Modelchecker and polyhedral invariant resources
The following variables limit the space and time used by the external Model Checker and polyhedral invariant packages. The **MC** variables control the Model Checker, and the **POLY** variables control the polyhedral invariant generator. Both of them will fail if any of the limits is exceeded.

- **STEP_MC_SPACE**/**STEP_POLY_SPACE**: Maximum amount of memory in Megabytes.
  Default: unlimited.

- **STEP_MC_TIME**/**STEP_POLY_TIME**: Maximum amount of user time in minutes.
  Default: unlimited.

- **STEP_MC_CPU**/**STEP_POLY_CPU**: Maximum amount of CPU time in minutes.
  Default: unlimited.

Finally, remember that the interrupt button can be used to interrupt lengthy computations in STEP.
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