

# Analyzing Private Network Data

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**Gerome Miklau**

Joint work with

**Michael Hay, Chao Li, David Jensen, Don Towsley**  
*University of Massachusetts, Amherst*

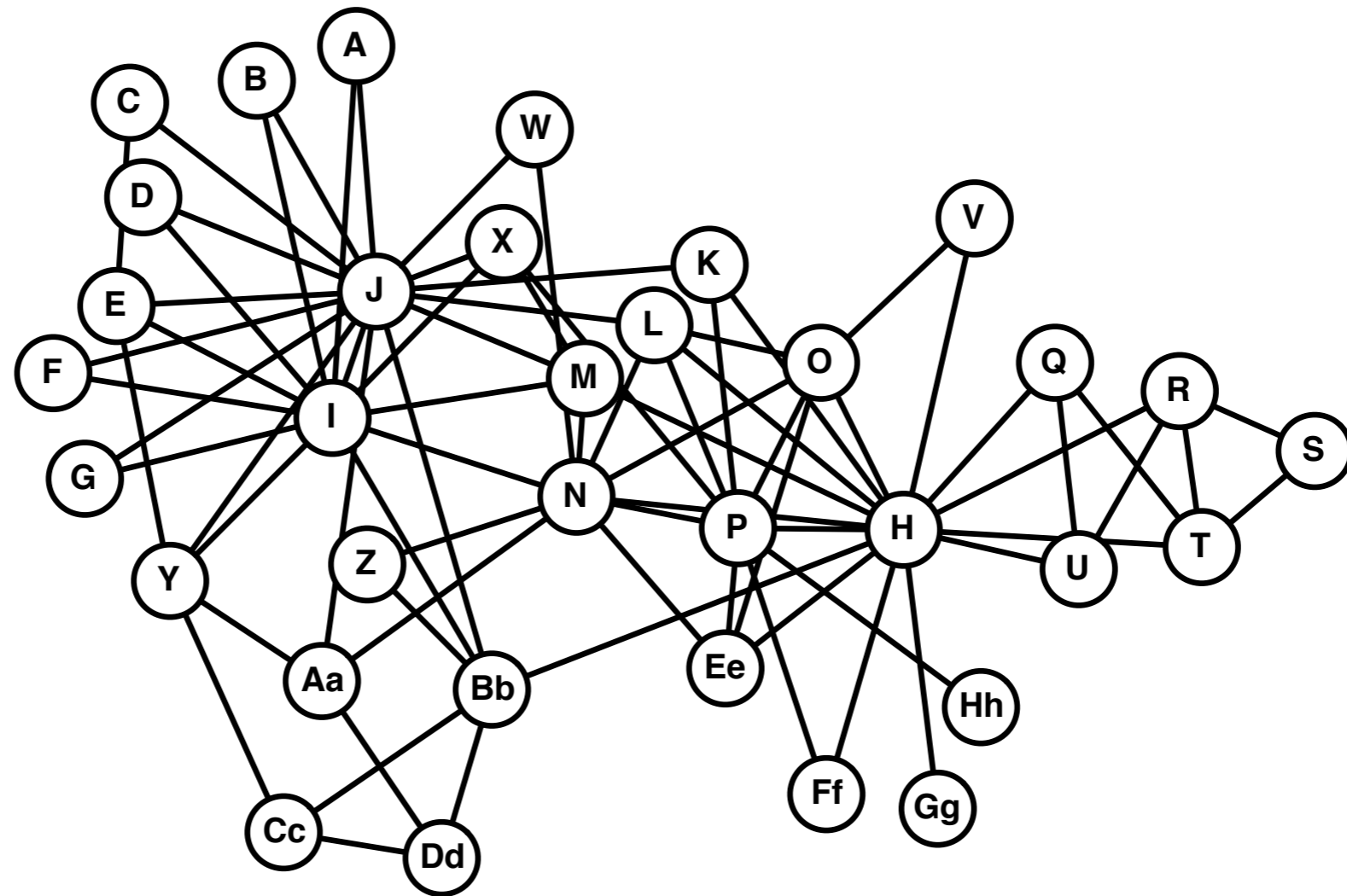
**Vibhor Rastogi, Dan Suciu**  
*University of Washington*



February 2010

# Friendship in a karate club

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## “Zachary’s Karate Club”

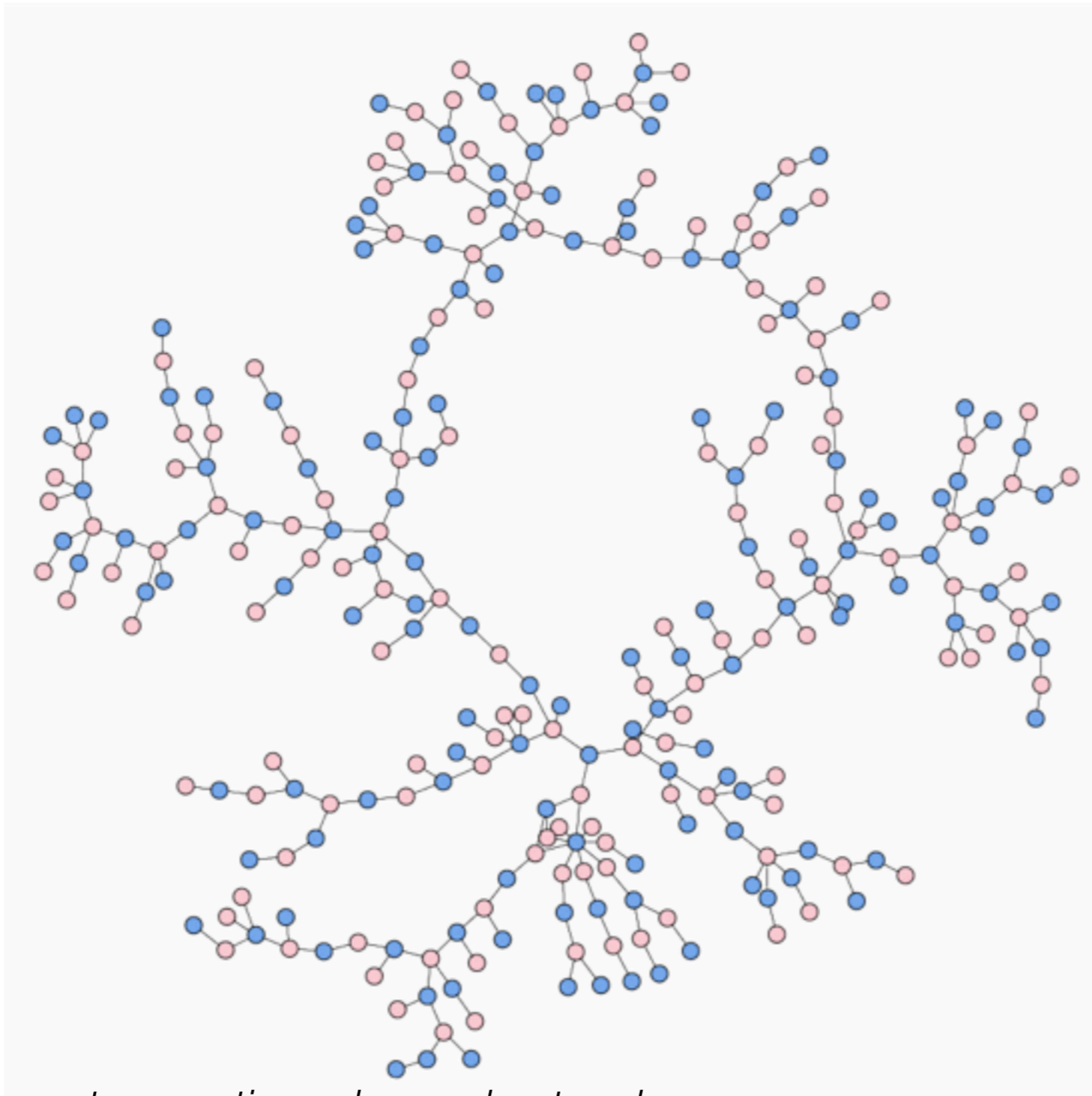
W. W. Zachary

*An information flow model for conflict and fission in small groups*

Journal of Anthropological Research, 1977

# Romantic connections in a high school

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Bearman, et al.

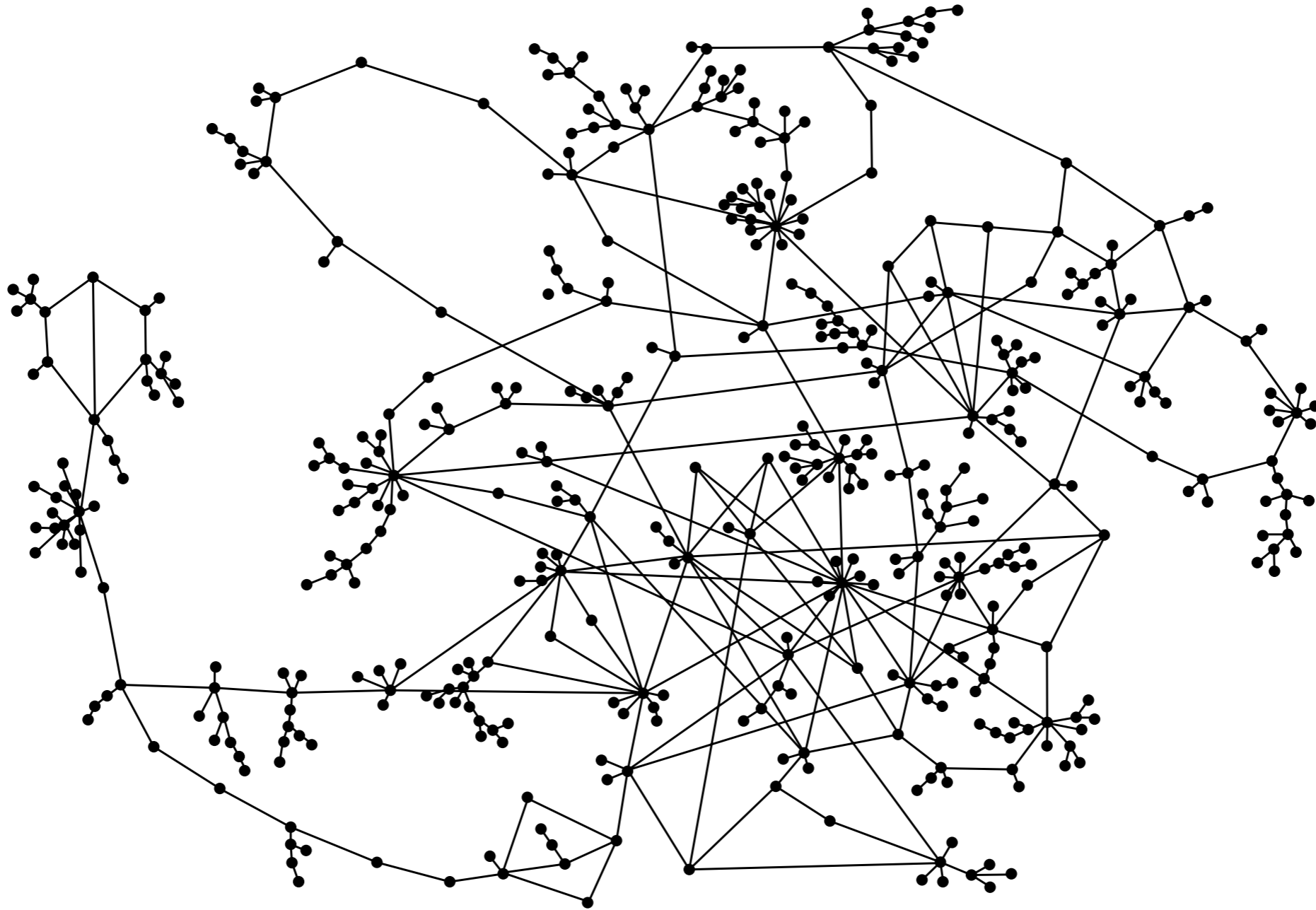
*The structure of adolescent romantic and sexual networks.*

American Journal of Sociology, 2004.

(Image drawn by Newman)

# Sexual and injecting drug partners

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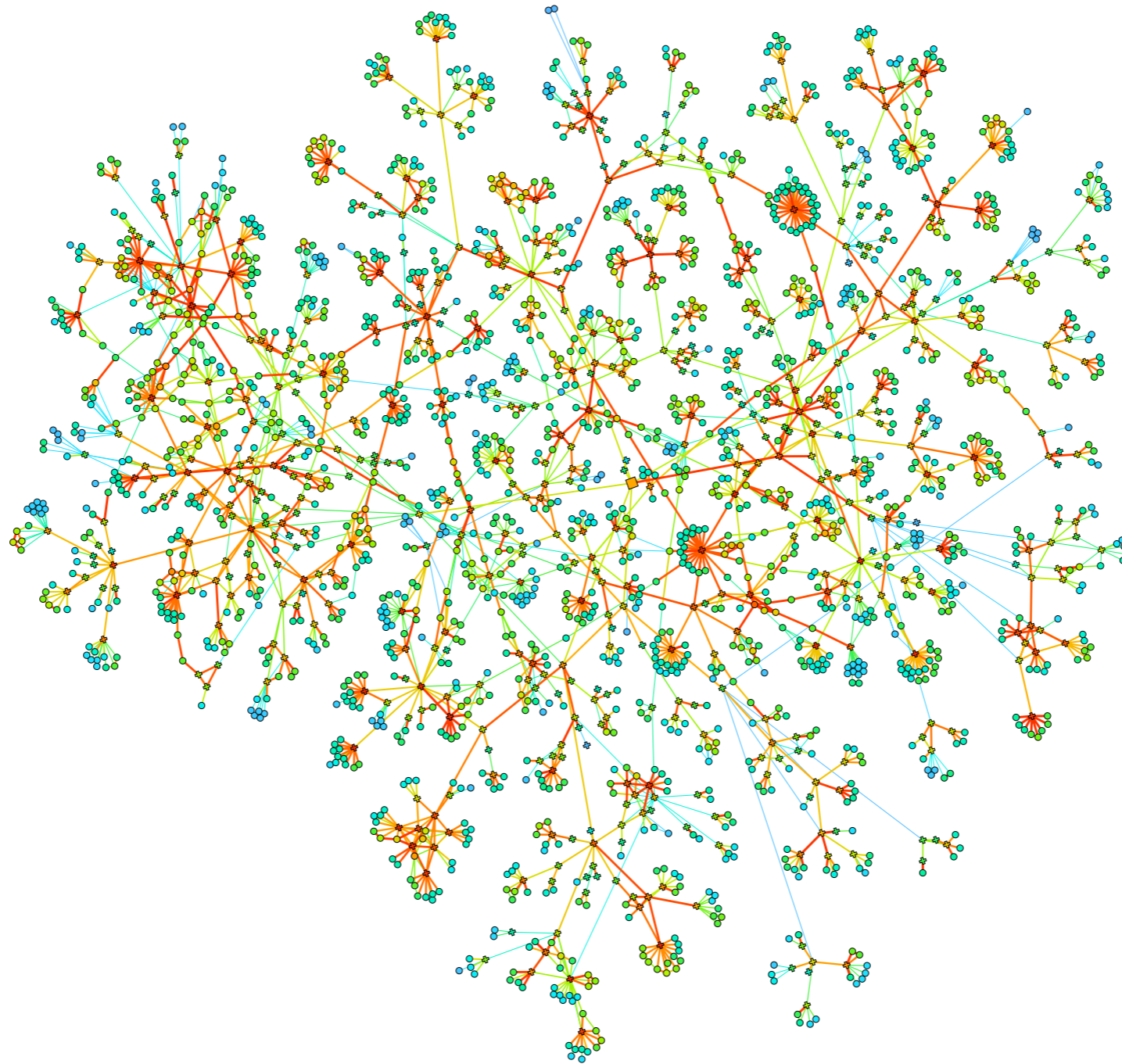


Potterat, et al.

*Risk network structure in the early epidemic phase of hiv transmission in colorado springs.*  
Sexually Transmitted Infections, 2002.

# Social ties derived from a mobile phone network

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J. Onnela et al.

*Structure and tie strengths in mobile communication networks,*  
Proceedings of the National Academy of Sciences, 2007

# Global instant messaging network

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**180 million nodes**  
**1.3 billion edges**

Leskovec, et al.

*Planetary-scale views on a large instant-messaging network.*

Conference on the World Wide Web, 2008.

# Privacy risk a major obstacle to network analysis

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## **Common outcomes include:**

- No availability
- Limited availability:
  - Only within institutions who own the data, or among limited set of researchers who have negotiated access.
- Availability, at a cost:
  - Privacy of participants may be violated, bias or inaccuracy in released data.

# Analysis of private networks

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Can we permit analysts to study networks without revealing sensitive information about participants?

Example analyses based on network topology:

- **Properties of the degree distribution**
- **Motif analysis**
- Community structure
- Processes on networks: routing, rumors, infection
- Resiliency / robustness



# Outline of the talk

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1. Existing approaches to protecting network data

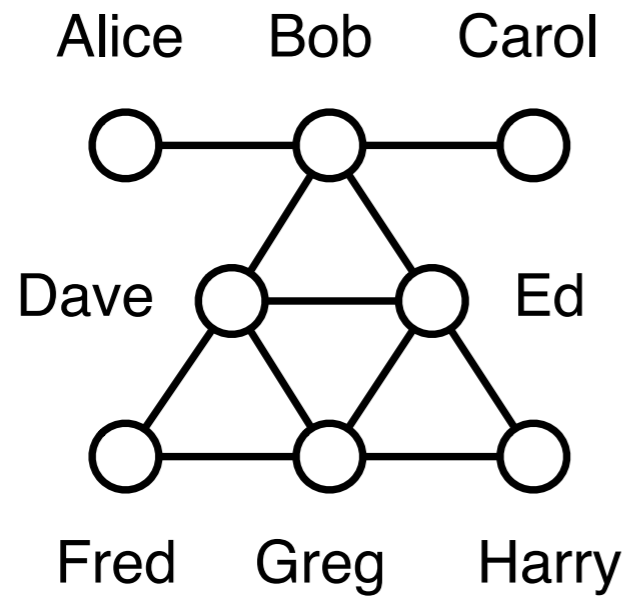
2. Background on differential privacy

3. Privately estimating the degree distribution

4. Privately counting motifs

5. Future goals and open questions

# Sensitive information in networks



## Nodes

ID	Age	HIV
Alice	25	Pos
Bob	19	Neg
Carol	34	Pos
Dave	45	Pos
Ed	32	Neg
Fred	28	Neg
Greg	54	Pos
Harry	49	Neg

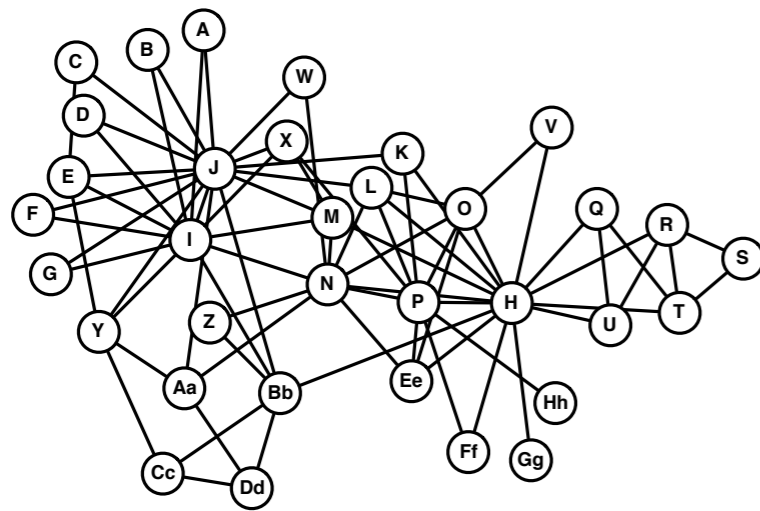
## Edges

ID1	ID2
Alice	Bob
Bob	Carol
Bob	Dave
Bob	Ed
Dave	Ed
Dave	Fred
Dave	Greg
Ed	Greg
Ed	Harry
Fred	Greg
Greg	Harry

# Naive anonymization

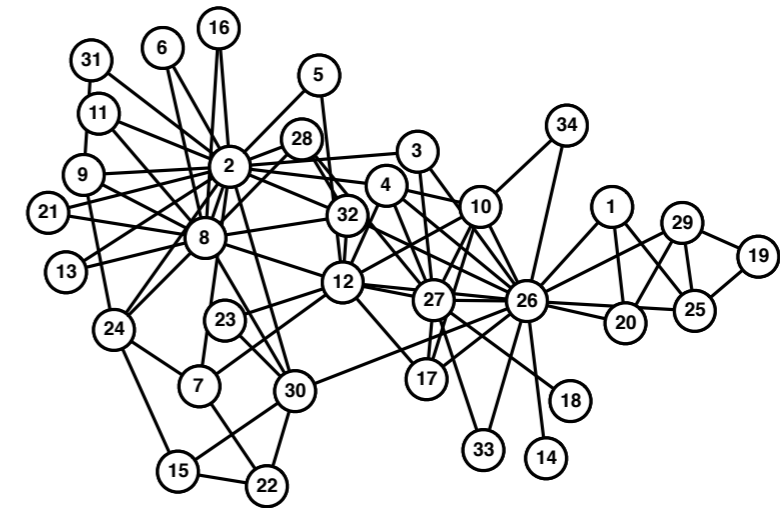
**DATA OWNER**

**ANALYST**



Original network

Naive  
Anonymization



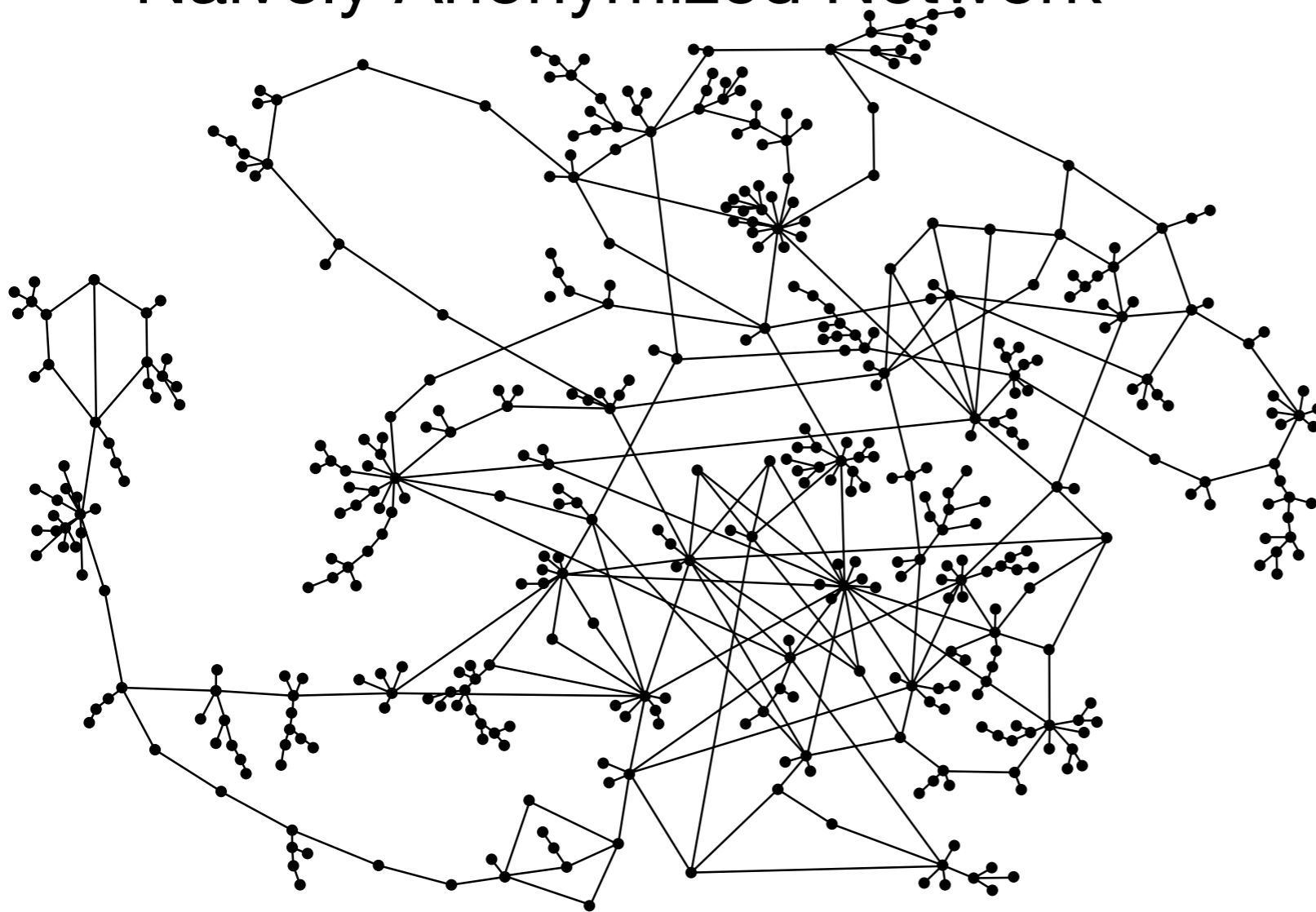
Naive anonymization

- Naive anonymization replaces identifiers with random numbers, releasing an isomorphic copy of the graph.
- Allows very accurate analysis of the topology... but not secure.

# Threat of re-identification

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## Naively Anonymized Network



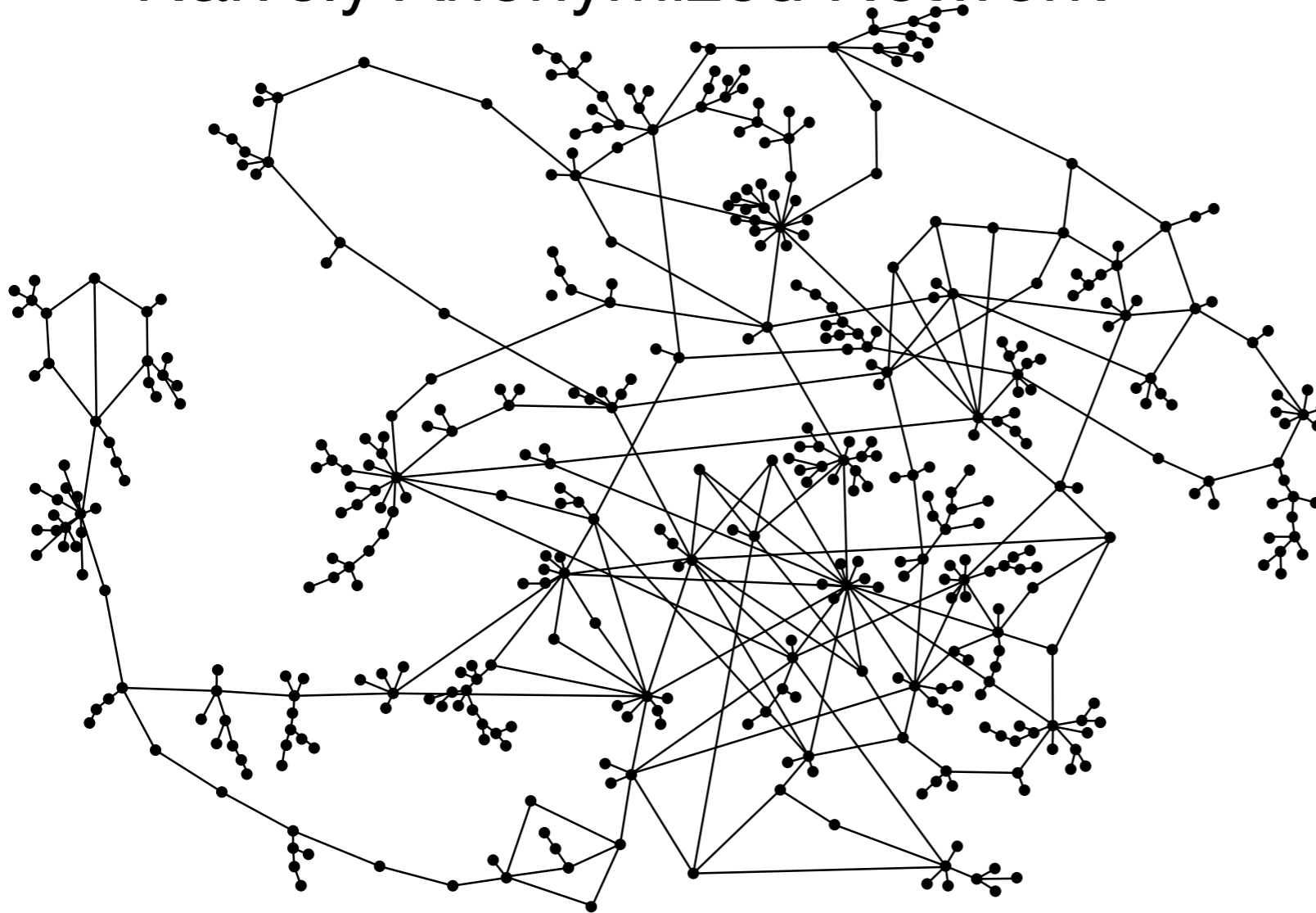
## Re-identification

Adversary acquires knowledge of network structure and uses it to re-identify individual

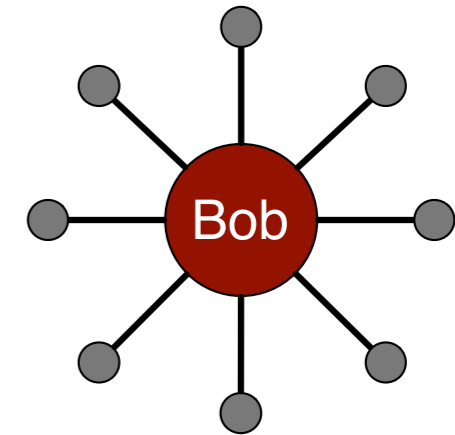
# Threat of re-identification

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## Naively Anonymized Network



## External information



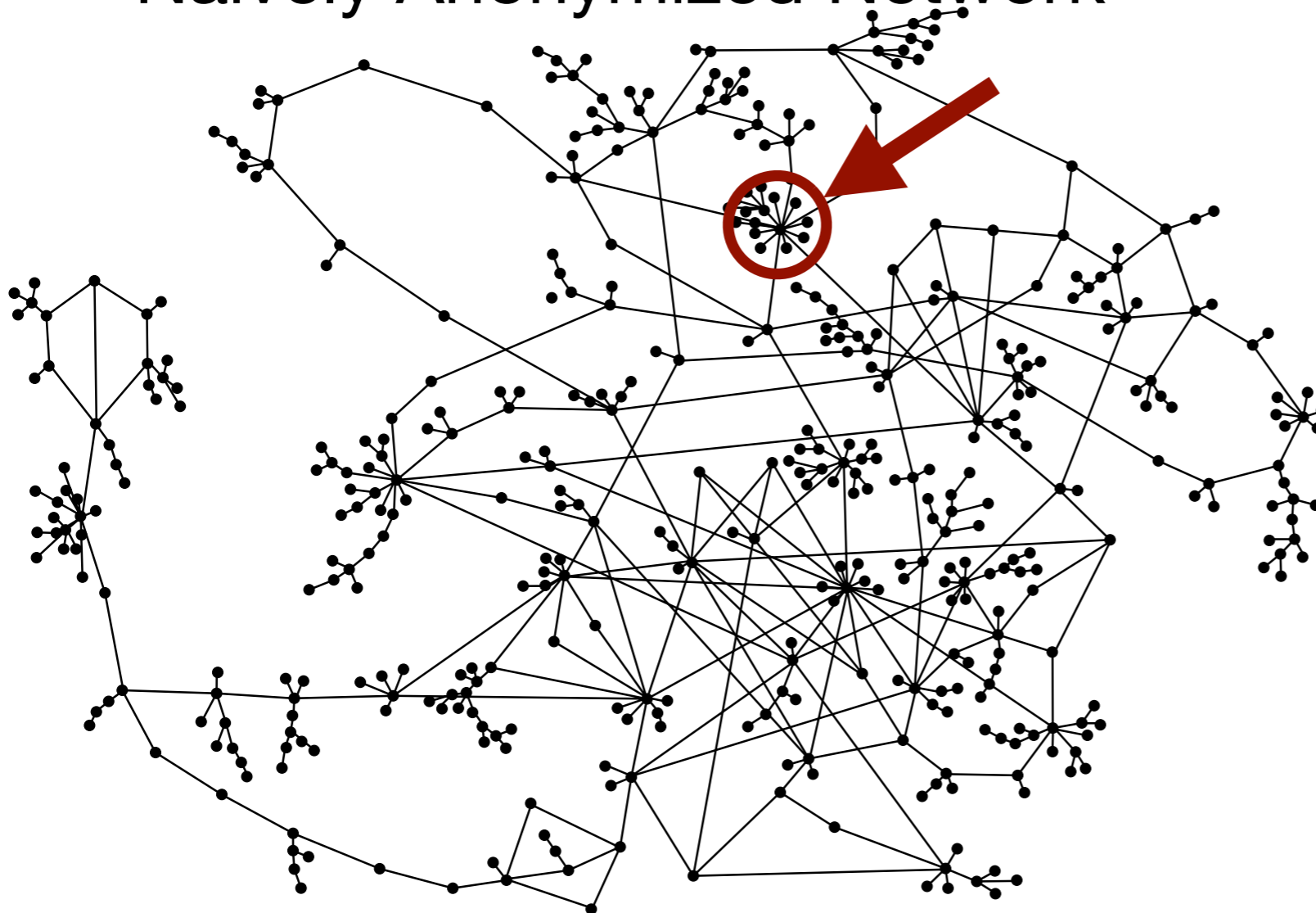
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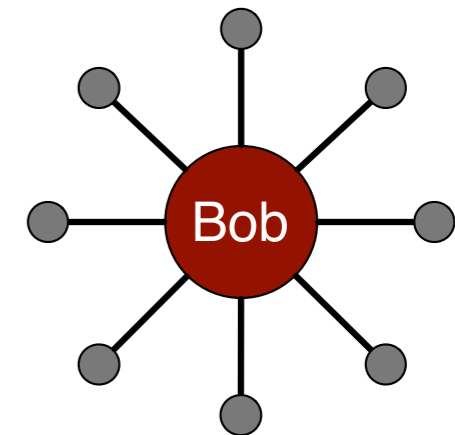
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## Naively Anonymized Network



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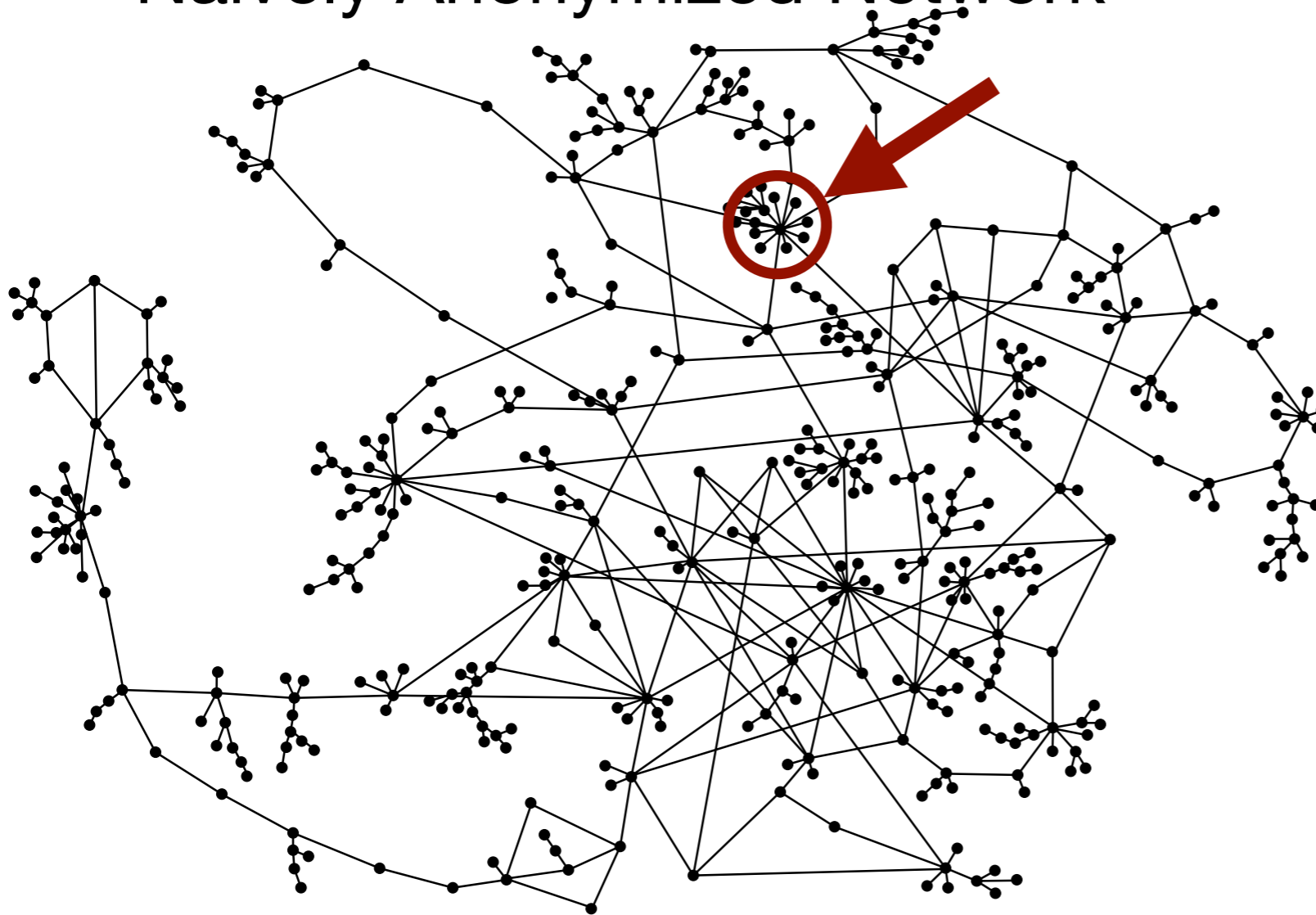
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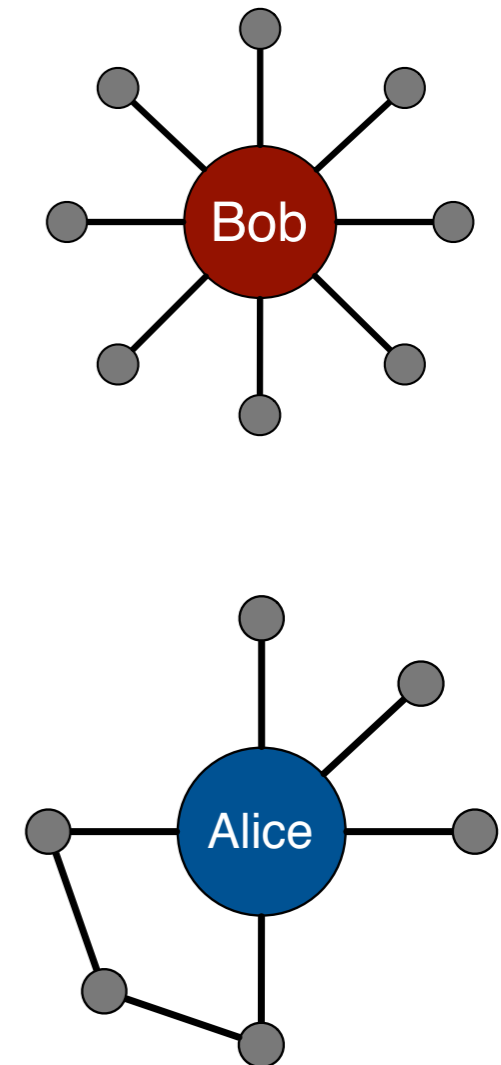
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## Naively Anonymized Network



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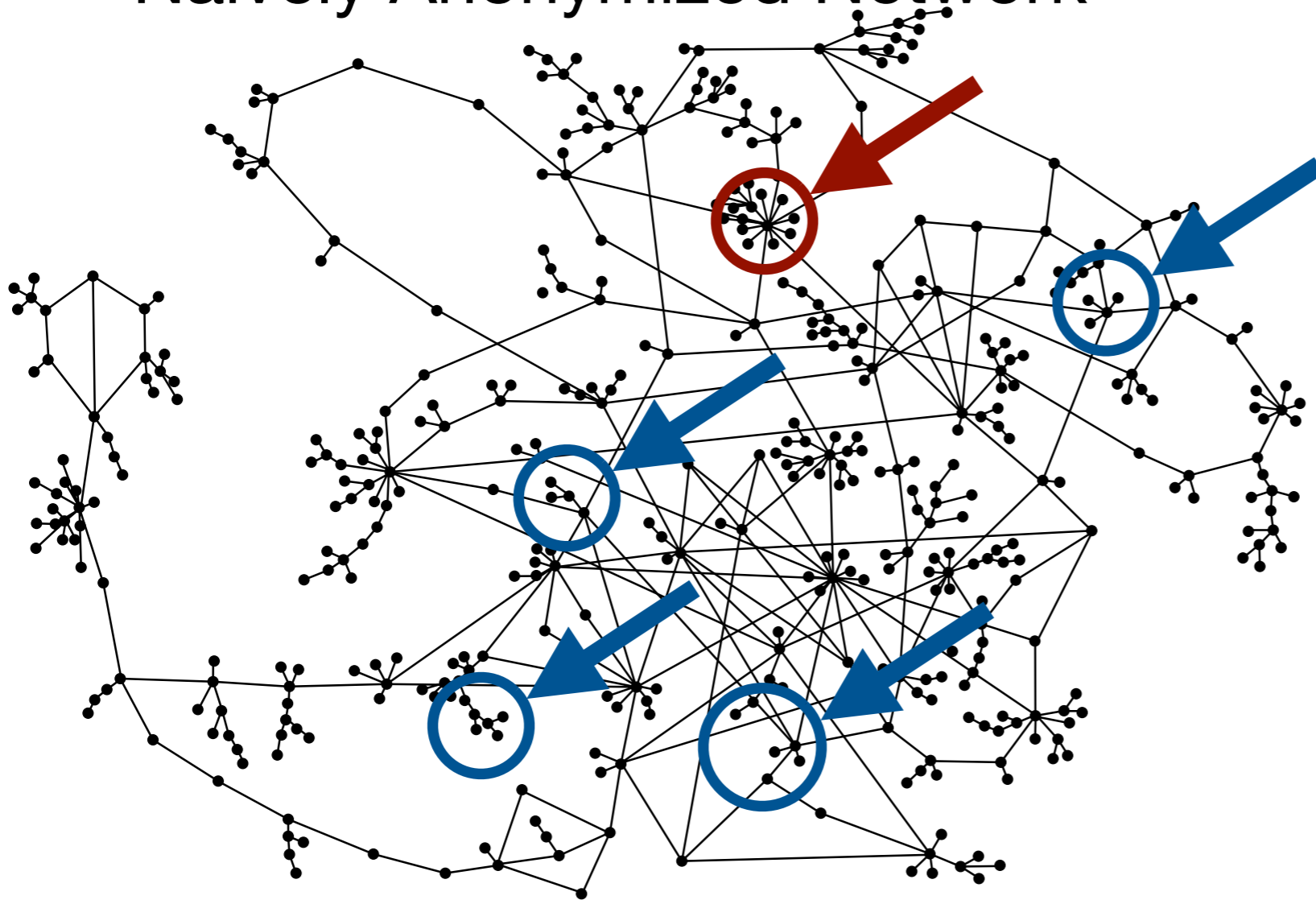


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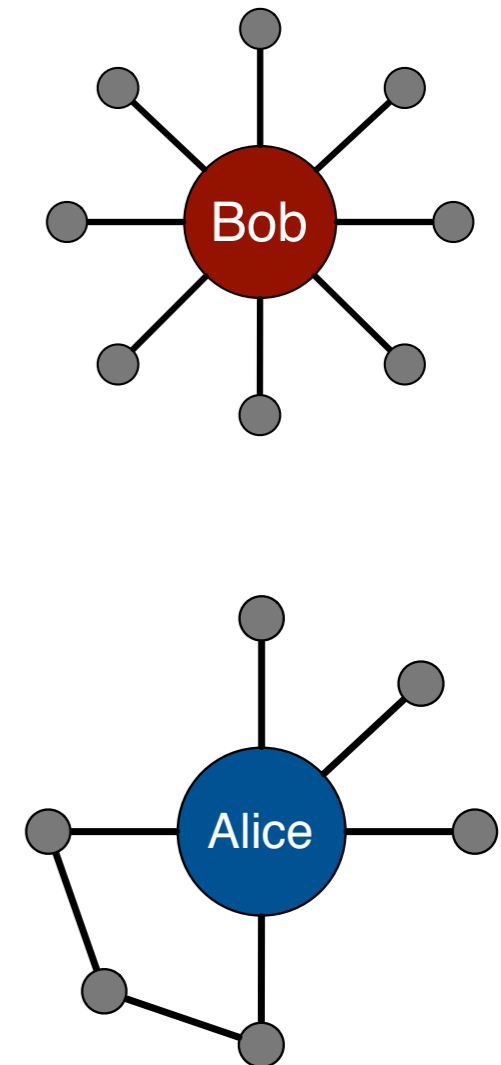
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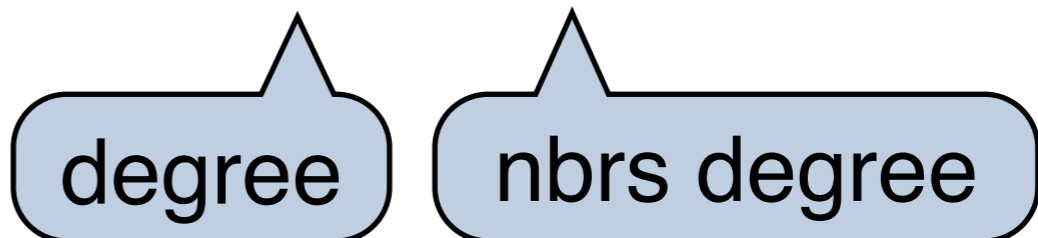
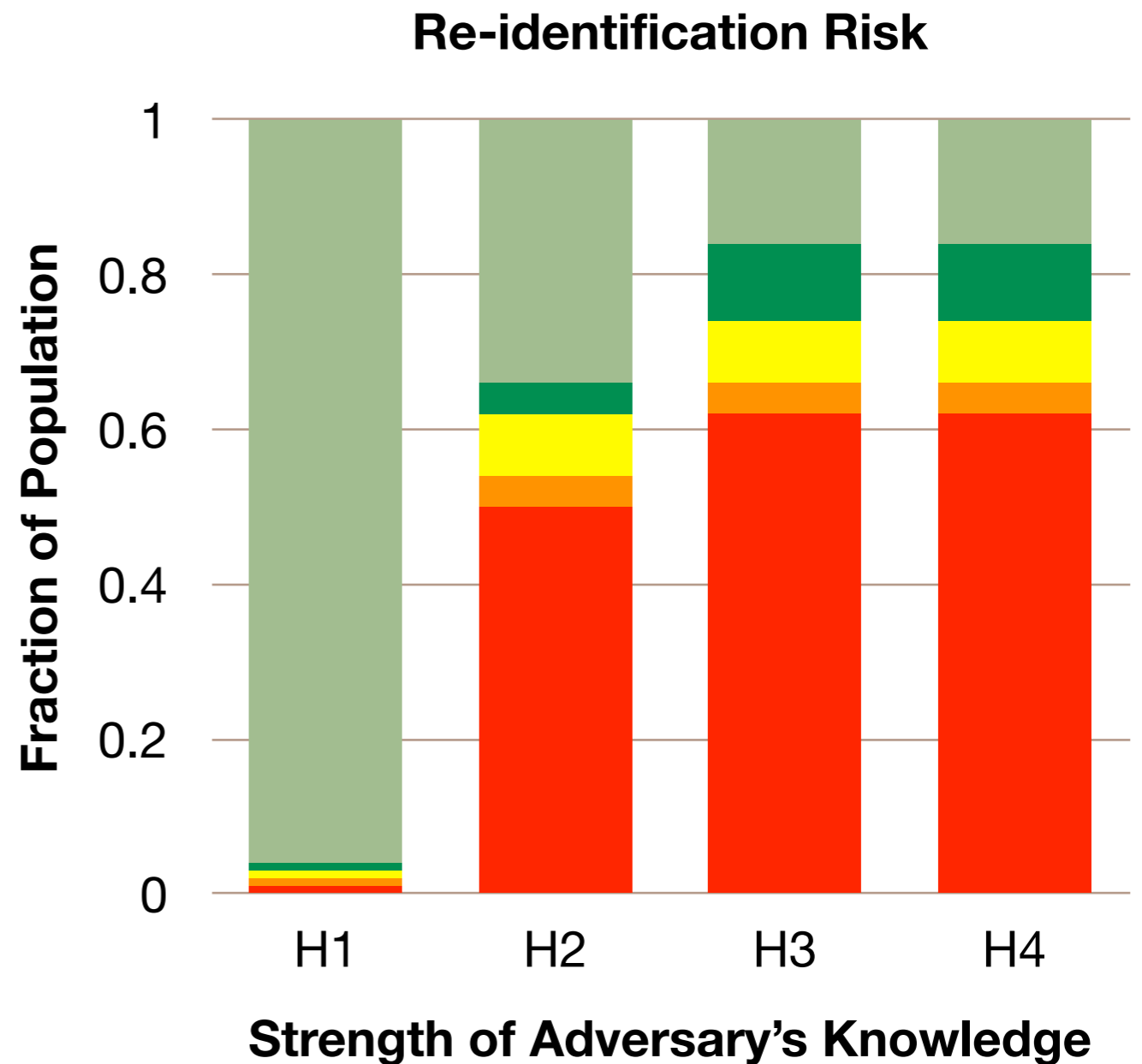
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# Local structure is highly identifying

**Friendster network**  
**~4.5 million nodes**



[Hay, VLDB 08]

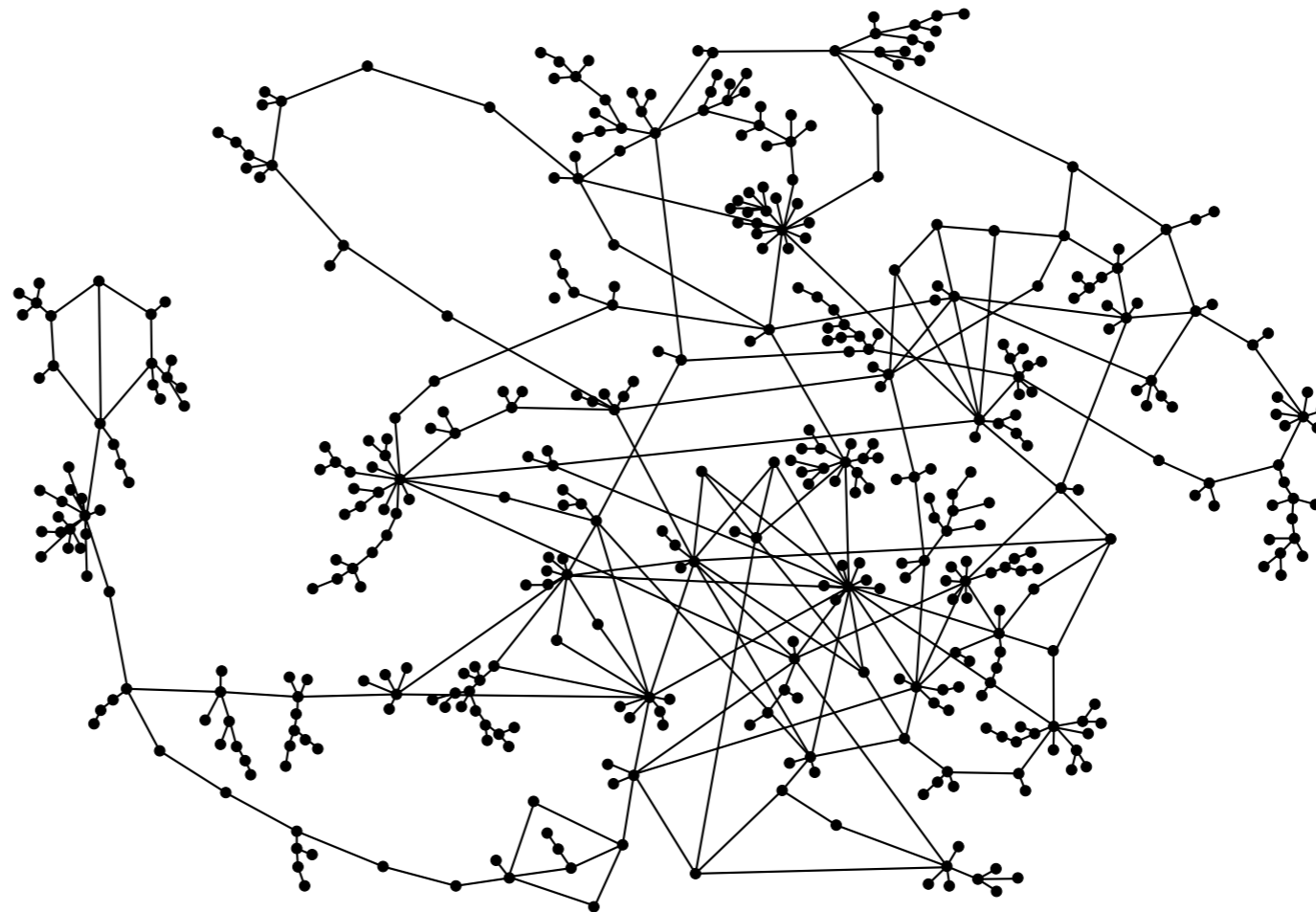
# Other attacks on naive anonymization

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## Active attack

Embed small random graph prior to anonymization.

[Backstrom,  
WWW 07]



## Auxiliary network attack

Use unanonymized public network with overlapping membership.

[Narayanan,  
OAKL 09]

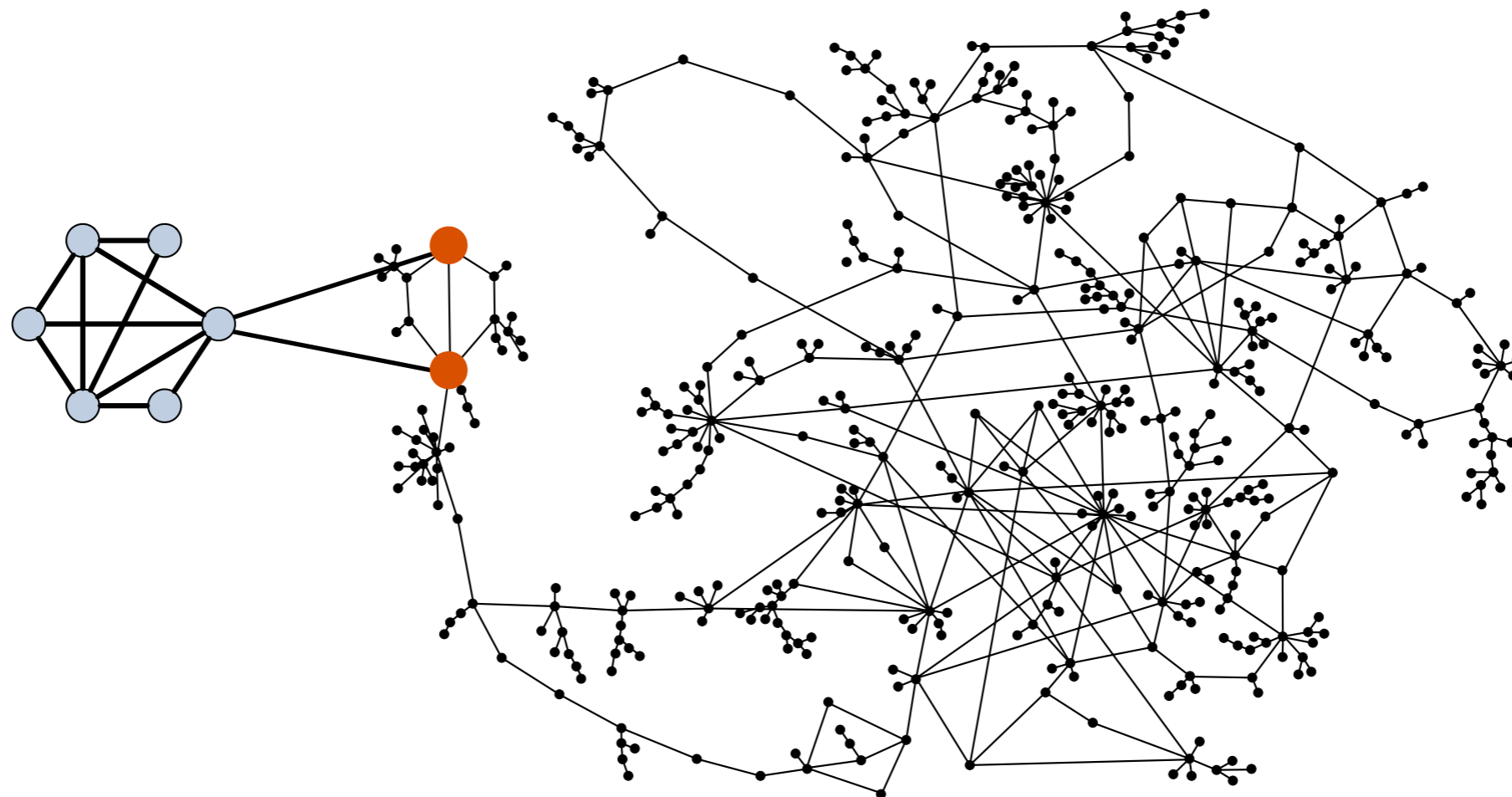
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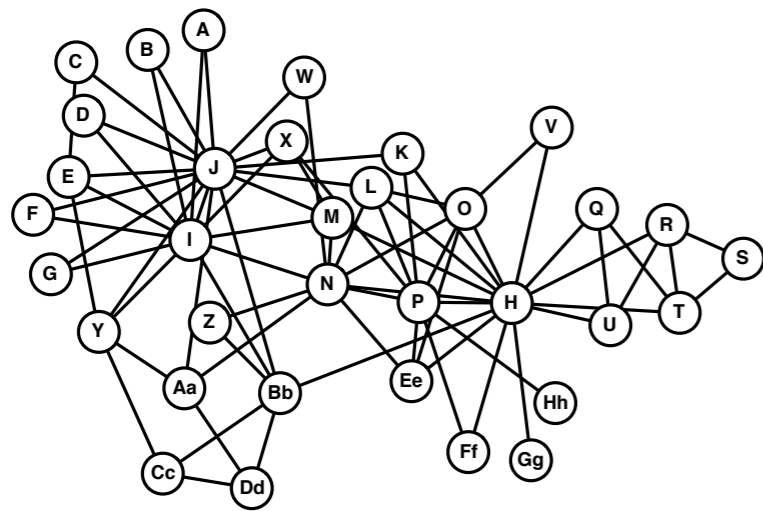
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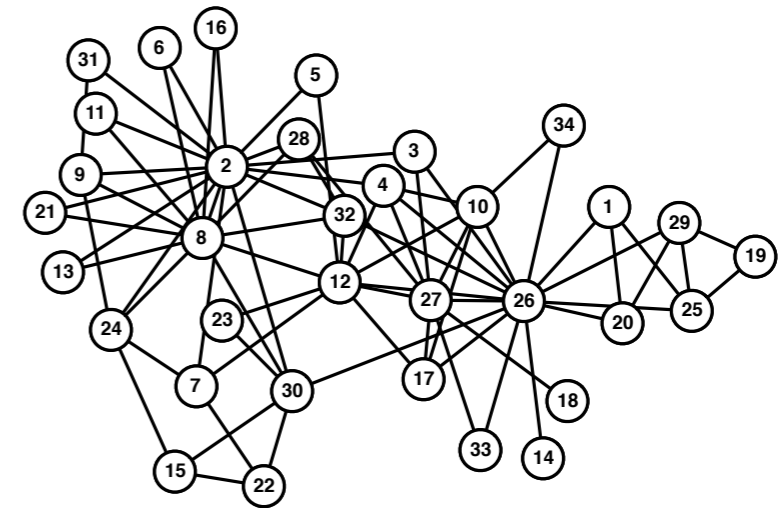
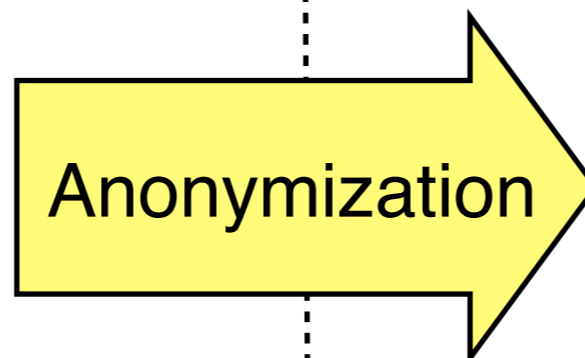
# Improved data publishing techniques

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Original network

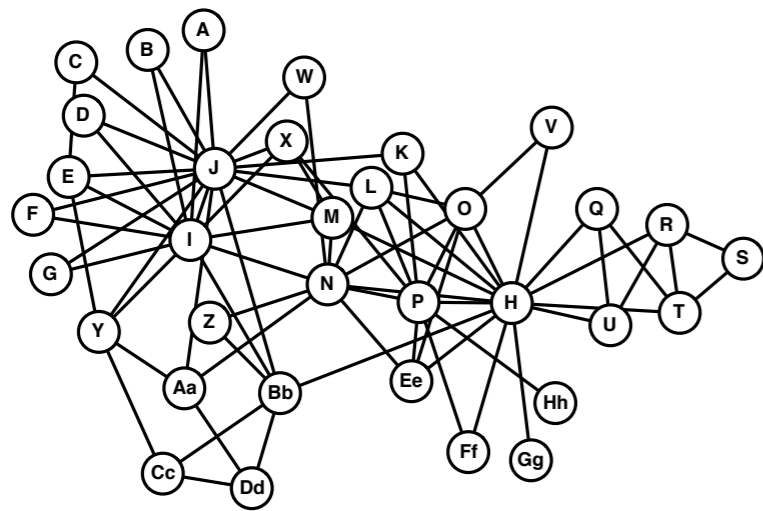


Naive anonymization

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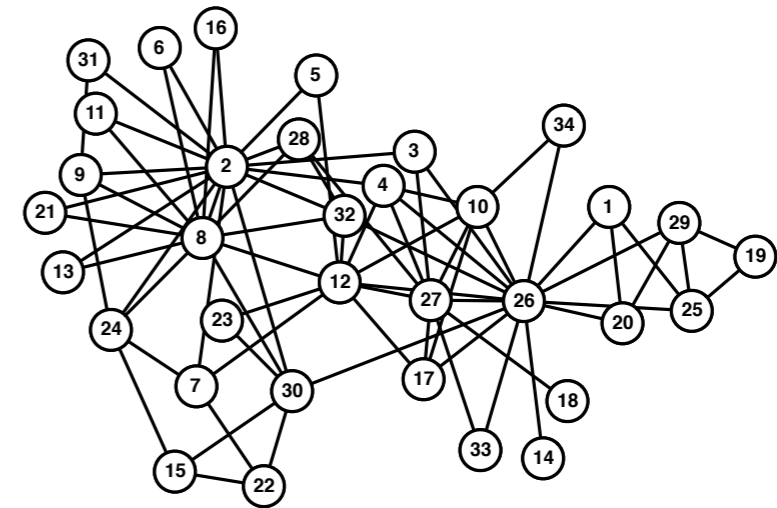
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Anonymization



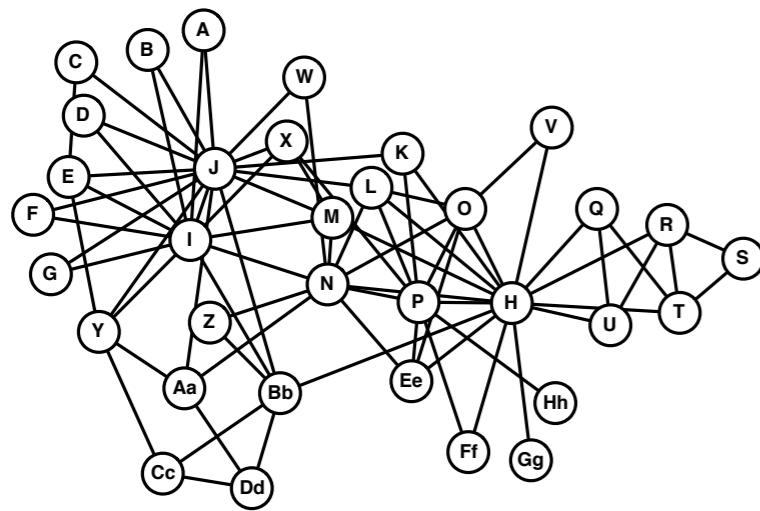
Naive anonymization

- Create topological similarity [Liu, SIGMOD 08] [Zhou, ICDE 08] [Zou, VLDB 09]

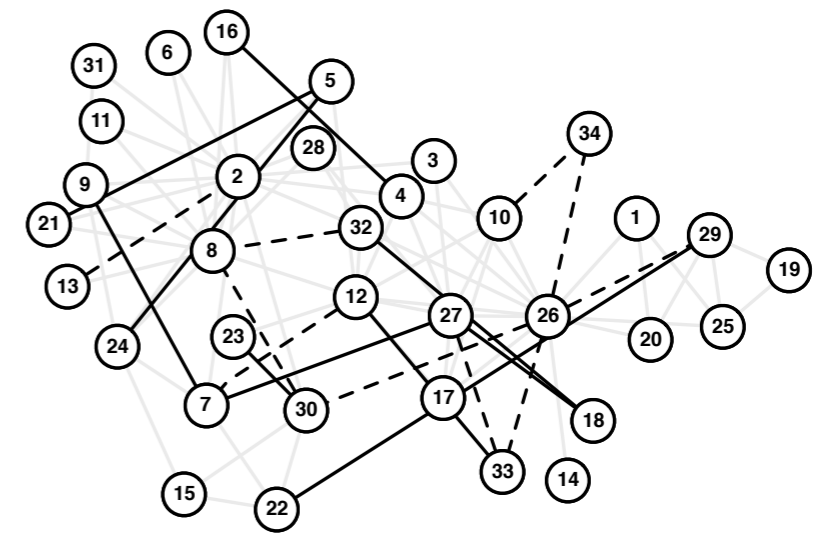
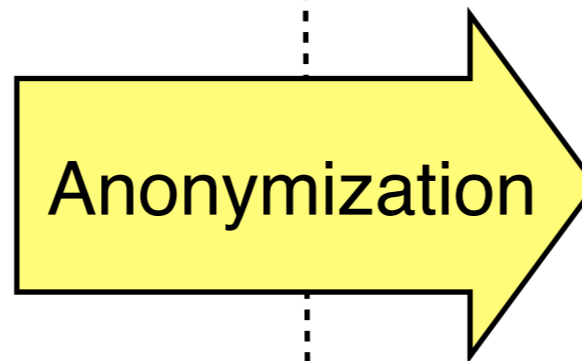
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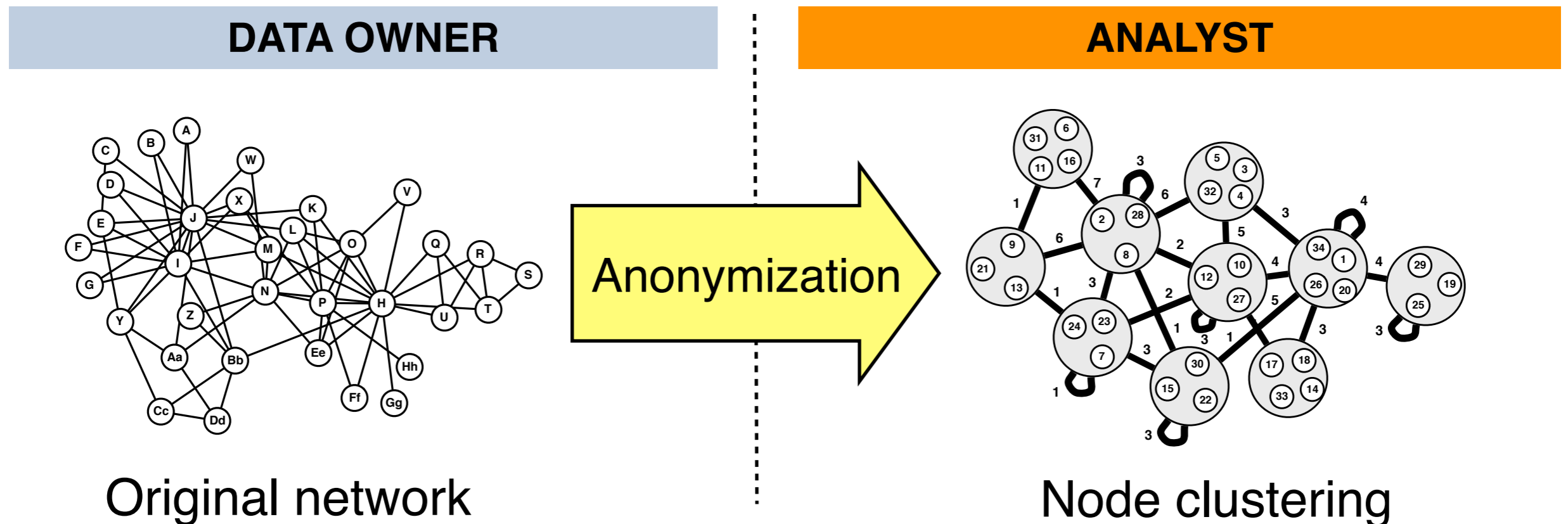
Original network



Randomized Edges

- Create topological similarity **[Liu, SIGMOD 08] [Zhou, ICDE 08] [Zou, VLDB 09]**
- Randomize edges **[Ying, SDM 2008]**

# Improved data publishing techniques

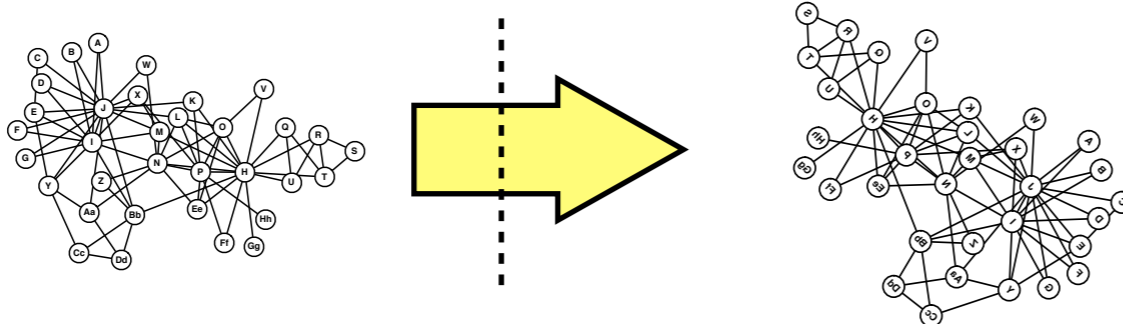


- Create topological similarity [Liu, SIGMOD 08] [Zhou, ICDE 08] [Zou, VLDB 09]
- Randomize edges [Ying, SDM 2008]
- Clustering/summarization [Campan, PinKDD 08] [Hay, VLDB 08] [Cormode, VLDB 08] [Cormode, VLDB 09]

# Data publishing v. output perturbation

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- Data publishing

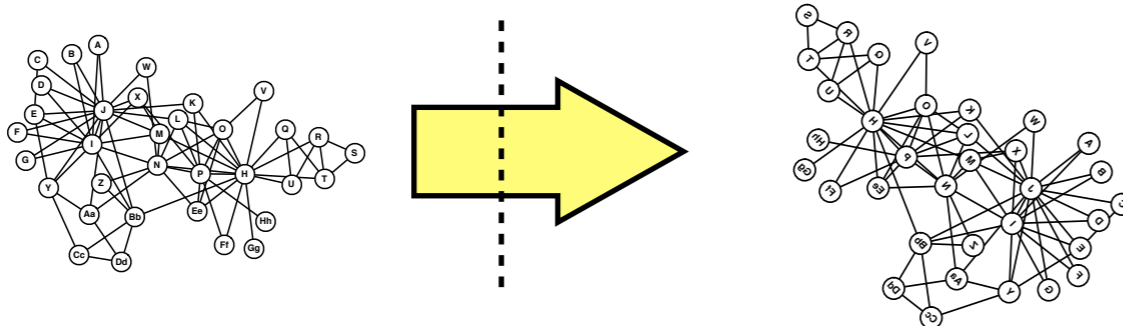




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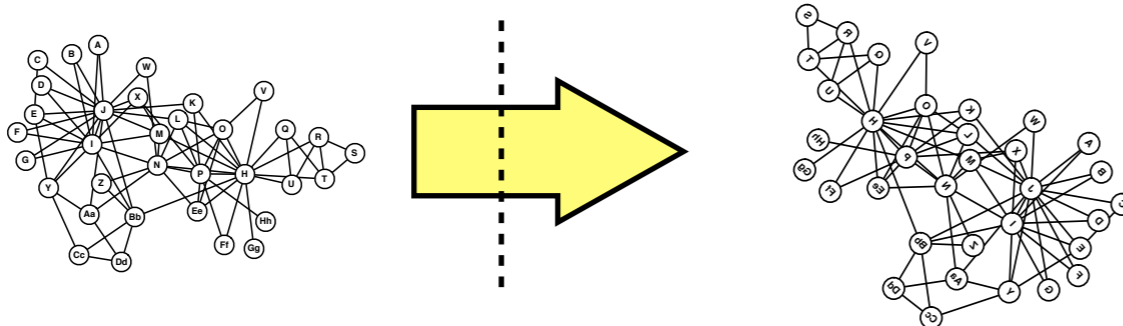
- Data publishing



<b>Ease of use</b>	good
<b>Privacy</b>	weak guarantees
<b>Accuracy</b>	no formal guarantees
<b>Scalability</b>	sometimes bad

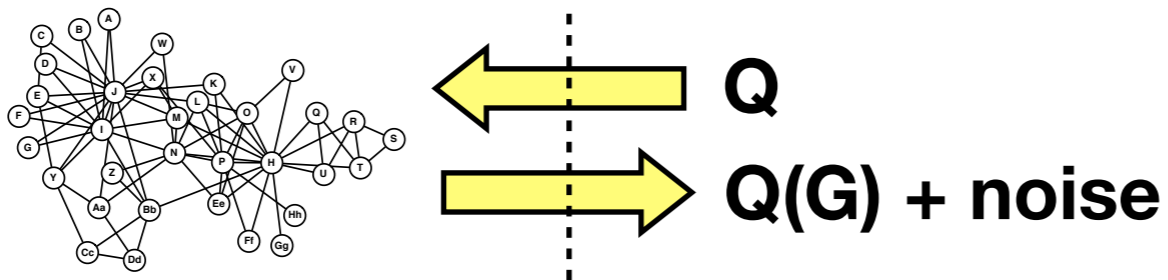
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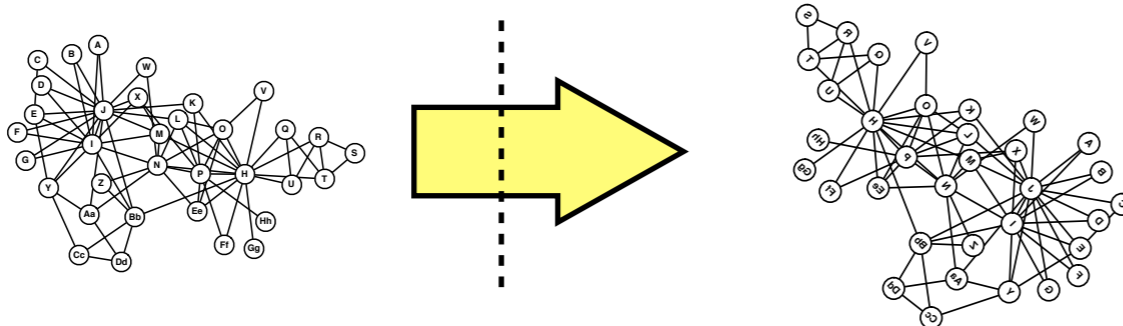
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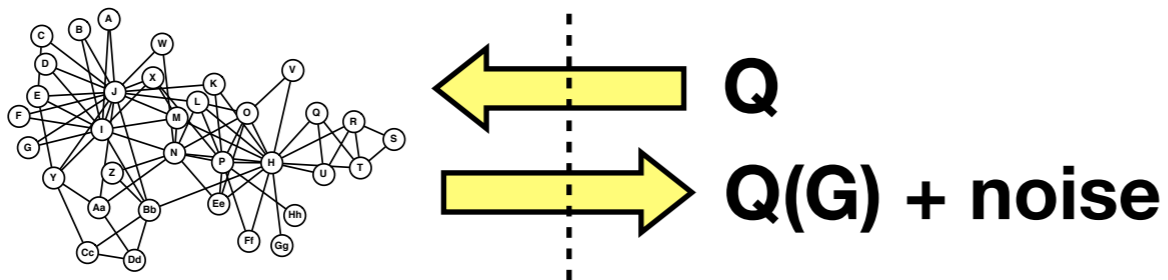
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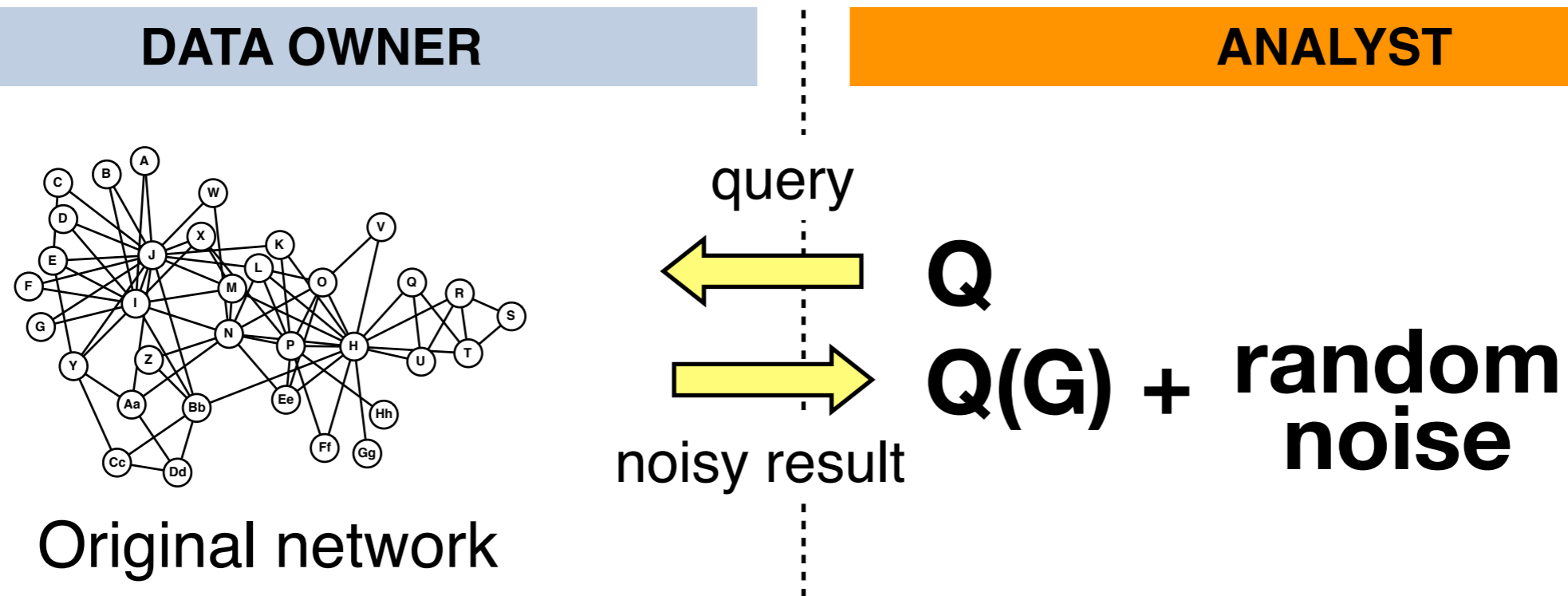
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- Output perturbation



<b>Ease of use</b>	bad for practical analyses
<b>Privacy</b>	formal guarantees
<b>Accuracy</b>	provable bounds
<b>Scalability</b>	very good

# Output perturbation



- Dwork, McSherry, Nissim, Smith [Dwork, TCC 06] have described an output perturbation mechanism satisfying *differential privacy*.
- Comparatively few results for graph data.

# Outline

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1. Existing approaches to protecting network data
2. Background on differential privacy
3. Privately estimating the degree distribution
4. Privately counting motifs
5. Future goals and open questions

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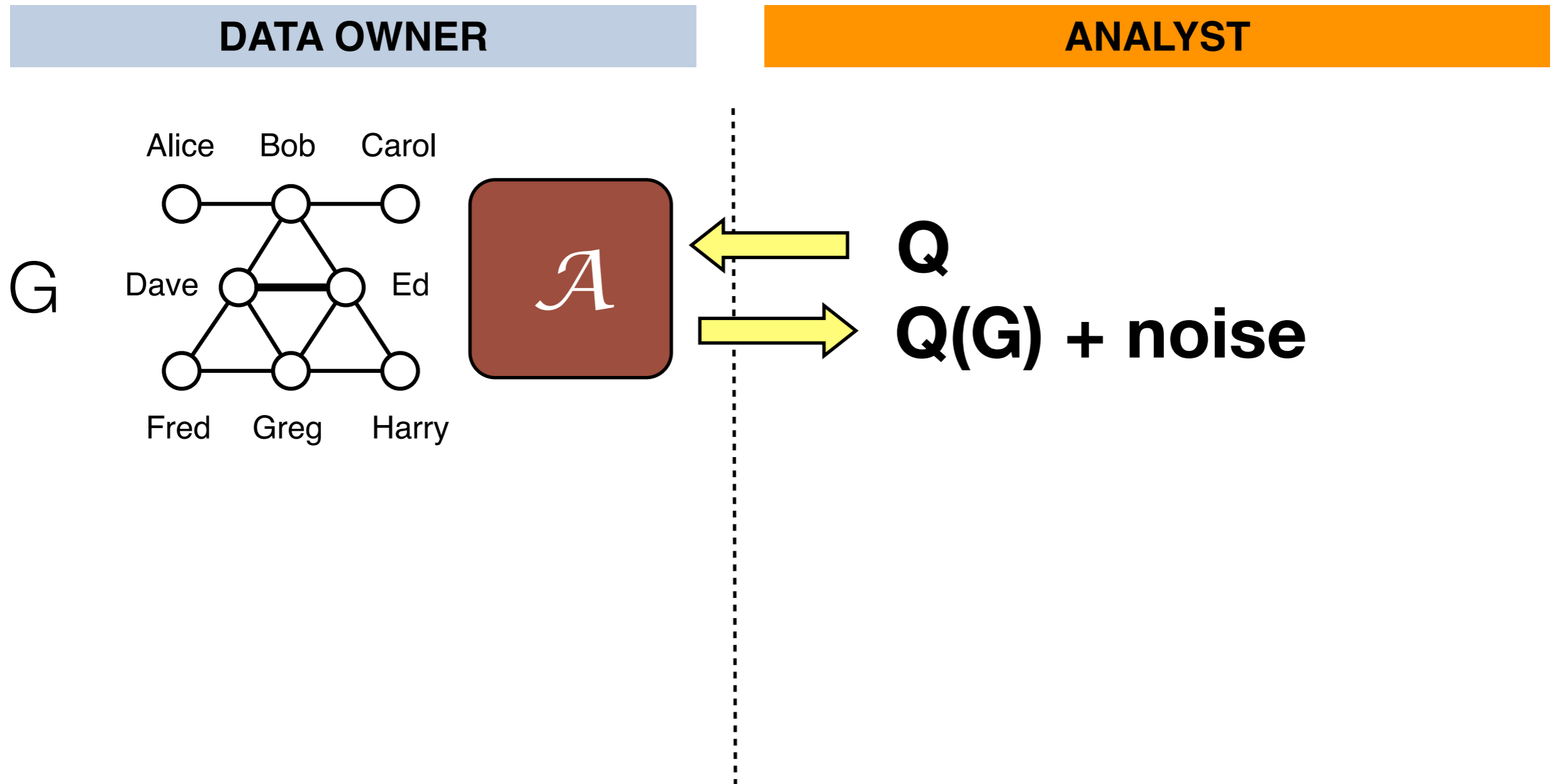
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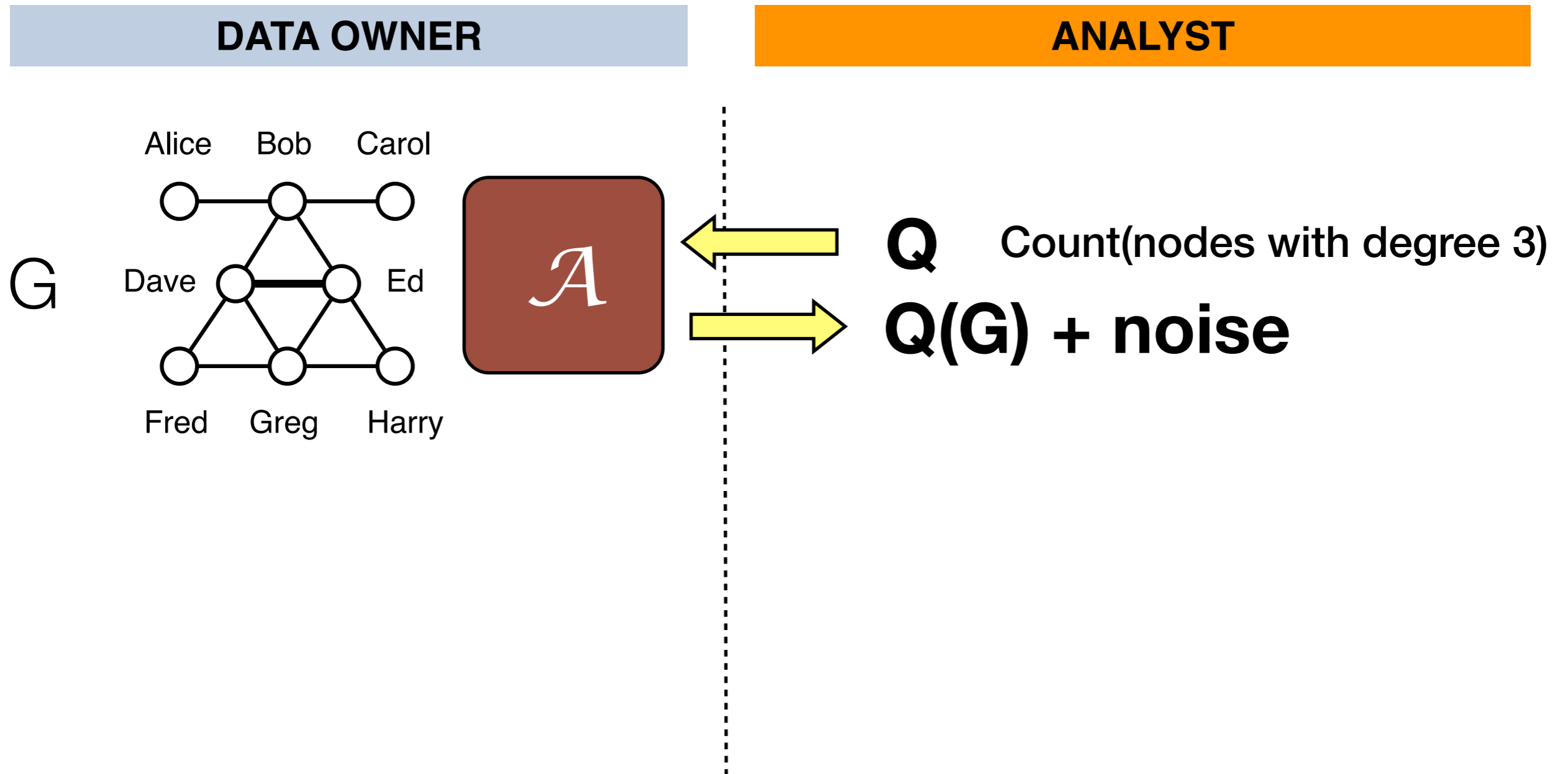
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# The differential guarantee

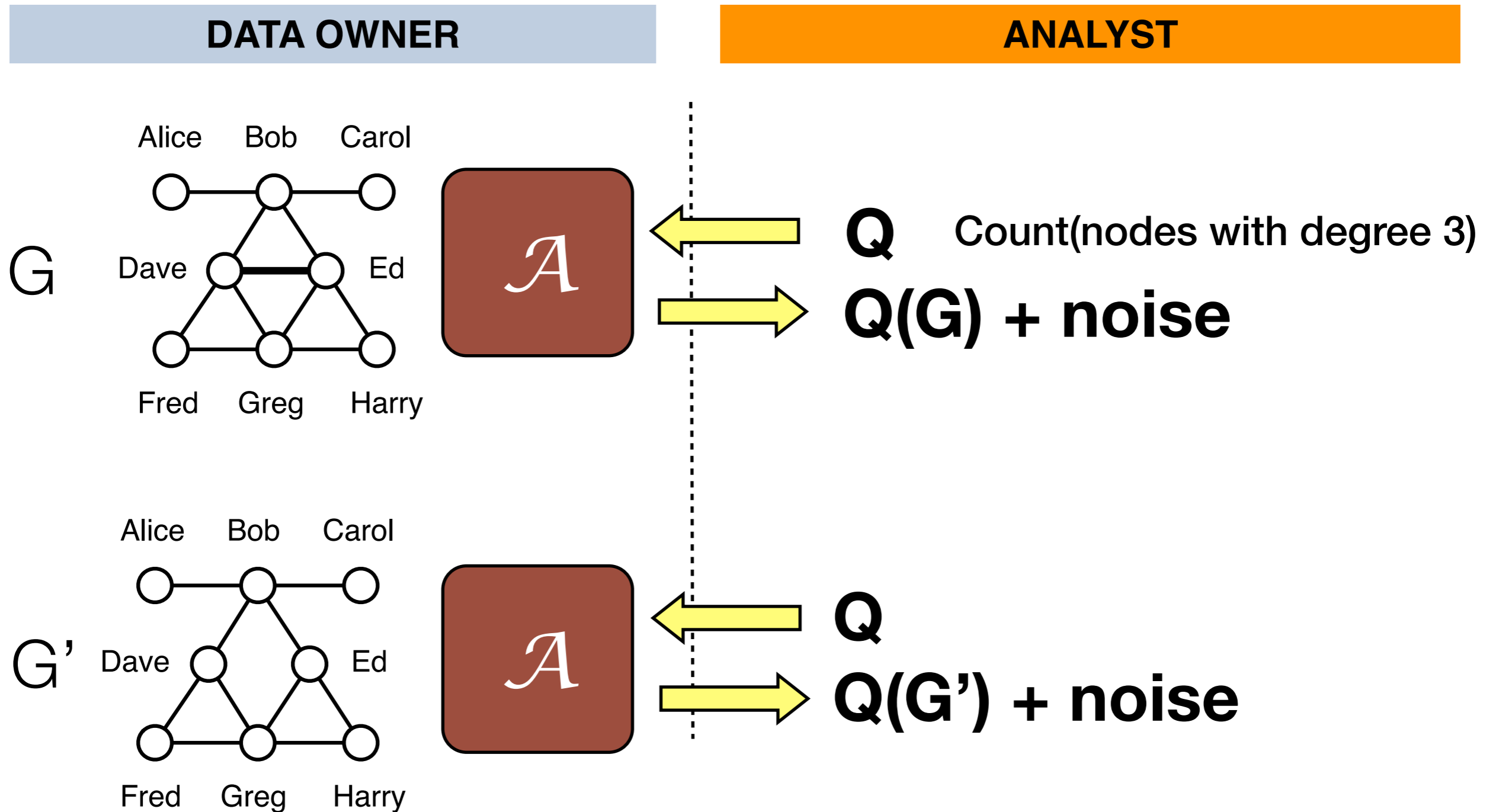


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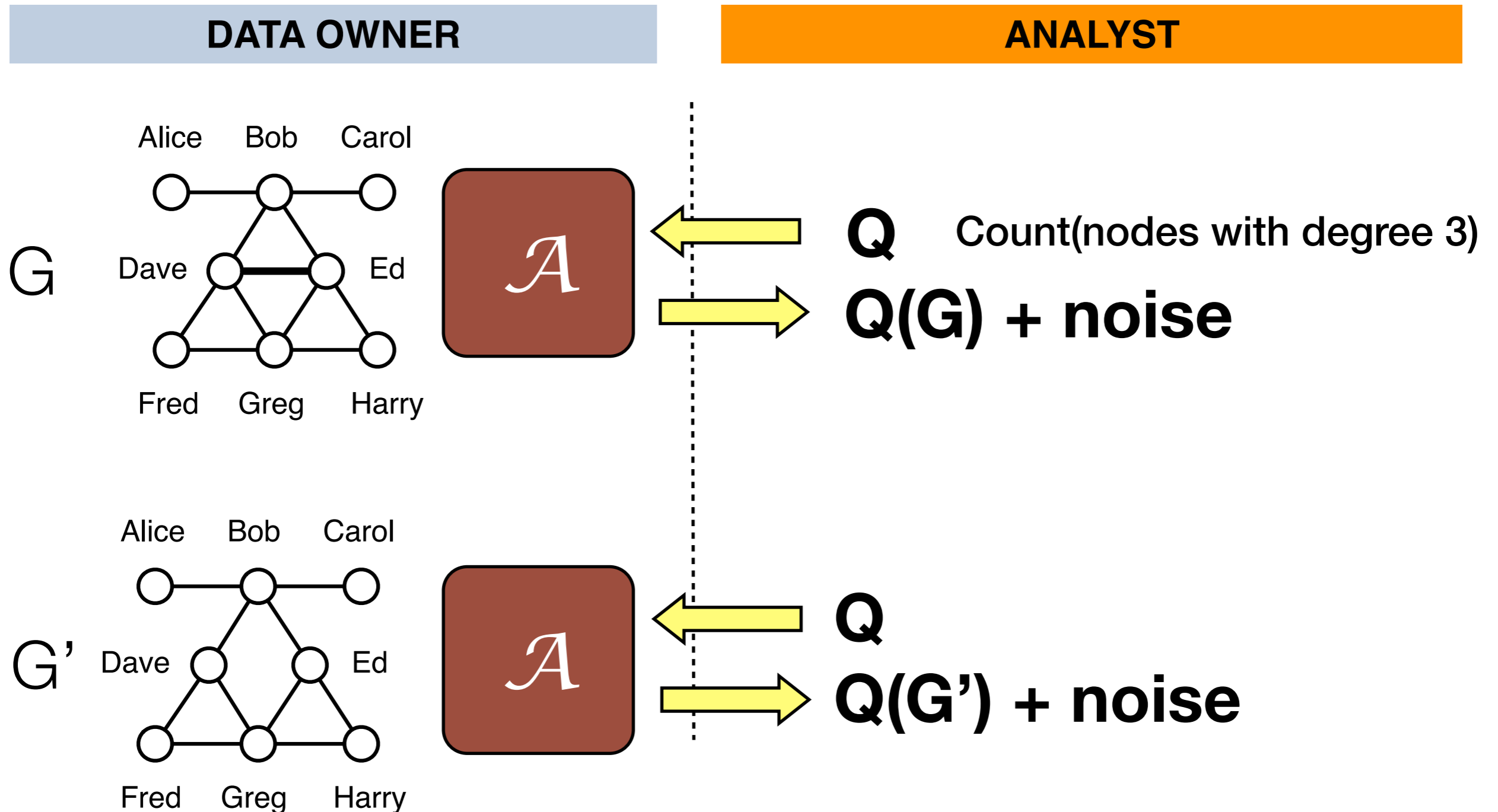




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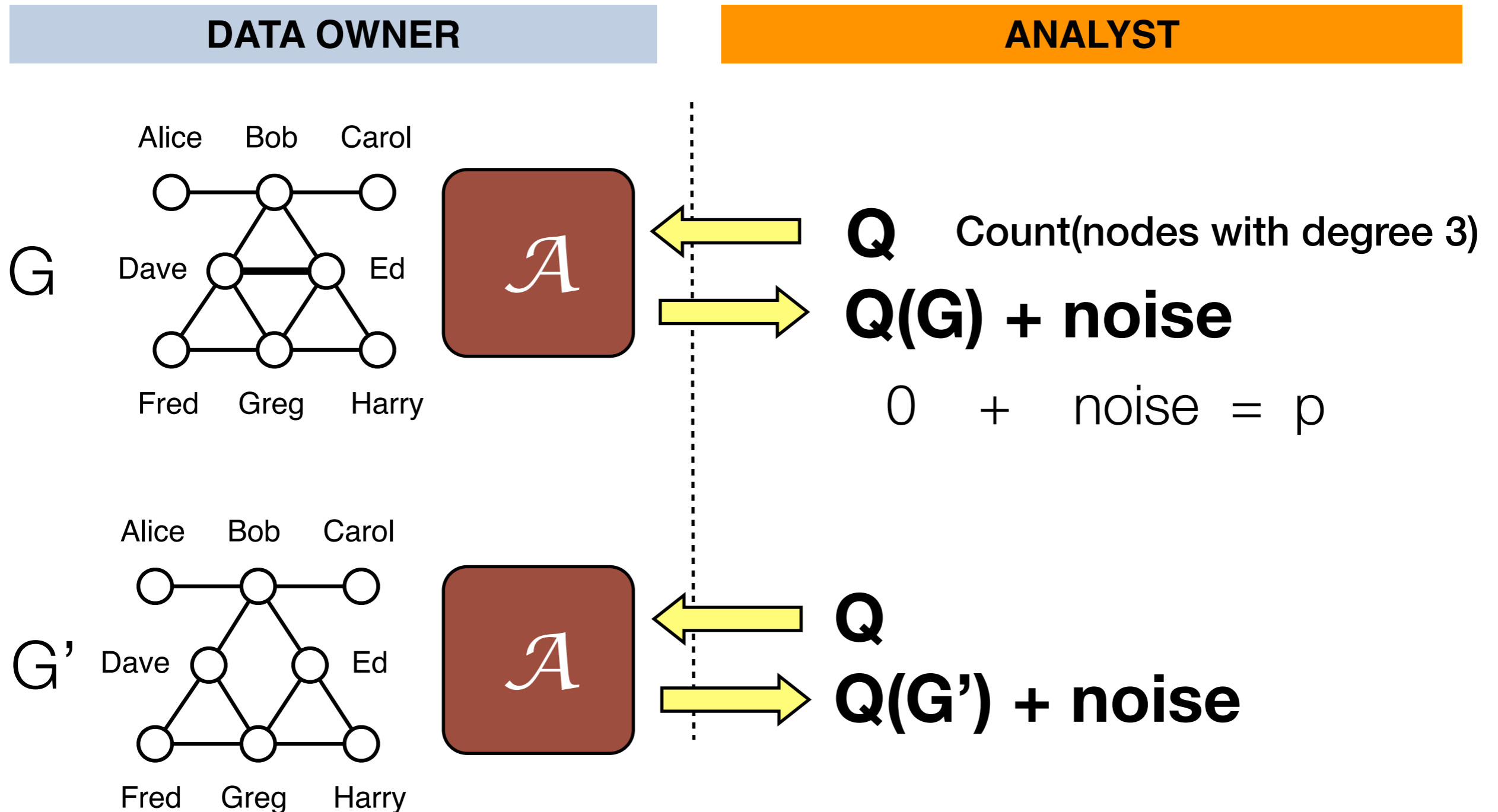


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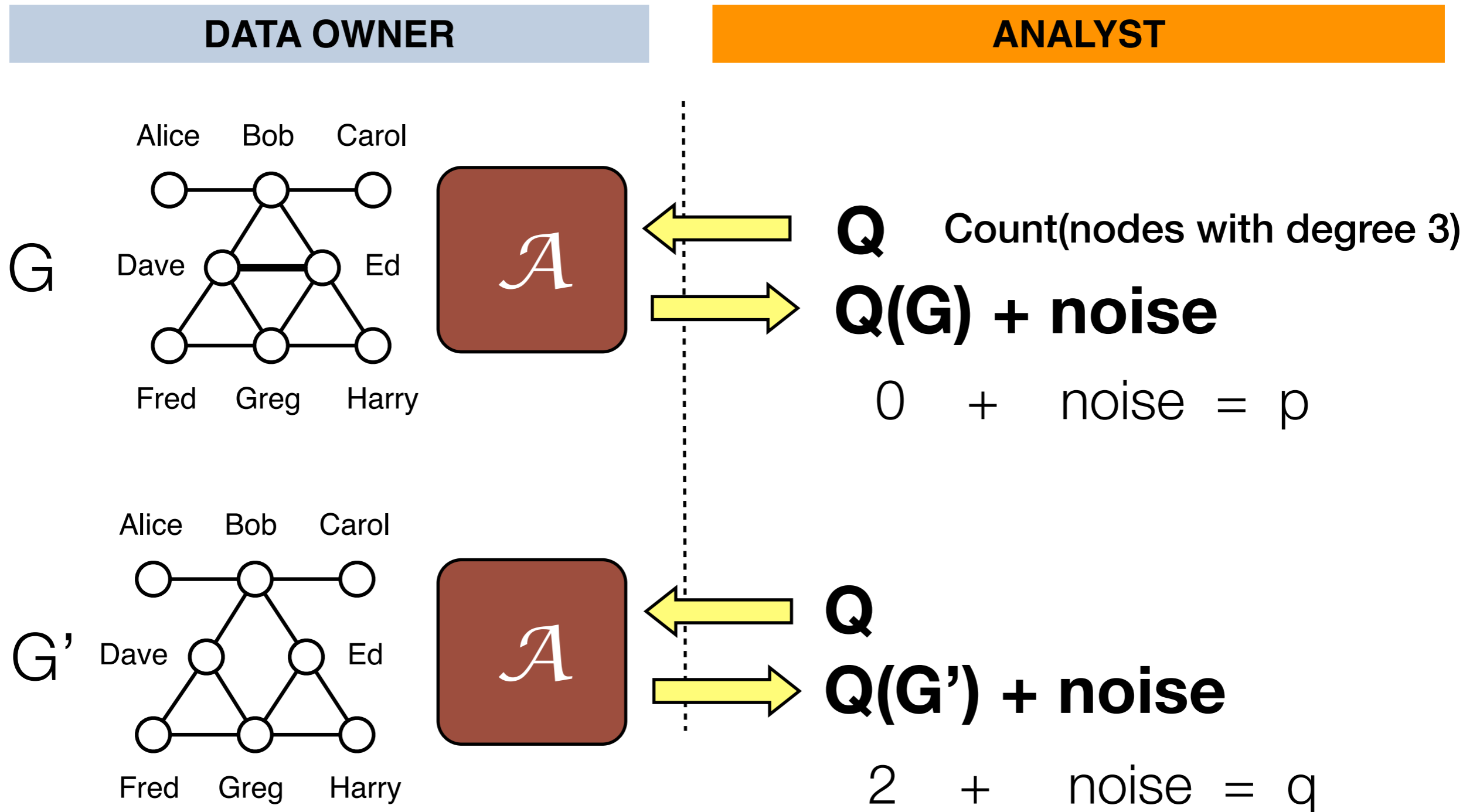
Two graphs are **neighbors** if they differ by at most one edge

# The differential guarantee



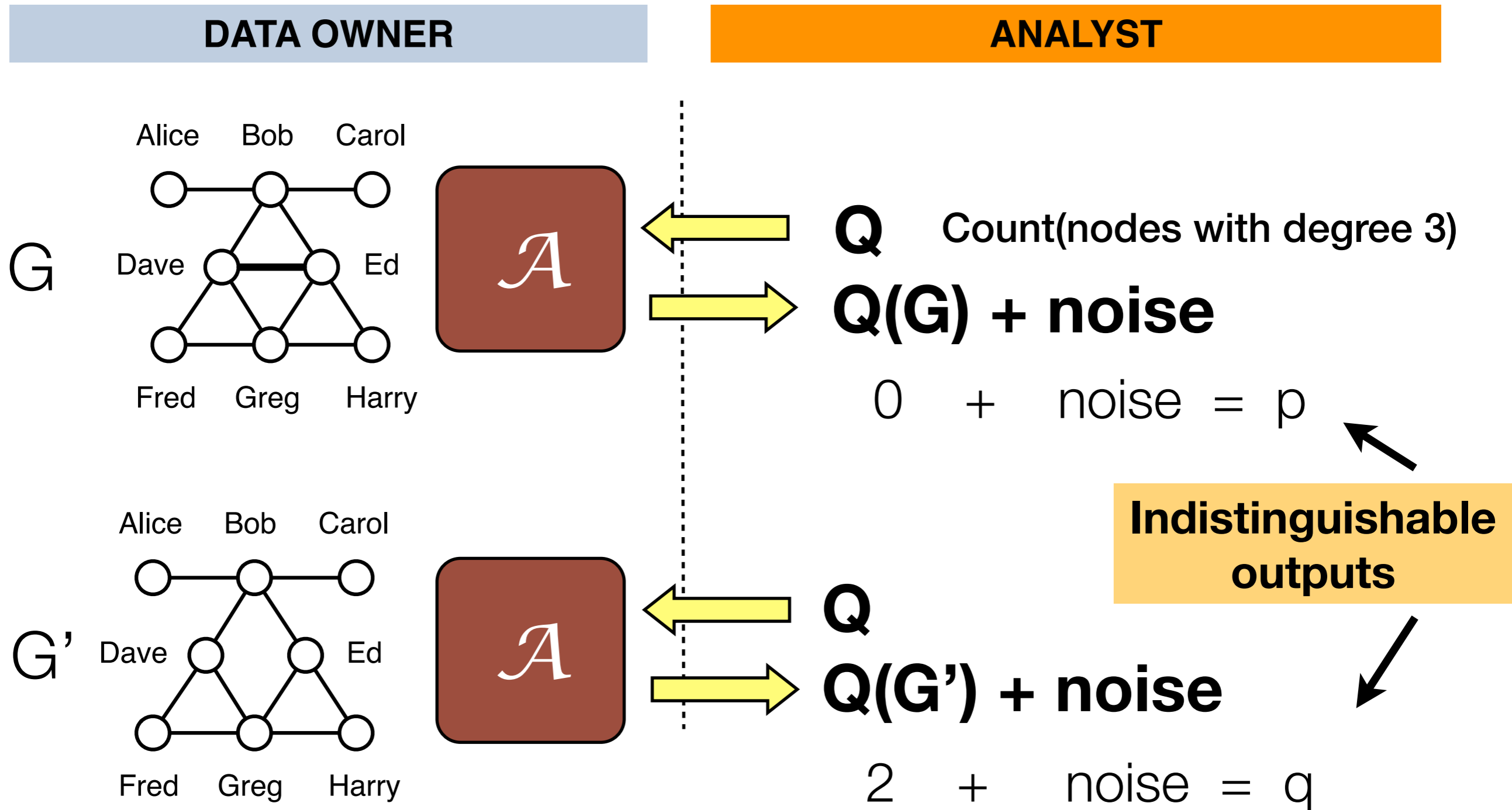
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# Differential privacy

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A randomized algorithm  $A$  provides  **$\epsilon$ -differential privacy** if:  
for all neighboring graphs  $G$  and  $G'$ , and  
for any set of outputs  $S$ :

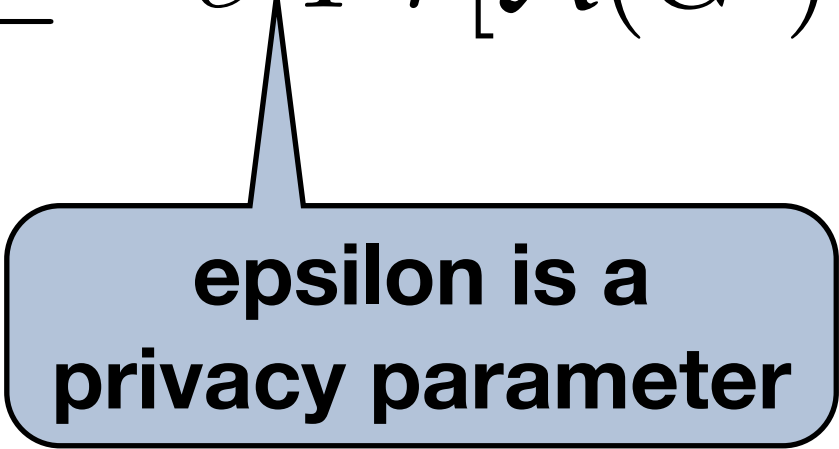
$$\Pr[\mathcal{A}(G) \in S] \leq e^\epsilon \Pr[\mathcal{A}(G') \in S]$$

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**epsilon is a  
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Epsilon is usually small: e.g. if  $\epsilon = 0.1$  then  $e^\epsilon \approx 1.10$

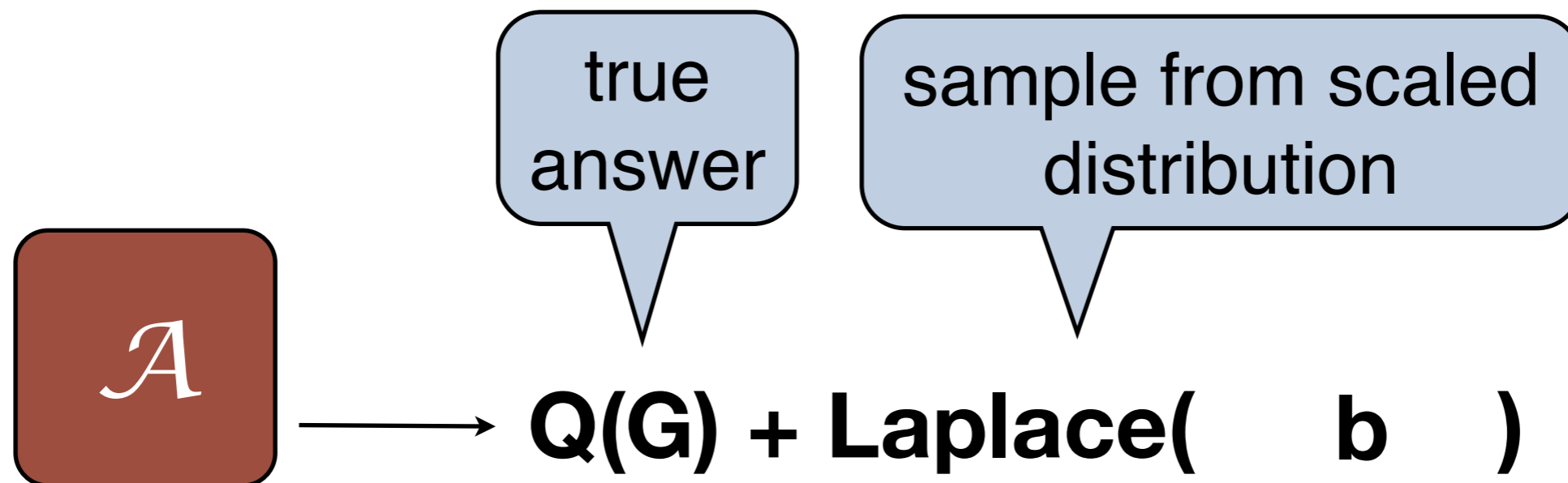
↓ epsilon = ↑ stronger privacy



# Calibrating noise

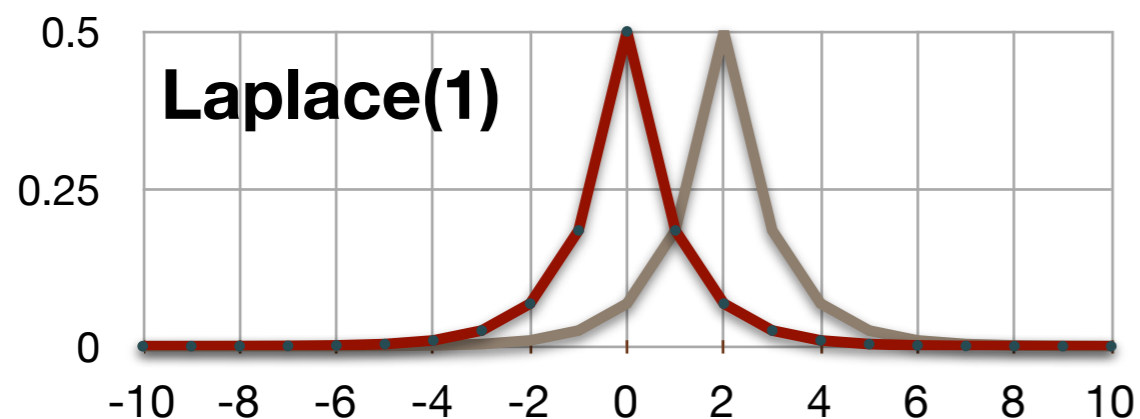
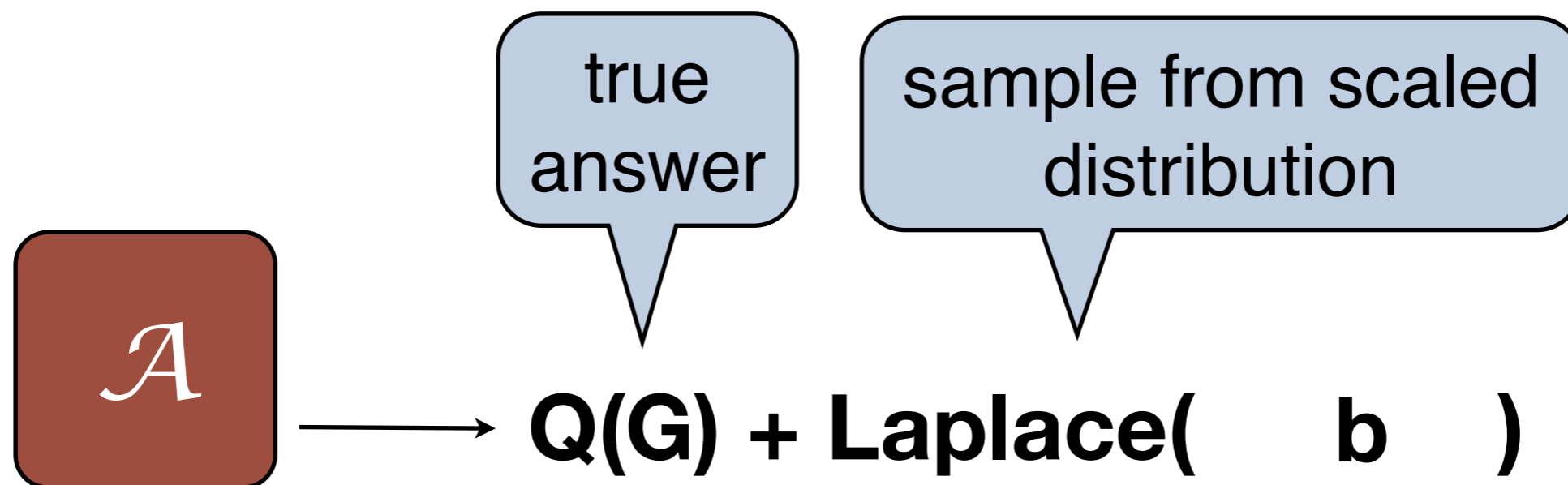
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- The following algorithm for answering  $Q$  is  $\epsilon$ -differentially private:



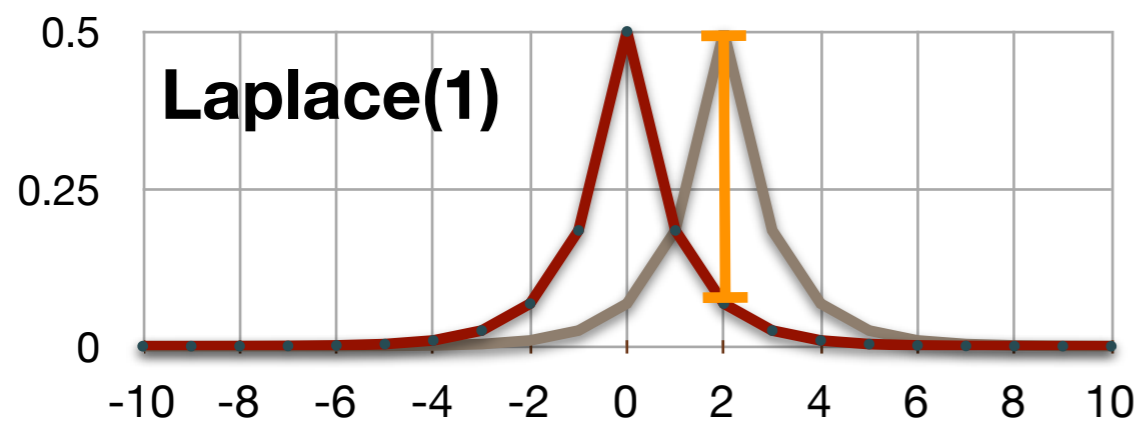
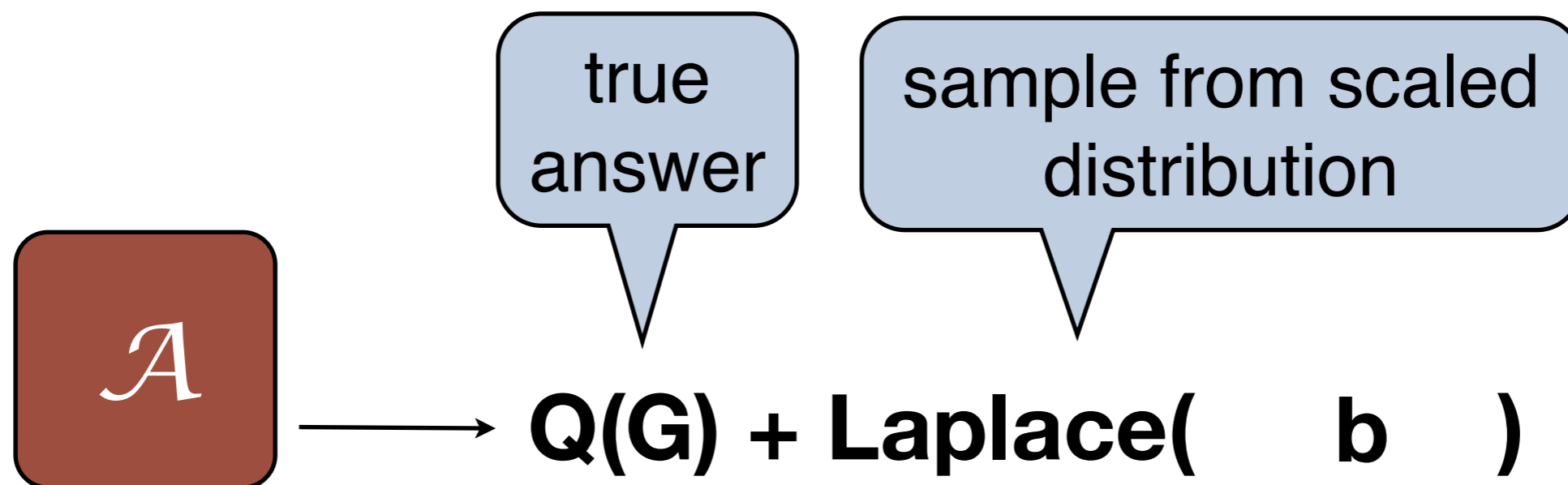
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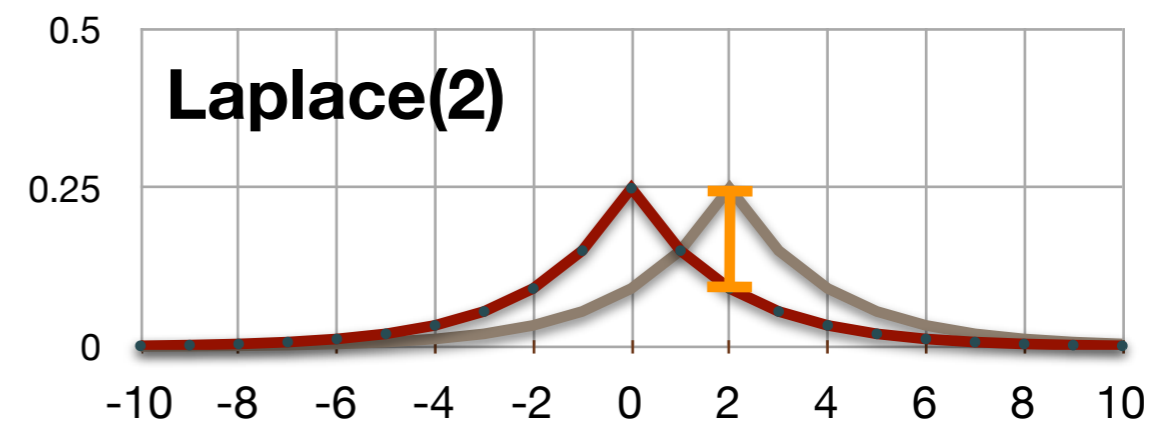
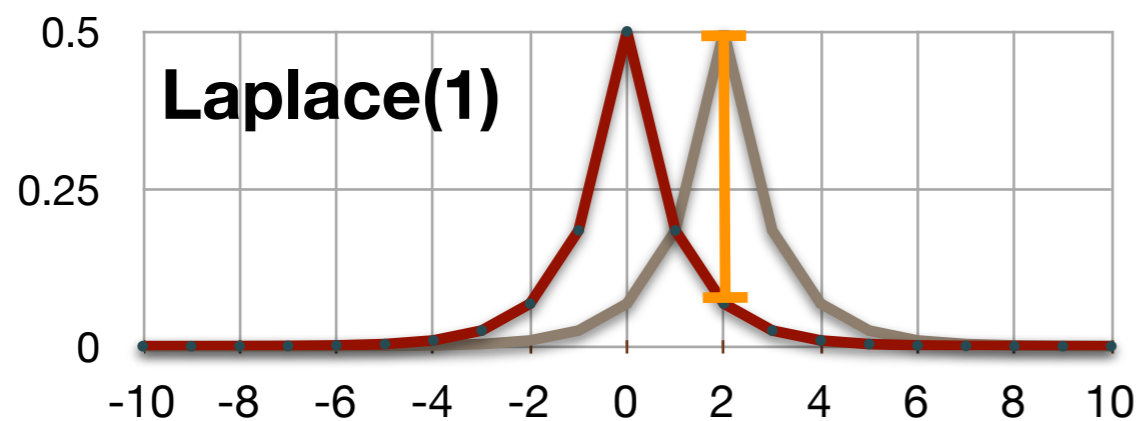
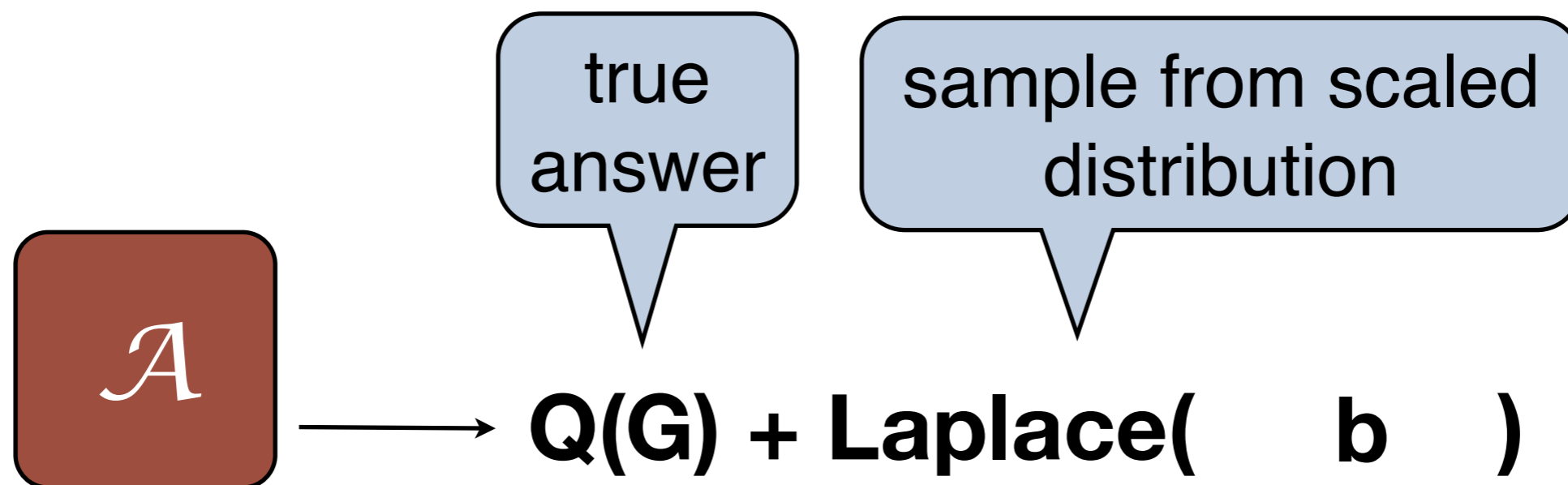
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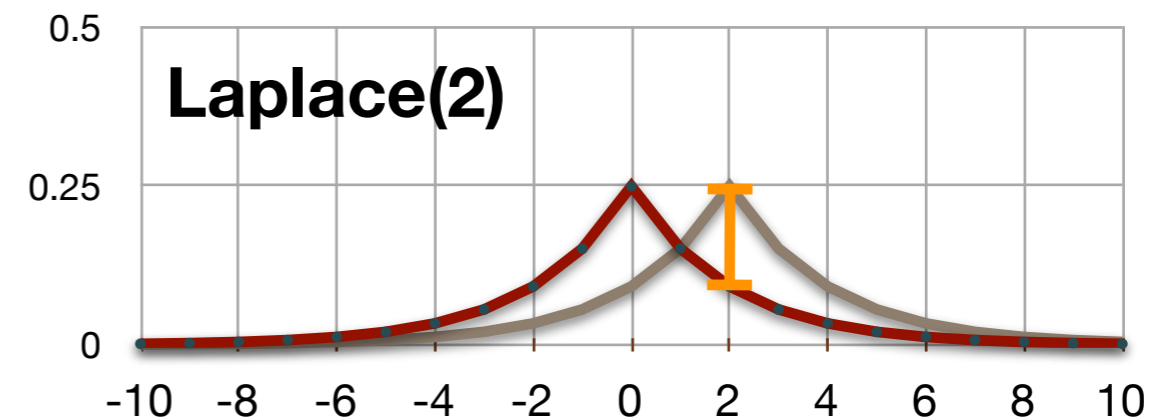
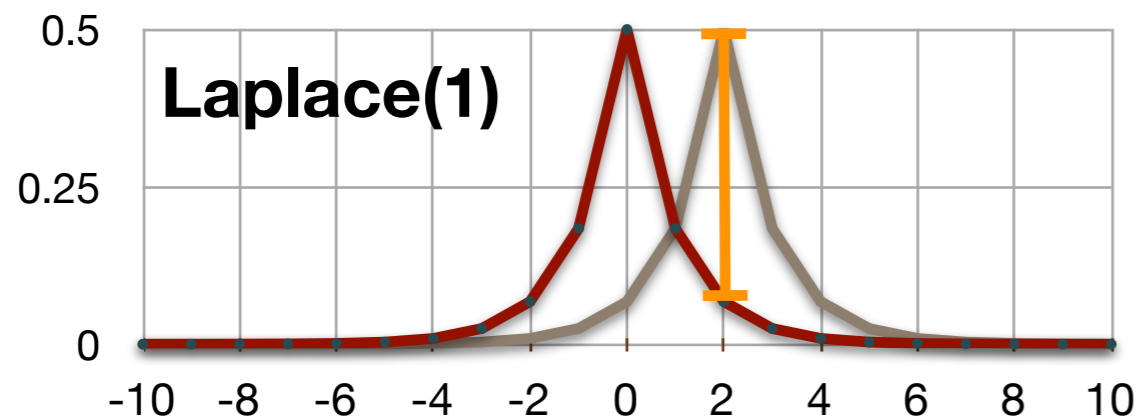
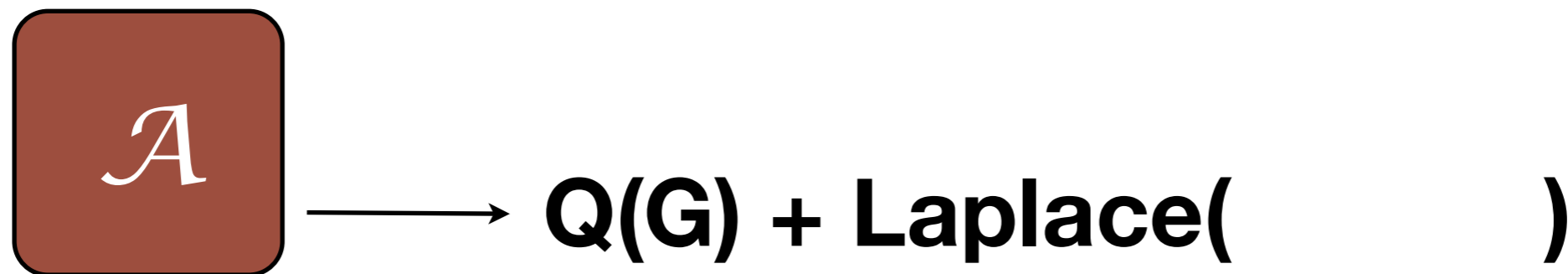
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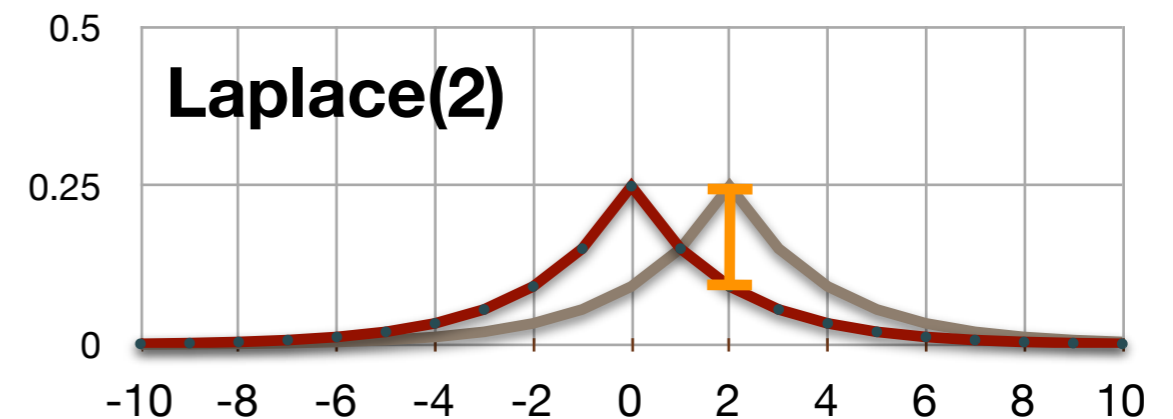
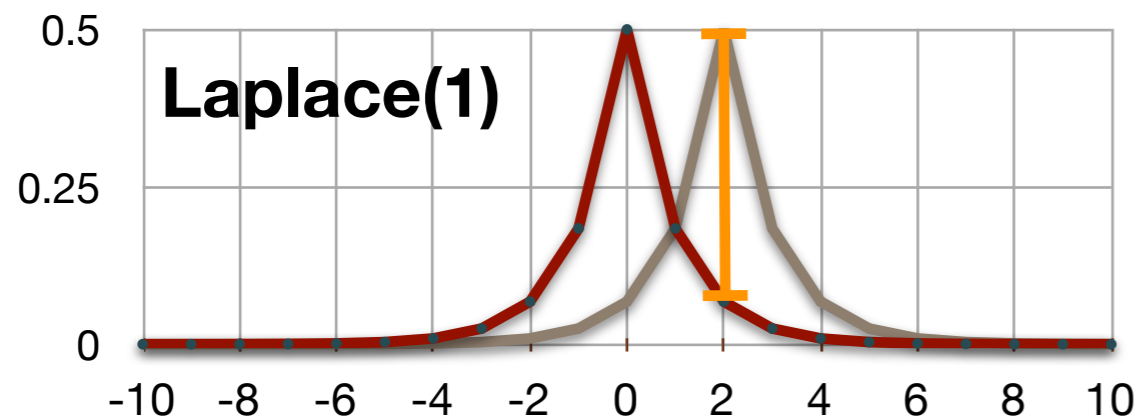
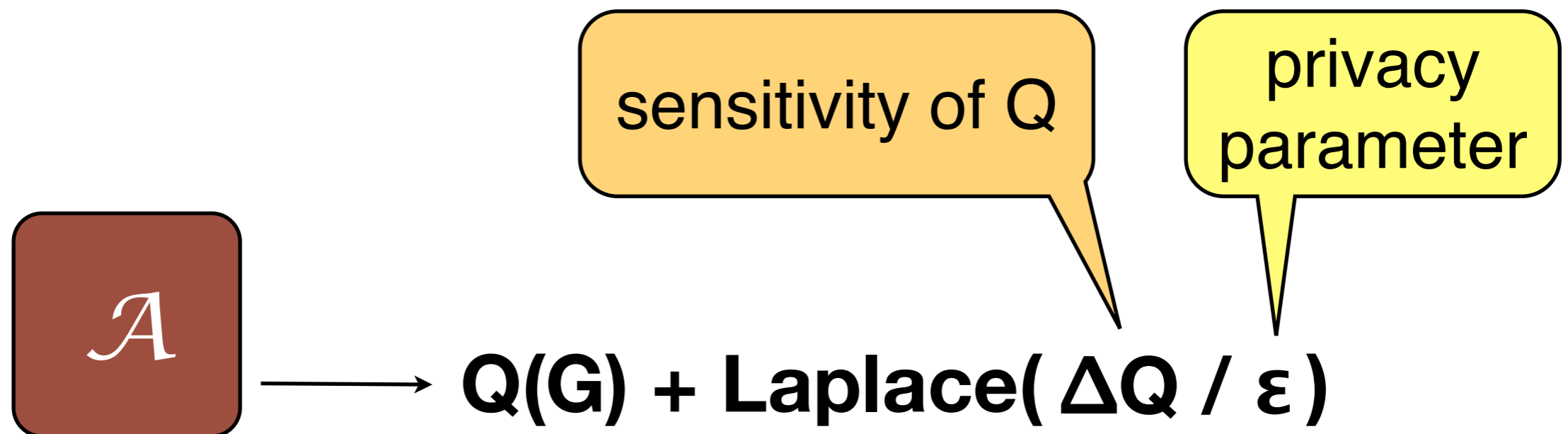
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# Examples of query sensitivity

---

The sensitivity of a query  $Q$  is

$$\Delta Q = \max_{G, G'} | Q(G) - Q(G') |$$

where  $G, G'$  are any two neighboring graphs

query

sensitivity truth

noisy answer

$\epsilon=0.5$

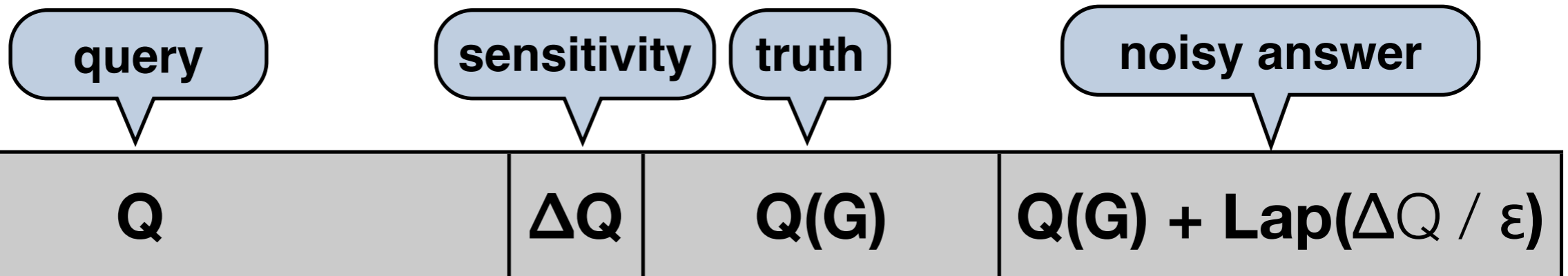
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query	sensitivity	truth	noisy answer
$Q$	$\Delta Q$	$Q(G)$	$Q(G) + \text{Lap}(\Delta Q / \epsilon)$
<b>deg<sub>A</sub></b> (degree of node A)	1	<b>deg<sub>Dave</sub>(G) = 4</b>	4+Lap(2)

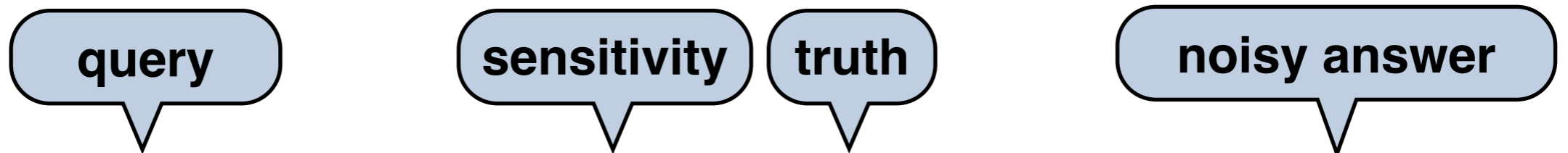
$\epsilon=0.5$

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where  $G, G'$  are any two neighboring graphs



<b>Q</b>	<b><math>\Delta Q</math></b>	<b><math>Q(G)</math></b>	<b><math>Q(G) + \text{Lap}(\Delta Q / \epsilon)</math></b>
<b>deg<sub>A</sub></b> (degree of node A)	1	<b>deg<sub>Dave</sub>(G) = 4</b>	4+Lap(2)
<b>cnt<sub>i</sub></b> (# nodes with degree i)	2	<b>cnt<sub>4</sub>(G) = 4</b>	4+Lap(4)

$\epsilon=0.5$

# Multiple queries

---

The sensitivity of a query  $Q$  is

$$\Delta Q = \max_{G, G'} | Q(G) - Q(G') |$$

$L_1$  dist for  
vectors

where  $G, G'$  are any two neighboring graphs

query

sensitivity

truth

noisy answer

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$Q$

$\Delta Q$

$Q(G)$

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$Q$	$\Delta Q$	$Q(G)$	$Q(G) + \text{Lap}(\Delta Q / \epsilon)$
$[\text{deg}_A, \text{deg}_B, \text{deg}_C]$	2	$[1, 4, 1]$	$[1 + \text{Lap}(4), 4 + \text{Lap}(4), 1 + \text{Lap}(4)]$

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$[\text{cnt}_0, \text{cnt}_1, \text{cnt}_2]$	4	$[0, 2, 2]$	$[0 + \text{Lap}(8), 2 + \text{Lap}(8), 2 + \text{Lap}(8)]$

$\epsilon = 0.5$

# Differential privacy for networks

---

A participant's sensitive information is **not** a single edge.

- **edge  $\epsilon$ -differential privacy**: algorithm output is largely indistinguishable whether or not any **single edge** is present or absent.
- **k-edge  $\epsilon$ -differential privacy**: algorithm output is largely indistinguishable whether or not any **set of k edges** is present or absent.
- **node  $\epsilon$ -differential privacy**: algorithm output is largely indistinguishable whether or not any single **node (and all its edges)** is present or absent.

**Laplace( $\Delta Q / \epsilon$ )**



**Laplace( $\Delta Q k / \epsilon$ )**

Suppose  $\Delta Q=1$ . Then Laplace(100) satisfies:  
1-edge 0.01-differential privacy  
10-edge 0.1-differential privacy

# Outline

---

1. Existing approaches to protecting network data

2. Background on differential privacy

3. Privately estimating the degree distribution

4. Privately counting motifs

5. Future goals and open questions



# Outline

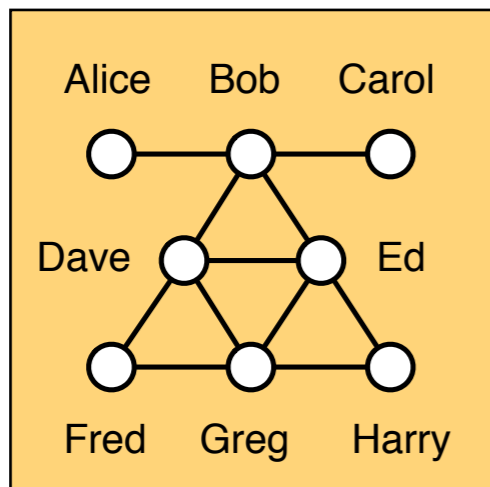
---

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# The degree sequence of a network

---

- Degree sequence: the list of degrees of each node in a graph.
- A widely studied property of networks.

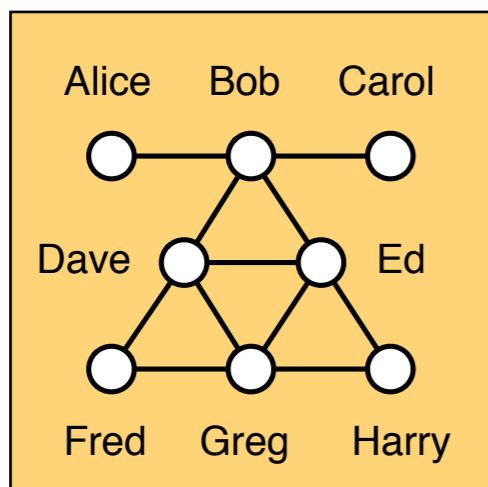


[1, 1, 2, 2, 4, 4, 4, 4]

# The degree sequence of a network

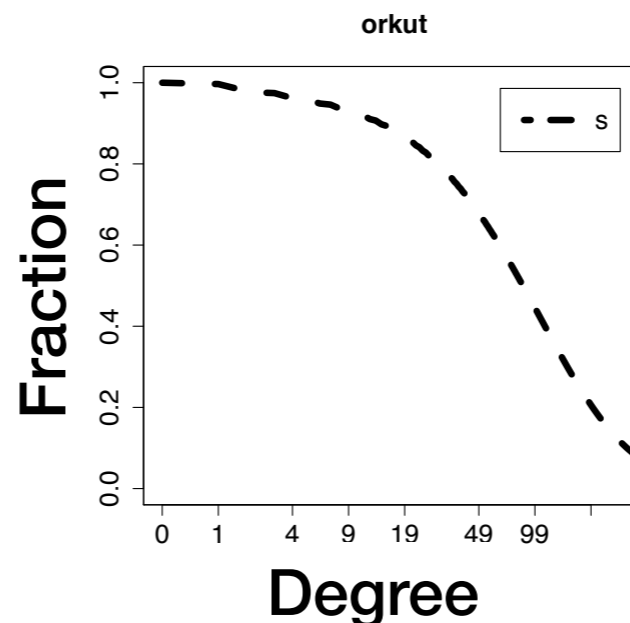
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- Degree sequence: the list of degrees of each node in a graph.
- A widely studied property of networks.



[1, 1, 2, 2, 4, 4, 4, 4]

Orkut  
crawl



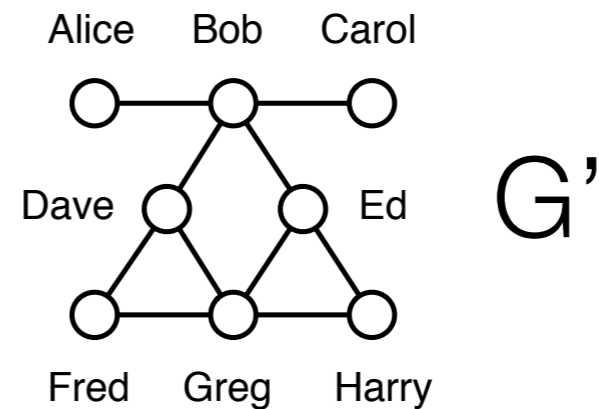
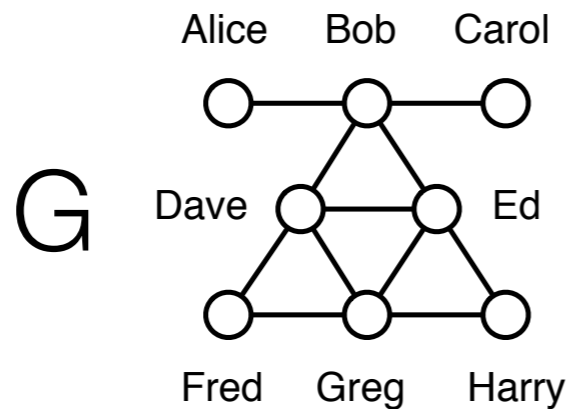
**Inverse  
cumulative  
distribution**

# The degree sequence is sensitive

---

- Why not release the true degree sequence of a network?
  - In extreme cases, the degree sequence can determine the structure of the graph --- no better than naive anonymization.
  - Background knowledge could lead to disclosures.
  - The degree sequence may not be the only statistic we release -- we must protect against combined disclosures.

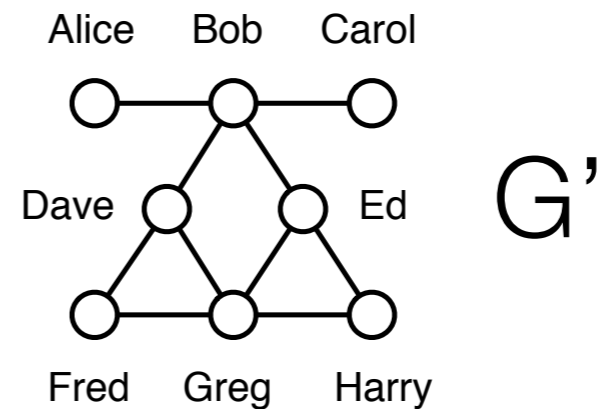
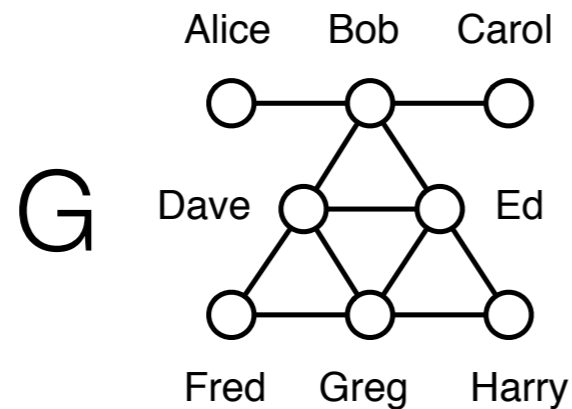
# Two basic queries for degrees



Degree of each node	
$\text{deg}_A$	degree of node A
<b>D</b>	$[\text{deg}_A, \text{deg}_B, \dots ]$

Frequency of each degree	
$\text{cnt}_i$	count of nodes with
<b>F</b>	$[\text{cnt}_0, \text{cnt}_1, \dots \text{cnt}_{n-1}]$

# Two basic queries for degrees



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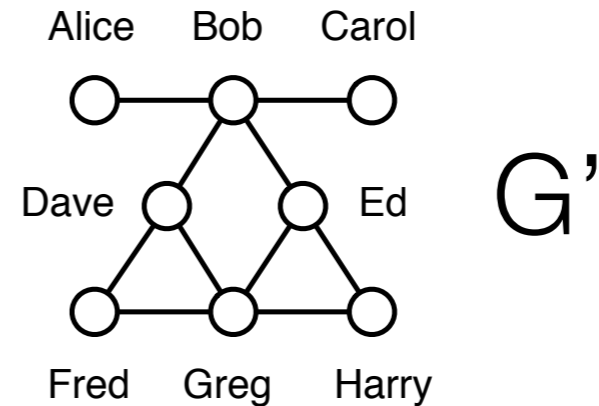
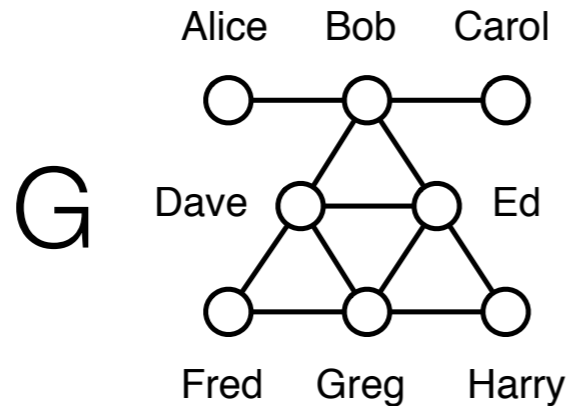
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$$D(G) = [1, 4, 1, 4, 4, 2, 4, 2]$$

$$D(G') = [1, 4, 1, \underline{3}, \underline{3}, 2, 4, 2]$$

$$\Delta D = 2$$

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$$F(G) = [0, 2, 2, 0, 4, 0, 0, 0]$$

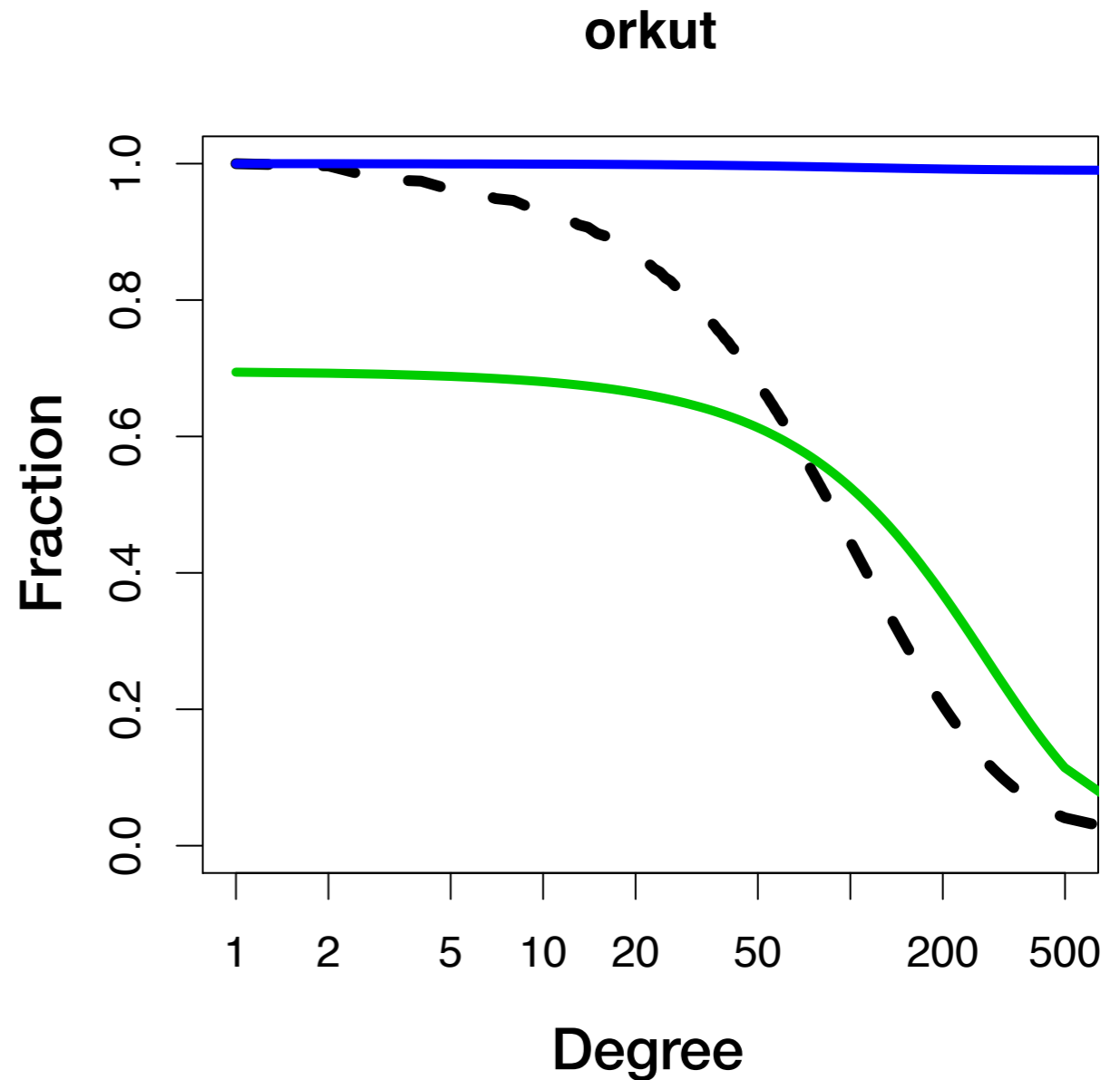
$$F(G') = [0, 2, 2, \underline{2}, \underline{2}, 0, 0, 0]$$

$$\Delta F = 4$$

# These queries are both flawed



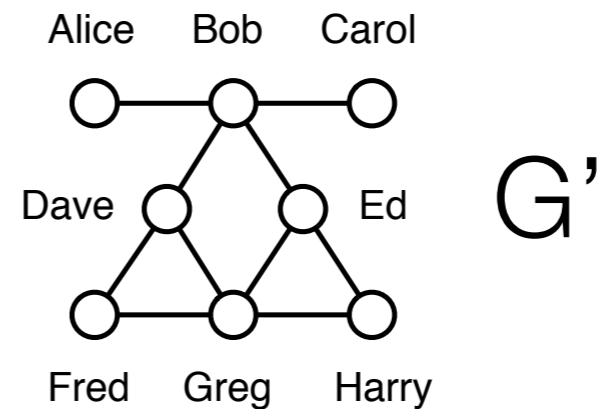
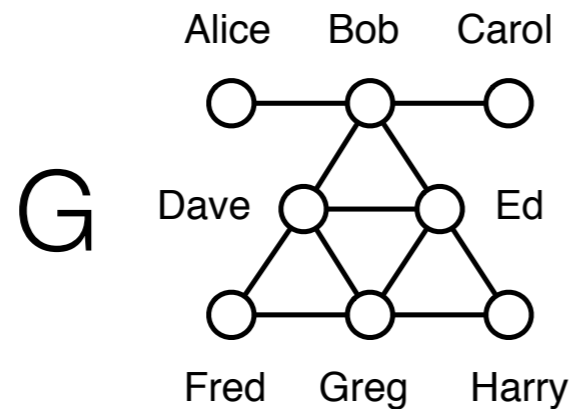
- D requires independent samples from Laplace( $2/\epsilon$ ) in each component.
- F requires independent samples from Laplace( $4/\epsilon$ ) in each component.
- Thus Mean Squared Error is  $O(n/\epsilon^2)$



( Laplace(b) has variance  $2b^2$  )



# An alternative query for degrees



Degree of each node	
$\text{deg}_A$	degree of node A
<b>D</b>	$[\text{deg}_A, \text{deg}_B, \dots]$

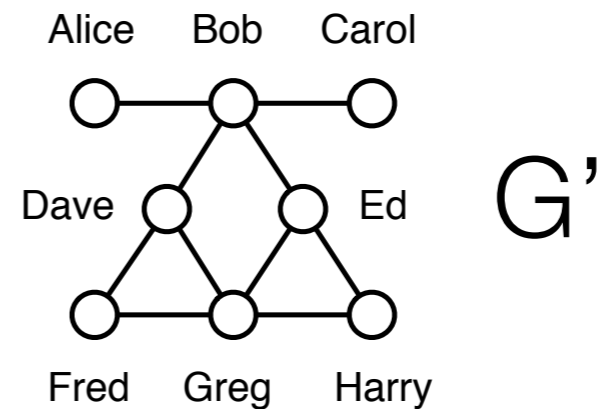
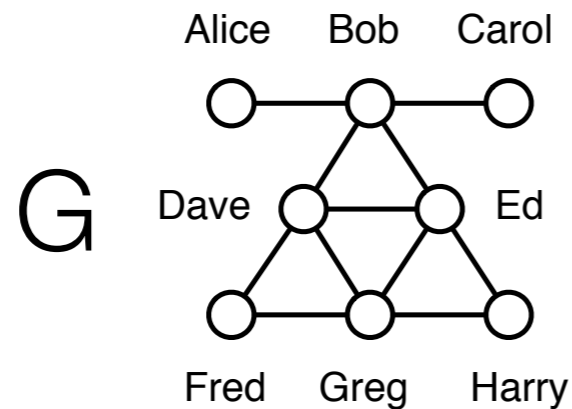
Degree of each node, ranked	
$\text{rnk}_i$	return the rank $i^{\text{th}}$ degree
<b>S</b>	$[\text{rnk}_1, \text{rnk}_2, \dots, \text{rnk}_n]$

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# An alternative query for degrees



## Degree of each node

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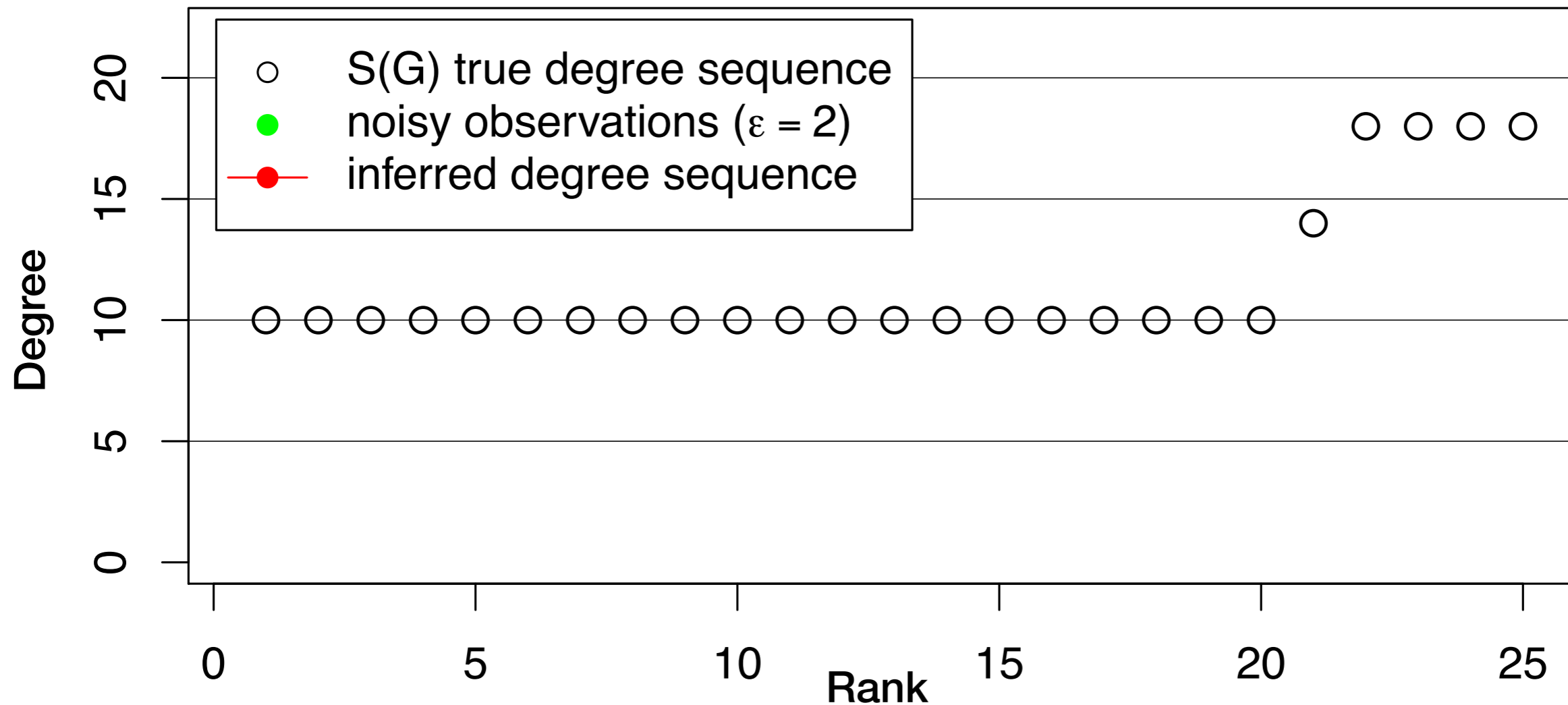
$$\Delta D = 2$$

$$S(G) = [1, 1, 2, 2, 4, 4, 4, 4]$$

$$S(G') = [1, 1, 2, 2, \underline{3}, \underline{3}, 4, 4]$$

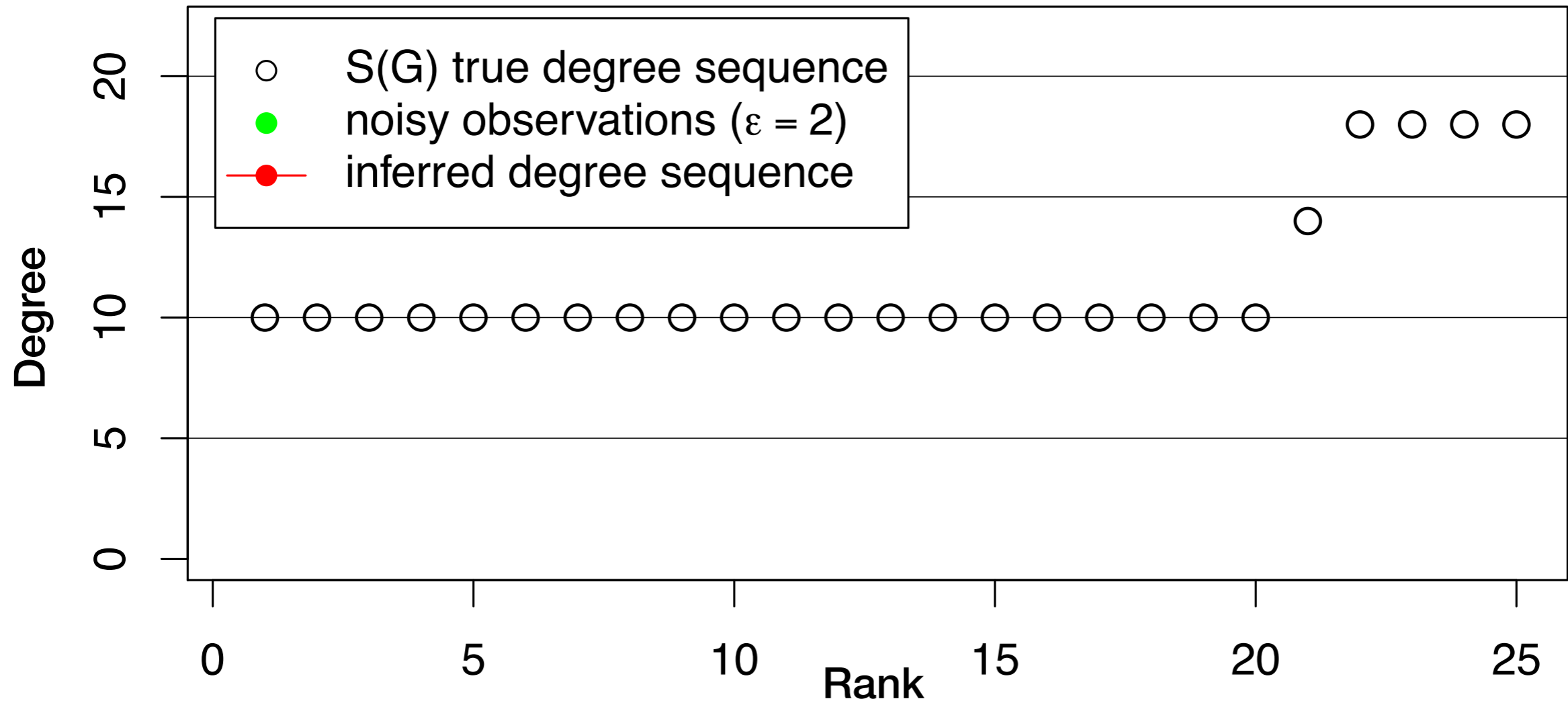
$$\Delta S = 2$$

# Using the sort constraint

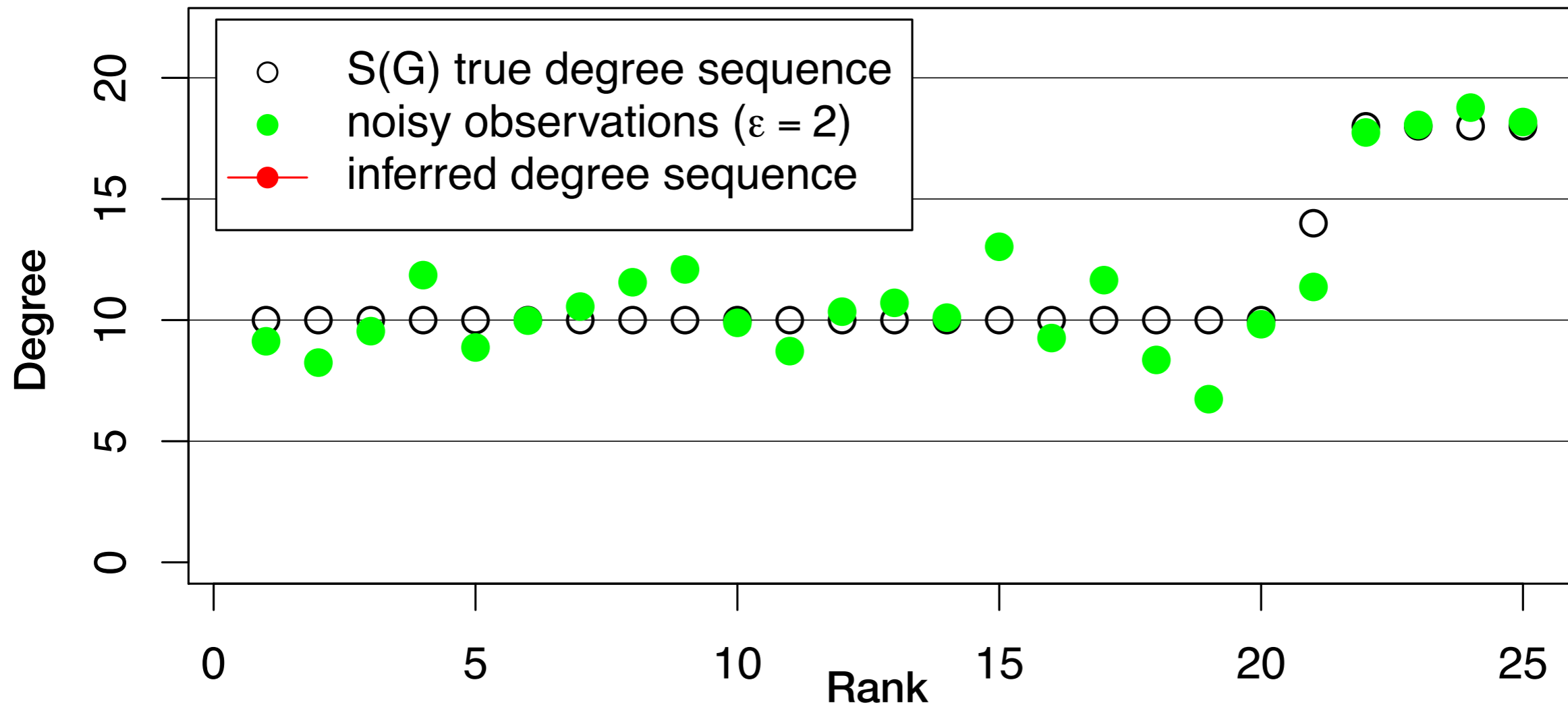


**$S(G) = [10, 10, \dots, 10, 10, 14, 18, 18, 18, 18]$**

# Using the sort constraint

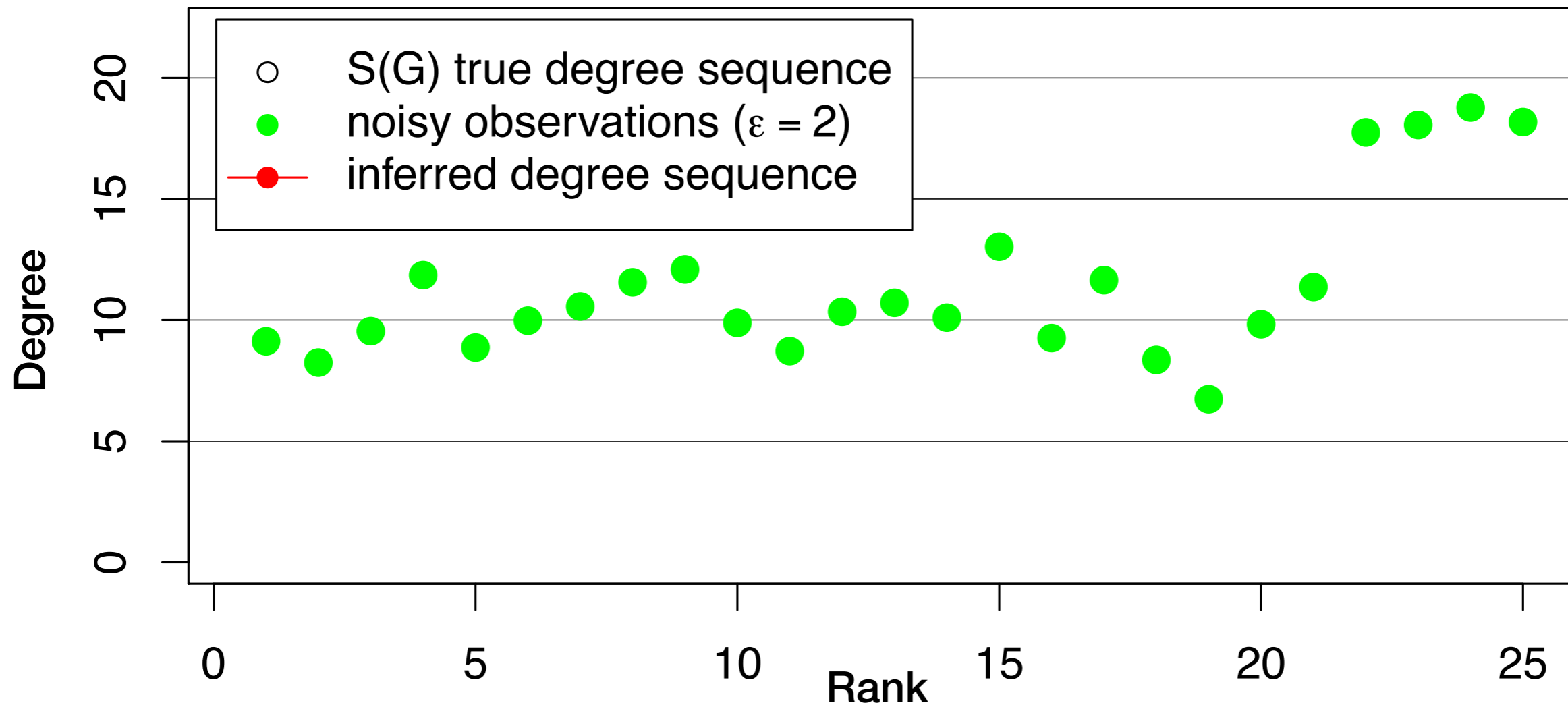


# Using the sort constraint



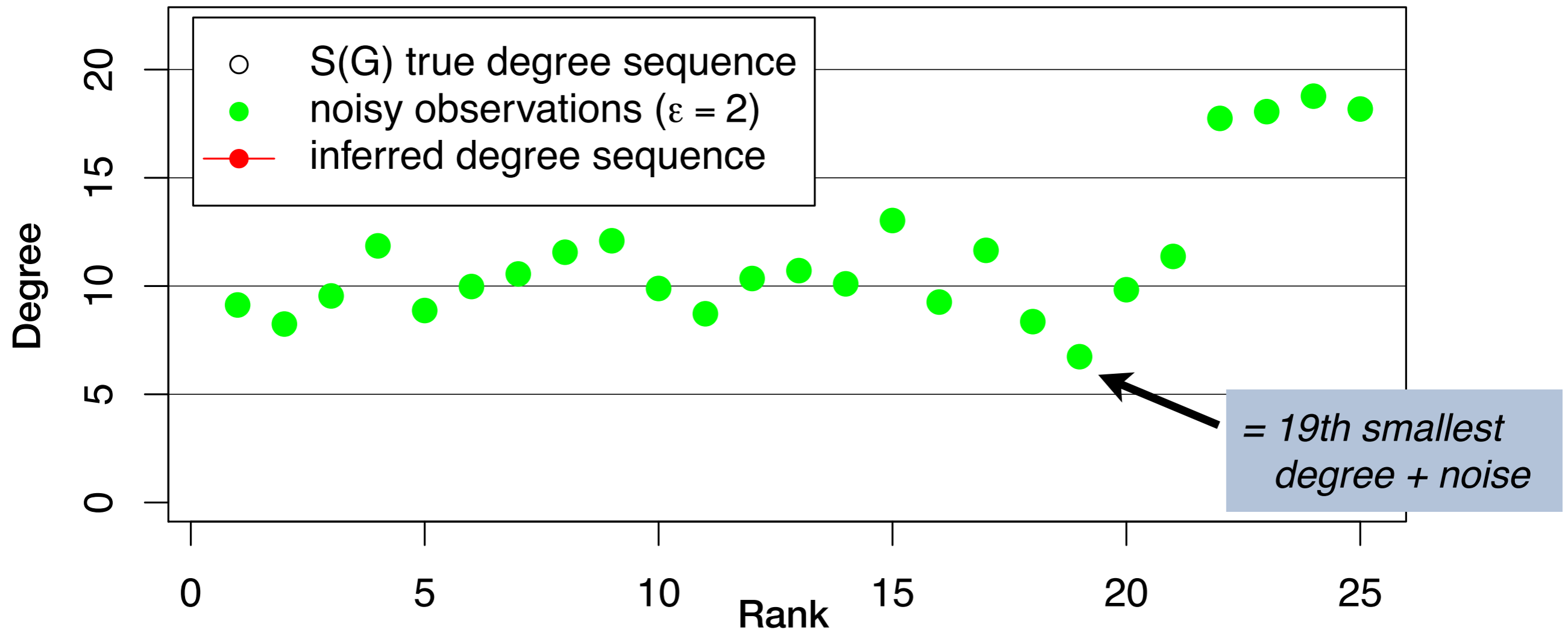
- The output of the sorted degree query is not (in general) sorted.
- We derive a new sequence by computing the **closest** non-decreasing sequence: i.e. minimizing L2 distance.

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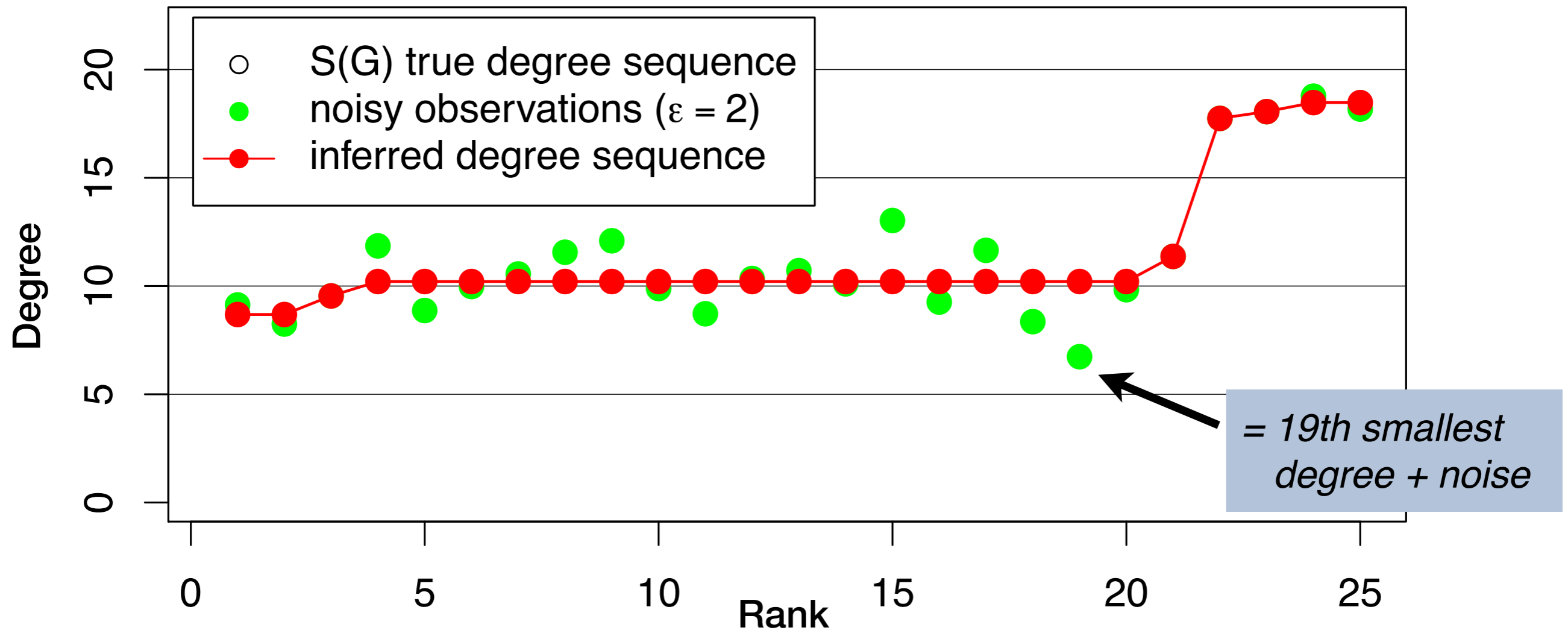
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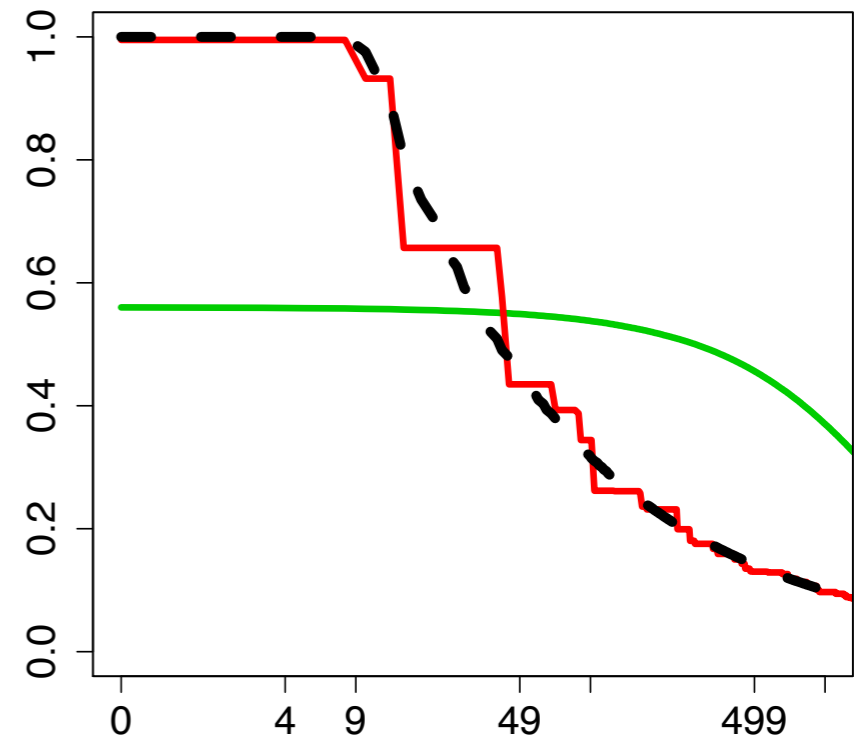
# Experimental results



power law,  $\alpha=1.5$ ,  $n=5M$

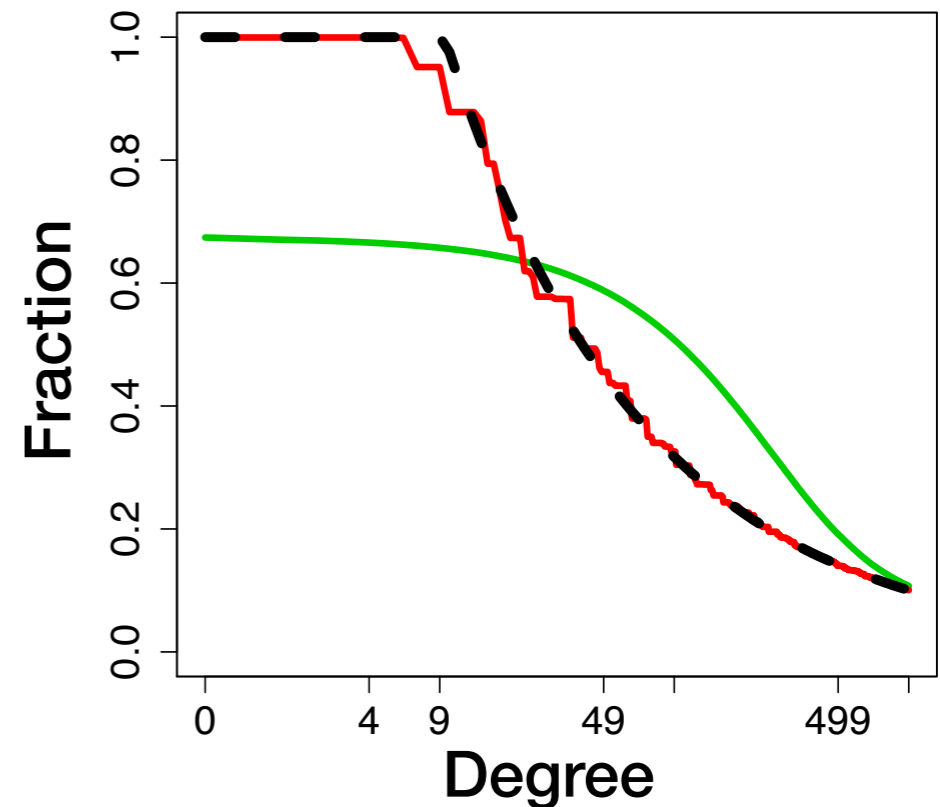
(100-edge,  
0.1-differential  
privacy)

$\epsilon=.001$

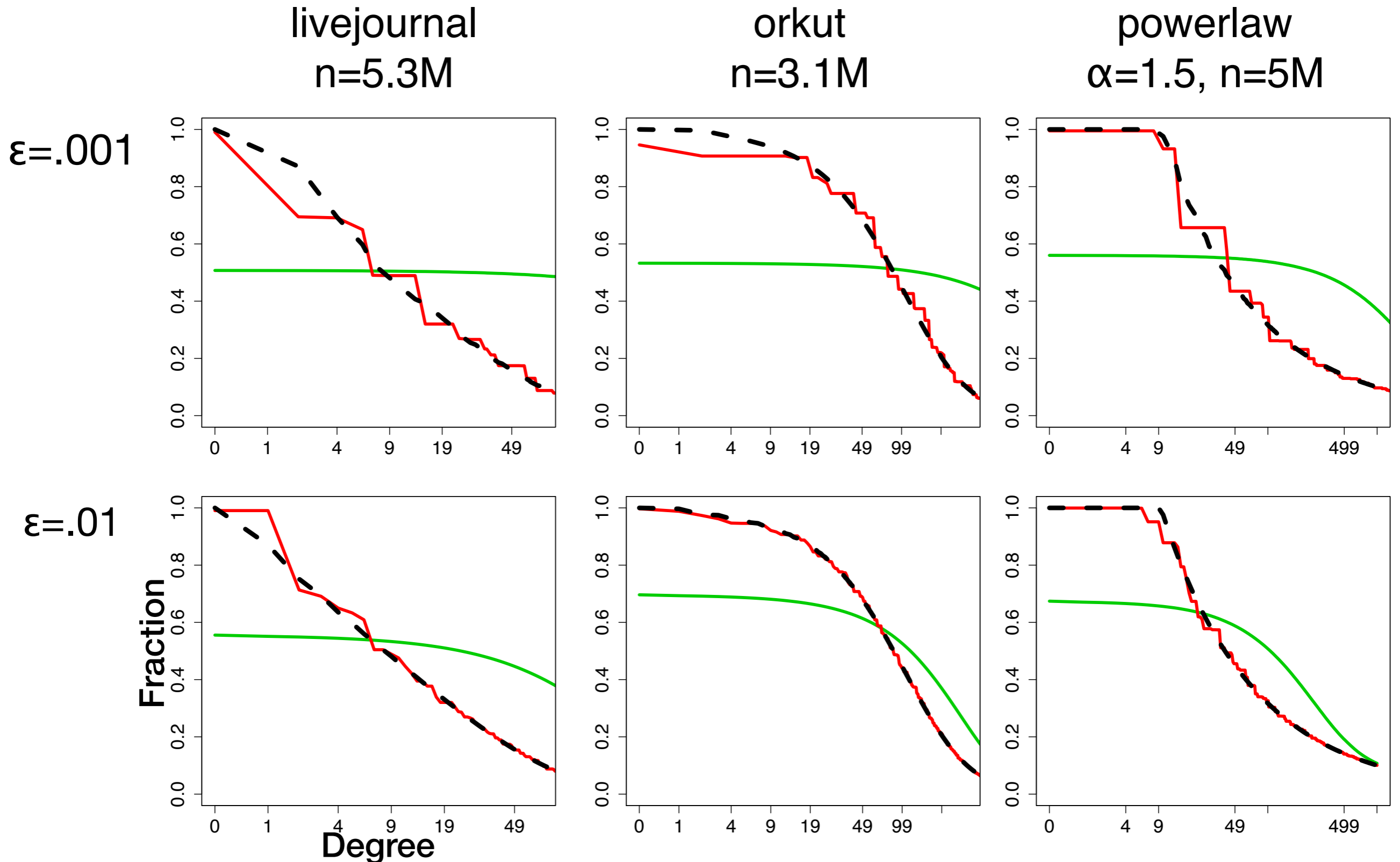


(10-edge,  
0.1-differential  
privacy)

$\epsilon=.01$



# Experimental results, continued



# Inference does **not** weaken privacy

DATA OWNER

ANALYST



# Inference does **not** weaken privacy

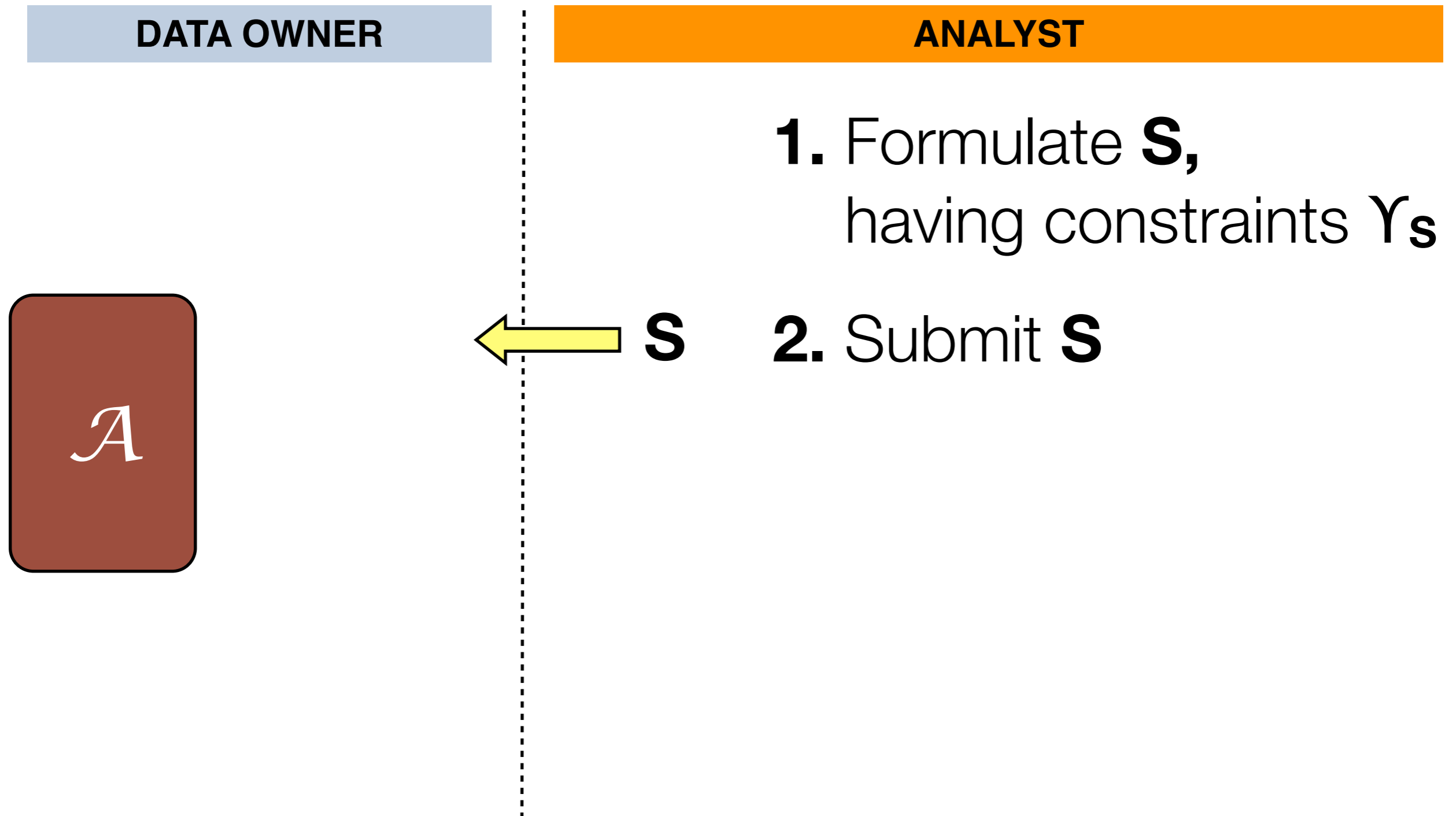
DATA OWNER

ANALYST

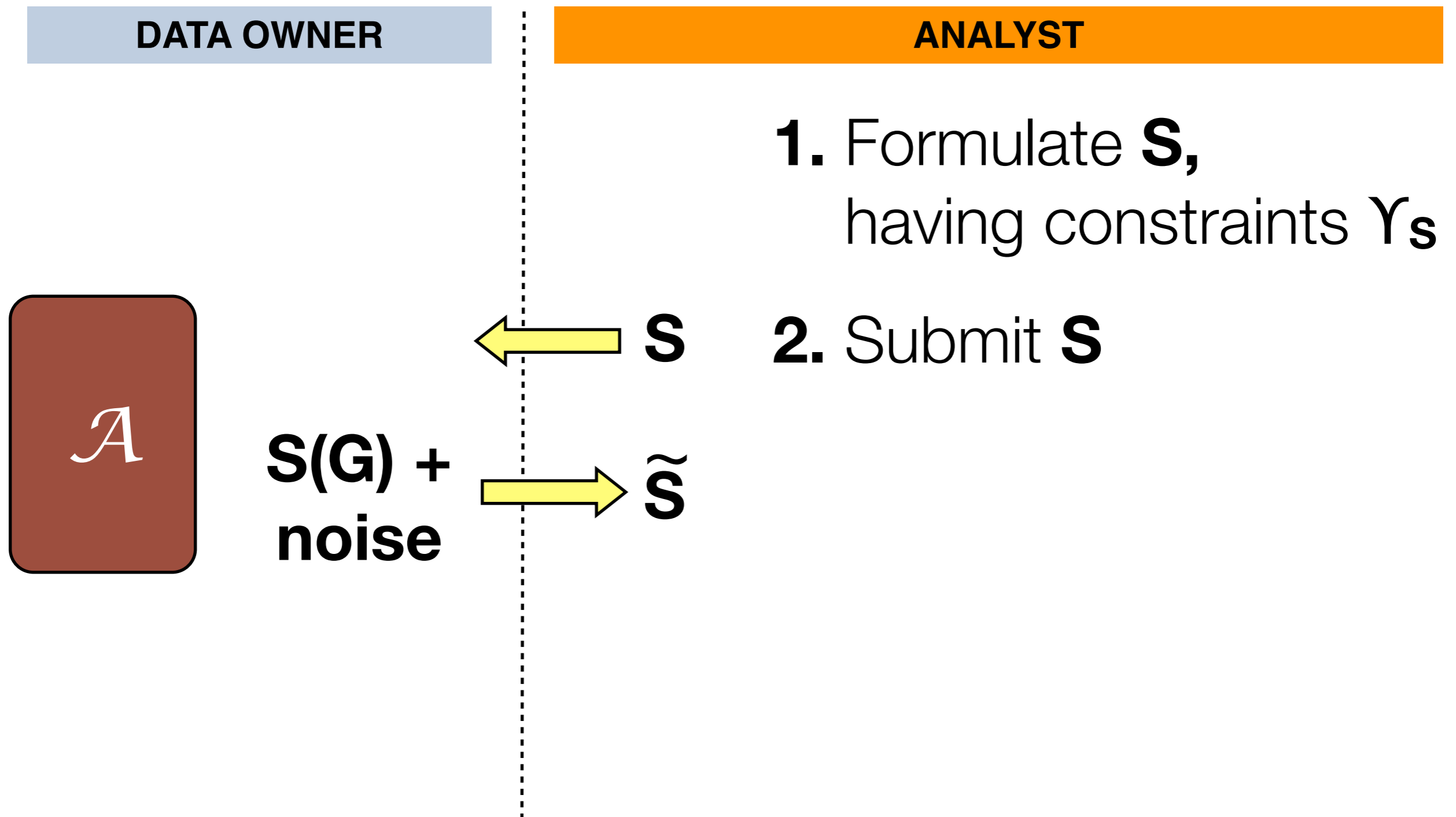


1. Formulate  $\mathbf{S}$ ,  
having constraints  $\Upsilon_{\mathbf{S}}$

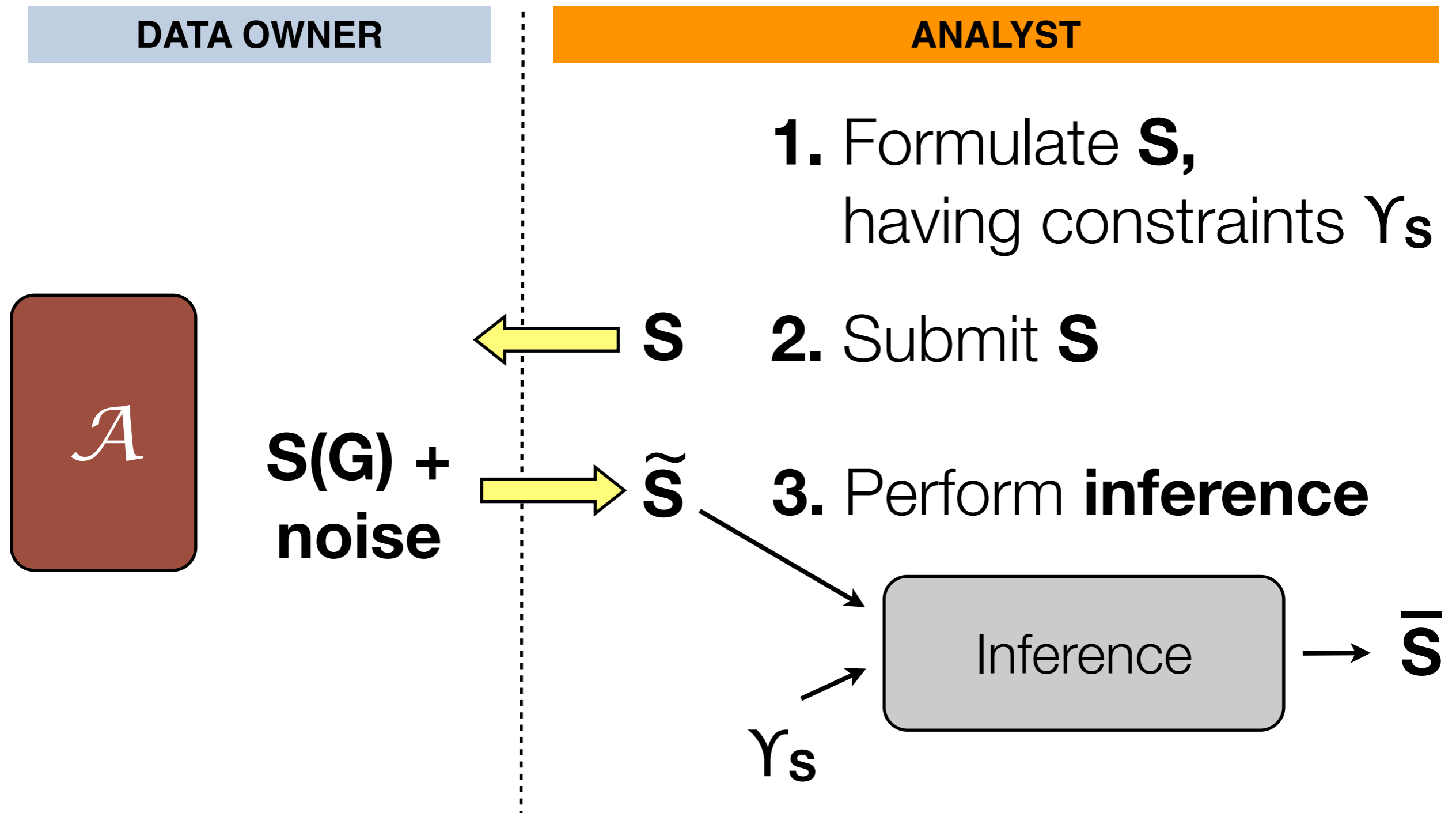
# Inference does **not** weaken privacy



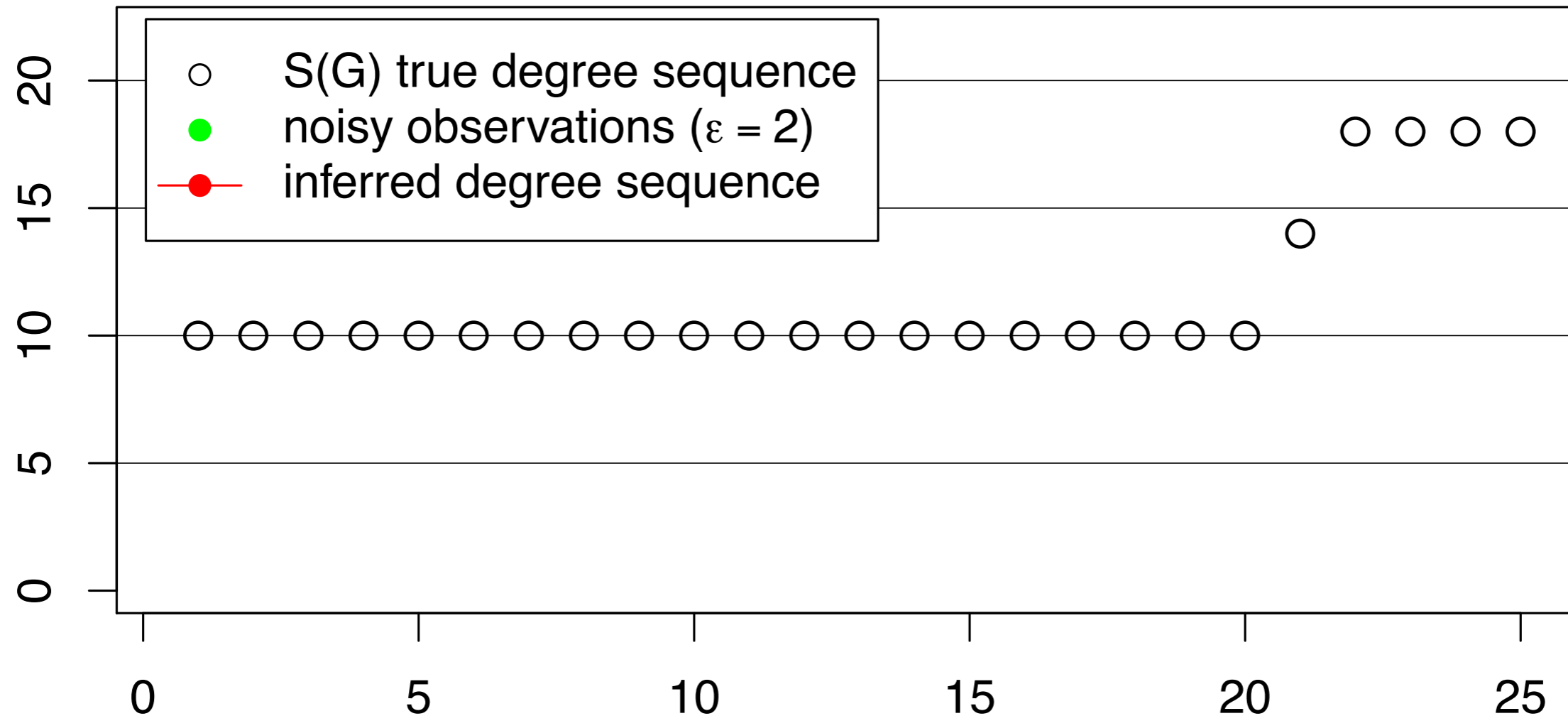
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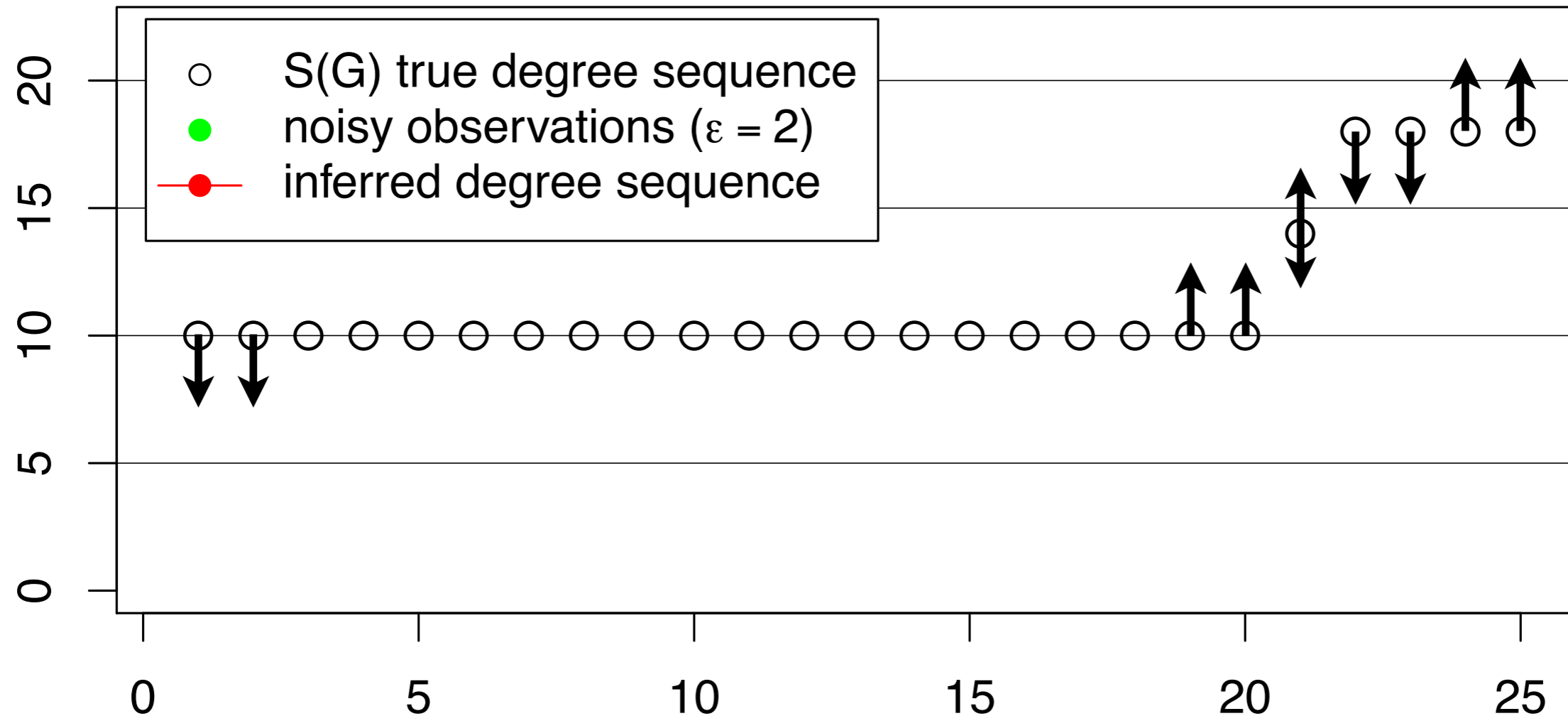
# After inference, noise only where needed



- Standard Laplace noise is sufficient *but not necessary* for differential privacy.
- By using inference, effectively apply a different noise distribution -- more noise where it is needed, less otherwise.
  - Improvement in accuracy will depend on sequence

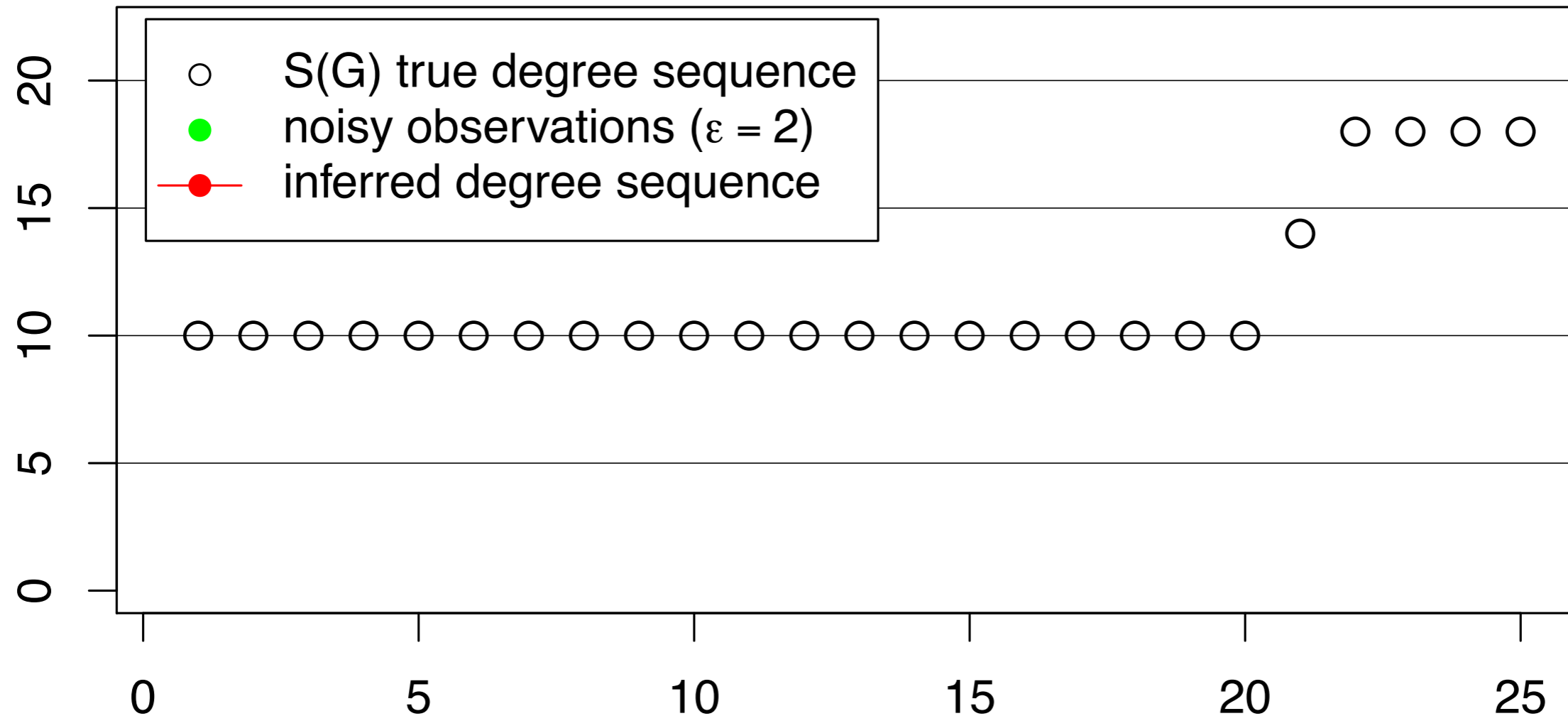


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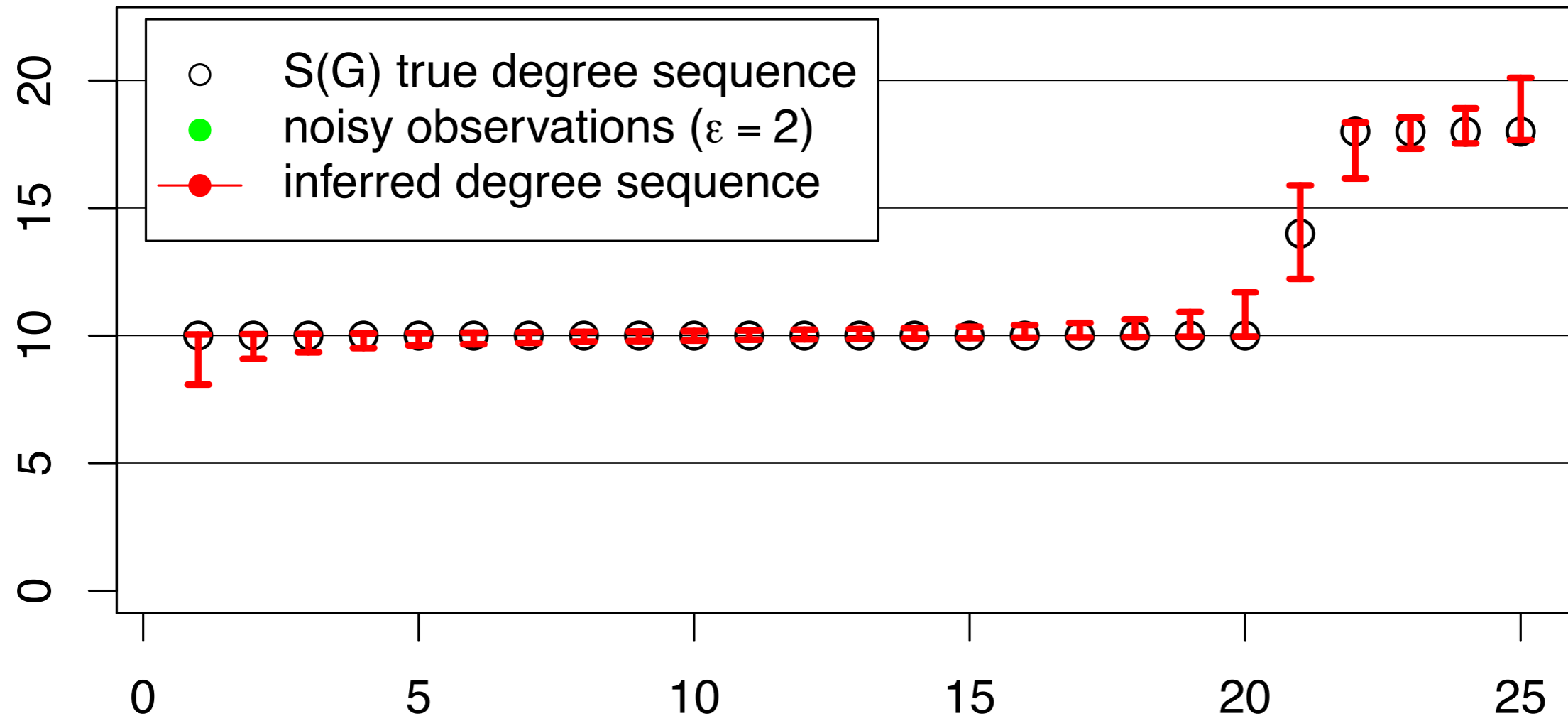
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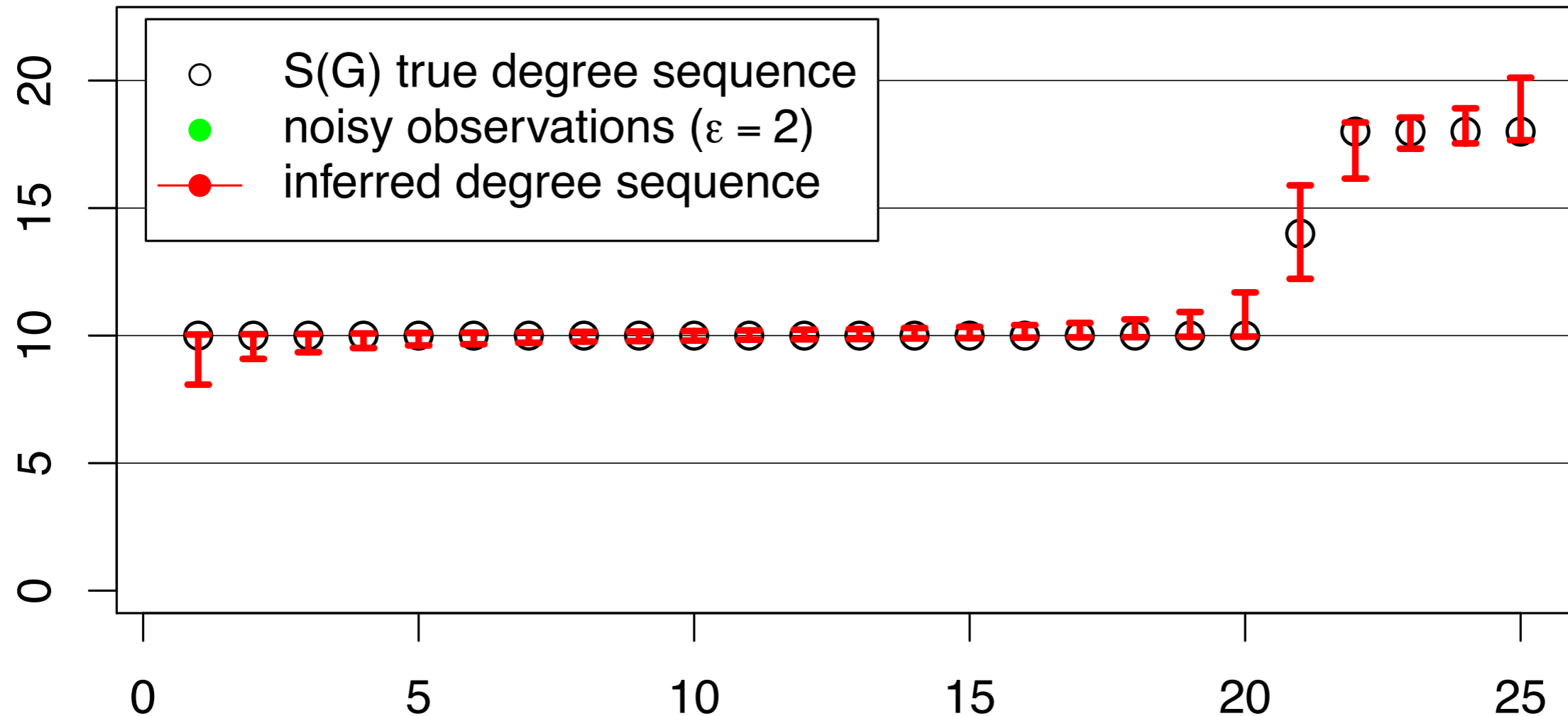
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# Accuracy is improved without sacrificing privacy!

---

- The accuracy achieved **depends on the input sequence.**

Mean Squared Error of Degree Sequence

Before inference,  $\tilde{\mathbf{S}} \quad \Theta(n/\epsilon^2)$

After inference,  $\bar{\mathbf{S}} \quad O(d \log^3 n / \epsilon^2)$

number of distinct degrees

- Performing inference is efficient: the sorted sequence which minimizes the L2 distance has an elegant closed form solution:
  - shown  $O(n^2)$  in [Hay, PVLDB 10]
  - improved to  $O(n)$  in [Hay, ICDM 09]

# Outline

---

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# Accurate motif analysis is hard

---

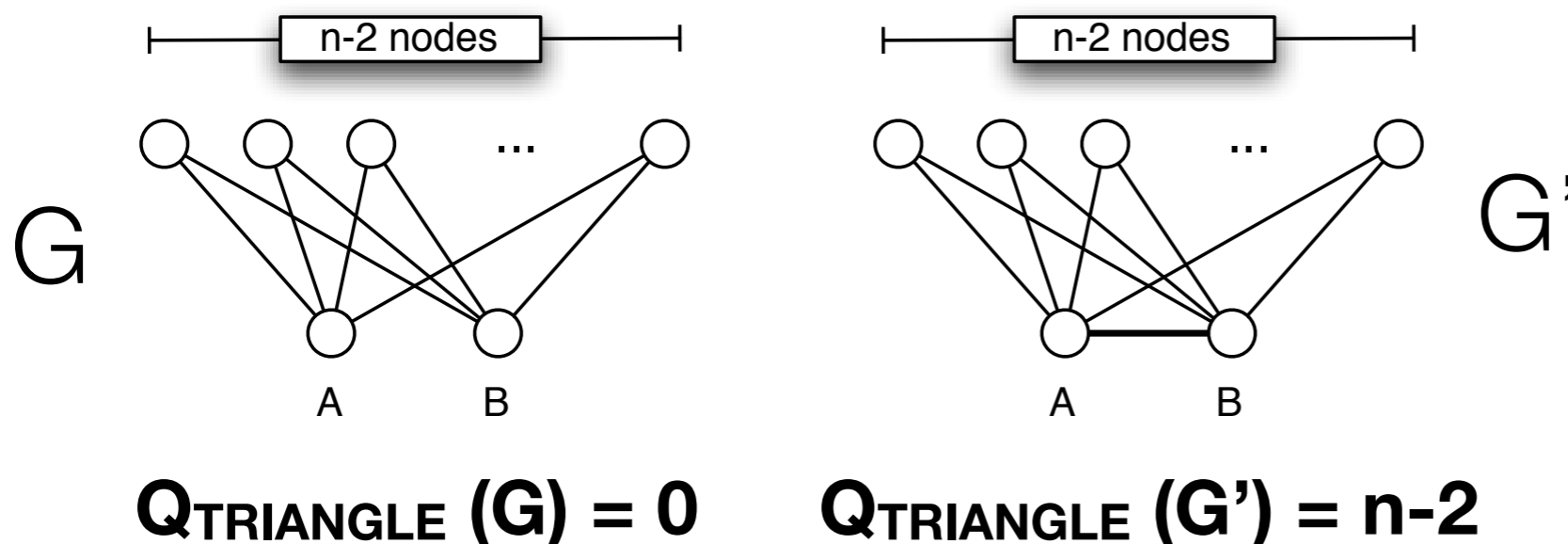
- Motif analysis measures the frequency of occurrence of small subgraphs in a network.
- Common example: **transitivity** in the network:
  - when A is friends with B and C, are B and C also friends?
  - **Q<sub>TRIANGLE</sub>**: return the number of triangles in the graph



# Accurate motif analysis is hard

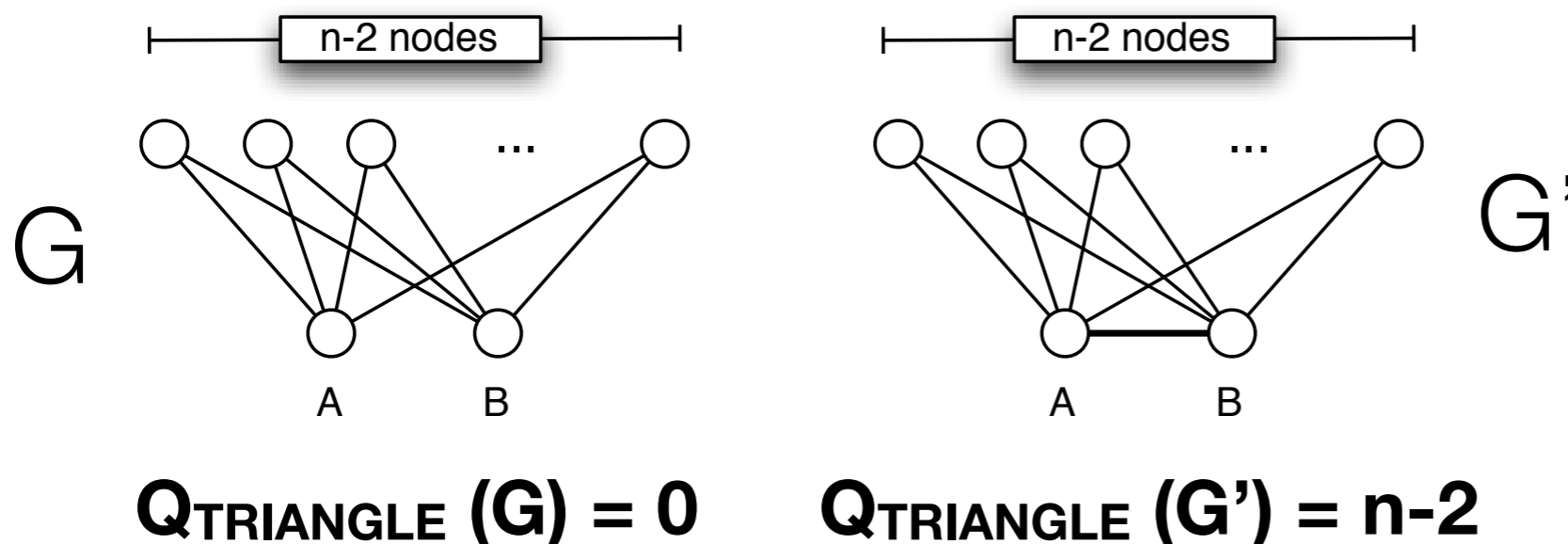
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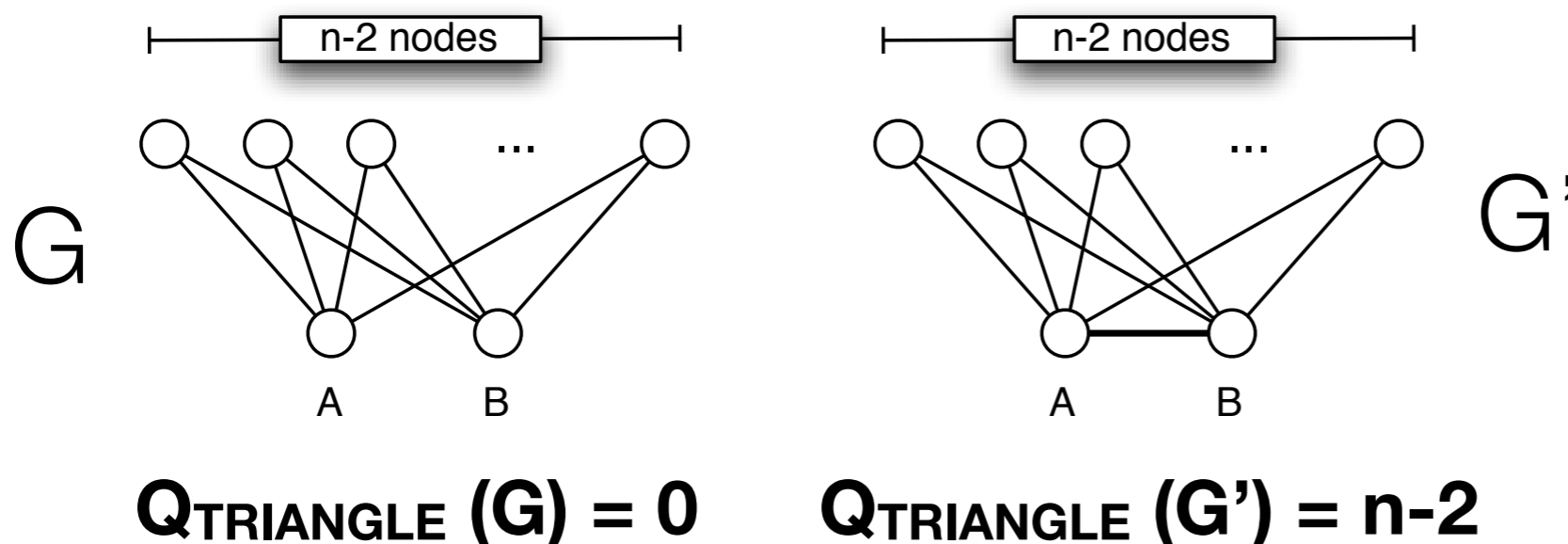
High Sensitivity:

$$\Delta Q_{\text{TRIANGLE}} = O(n)$$

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High Sensitivity:

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# Accurate motif analysis requires weakening privacy

---

- There exist output perturbation methods that achieve significantly better accuracy--expected error  $\Theta(\log^2 n)$  instead of  $\Theta(n)$  :
  - **[Rastogi, PODS 09]** Limiting assumptions on the prior knowledge of the adversary, and satisfying adversarial privacy.
    - works for general class of “motif” queries.
  - **[Nissim, STOC 07]** Under certain assumptions about the input graphs, and a modest relaxation of differential privacy:
    - works only for triangle queries (but could be extended).

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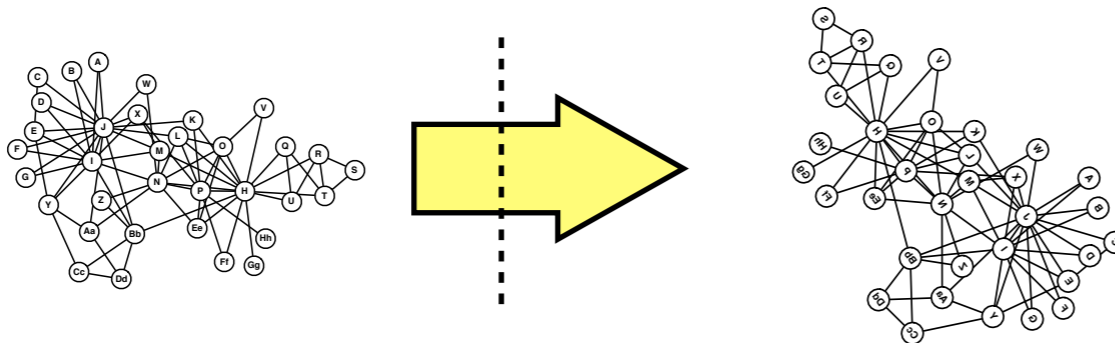
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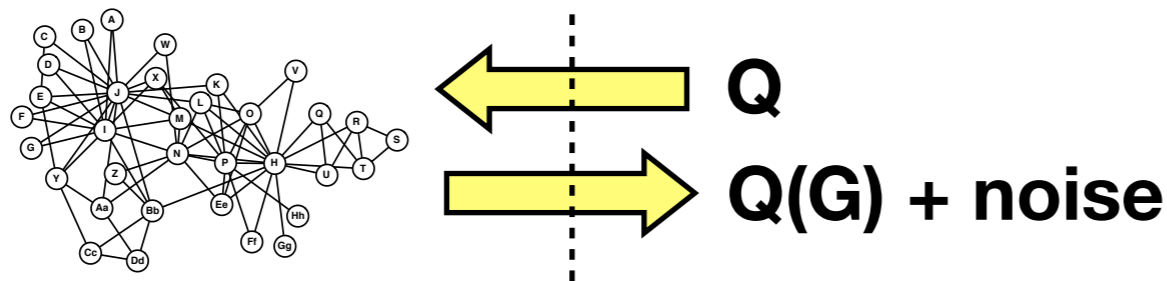
# Data publishing v. output perturbation

- Data publishing



<b>Ease of use</b>	good
<b>Privacy</b>	weak guarantees
<b>Accuracy</b>	no formal guarantees
<b>Scalability</b>	sometimes bad

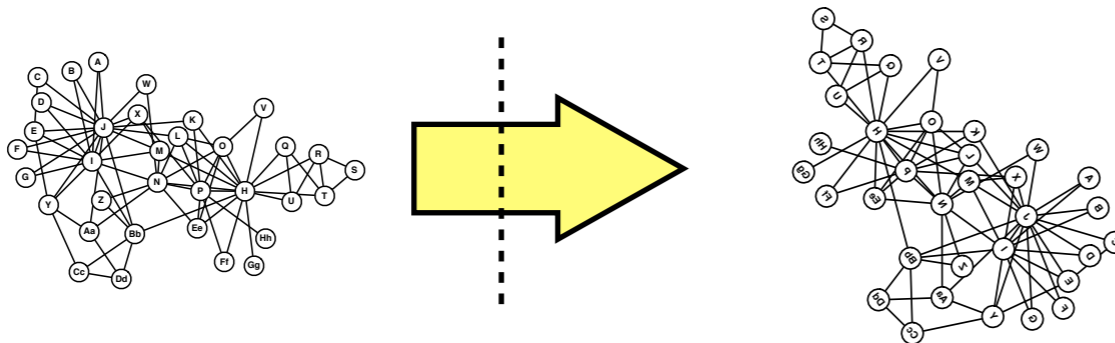
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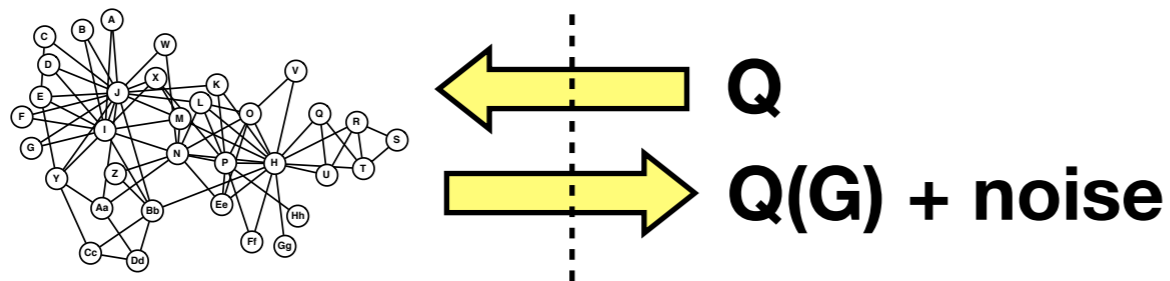
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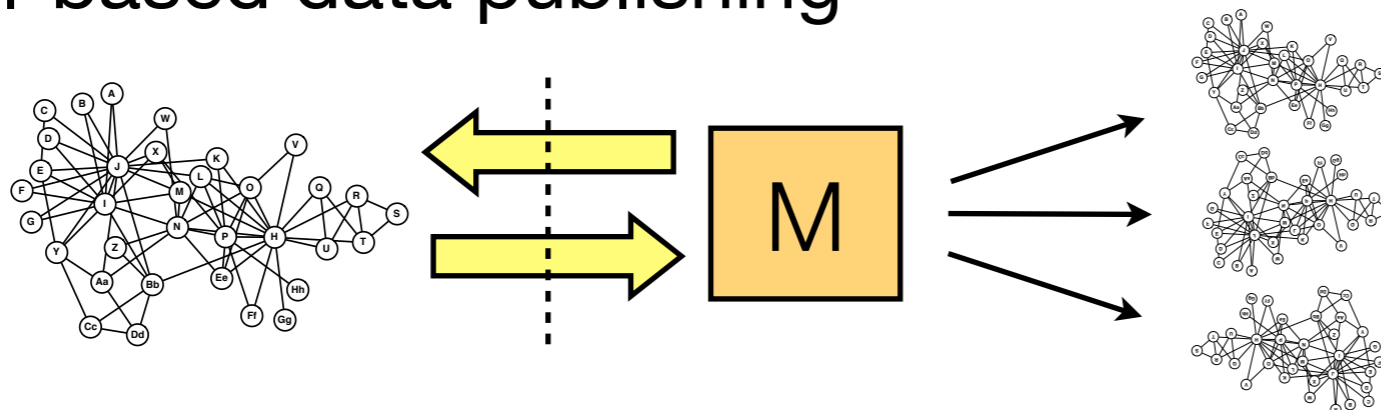
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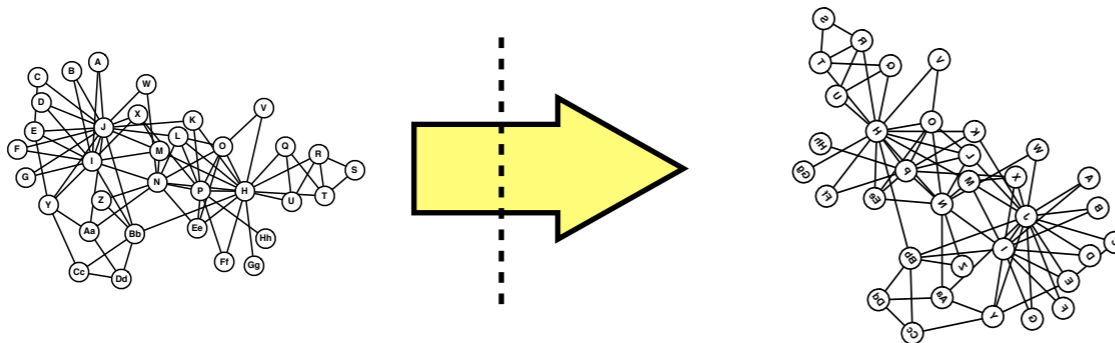
- Model-based data publishing





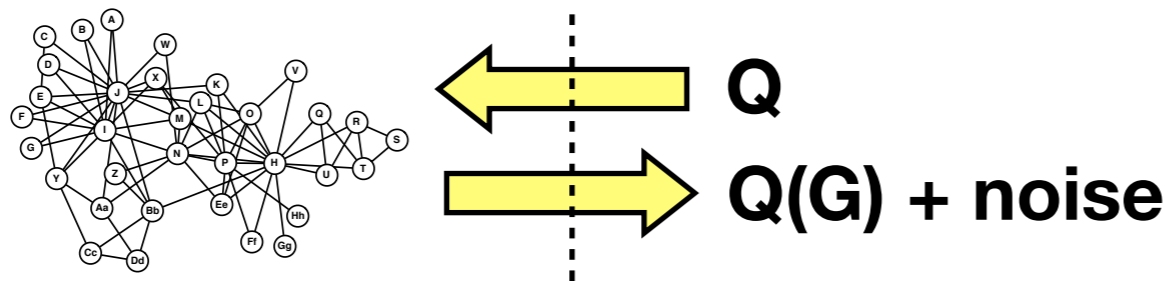
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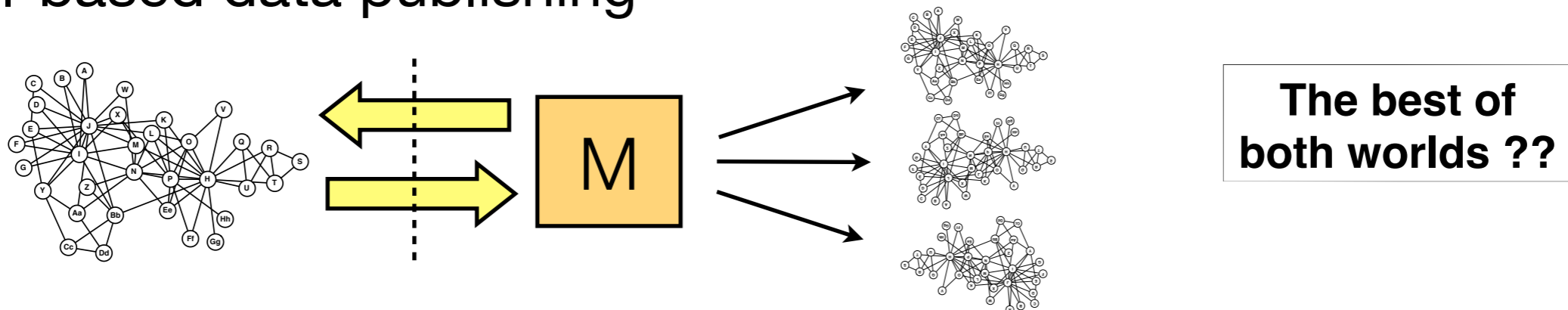
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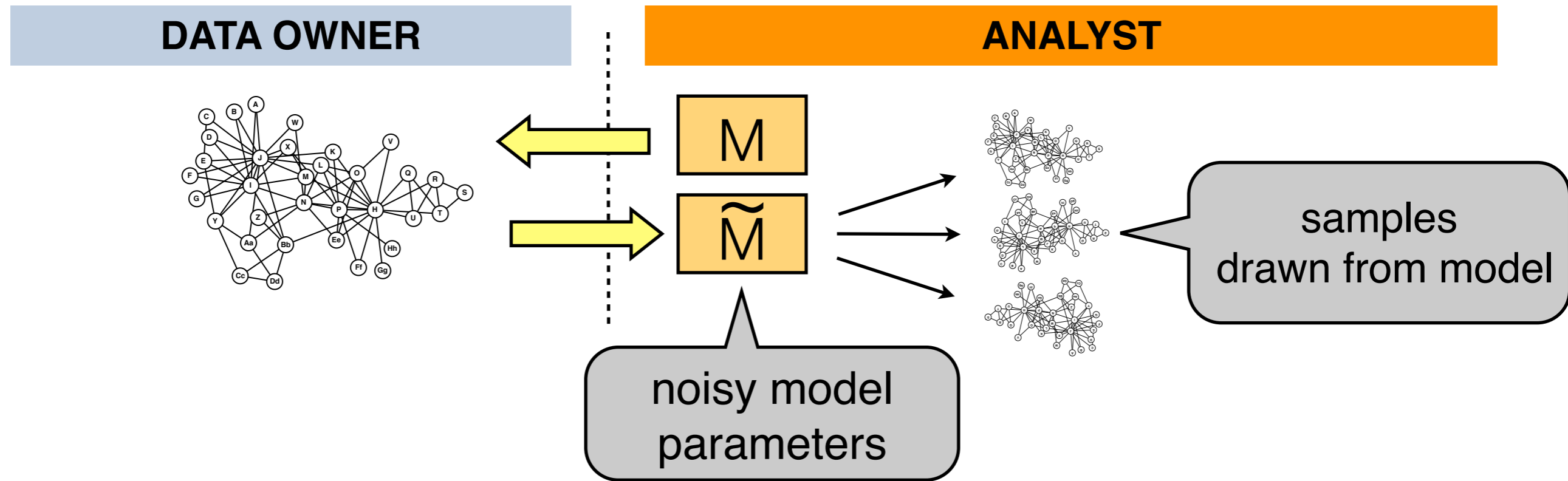


<b>Ease of use</b>	bad for practical
<b>Privacy</b>	formal guarantees
<b>Accuracy</b>	provable bounds
<b>Scalability</b>	very good

- Model-based data publishing

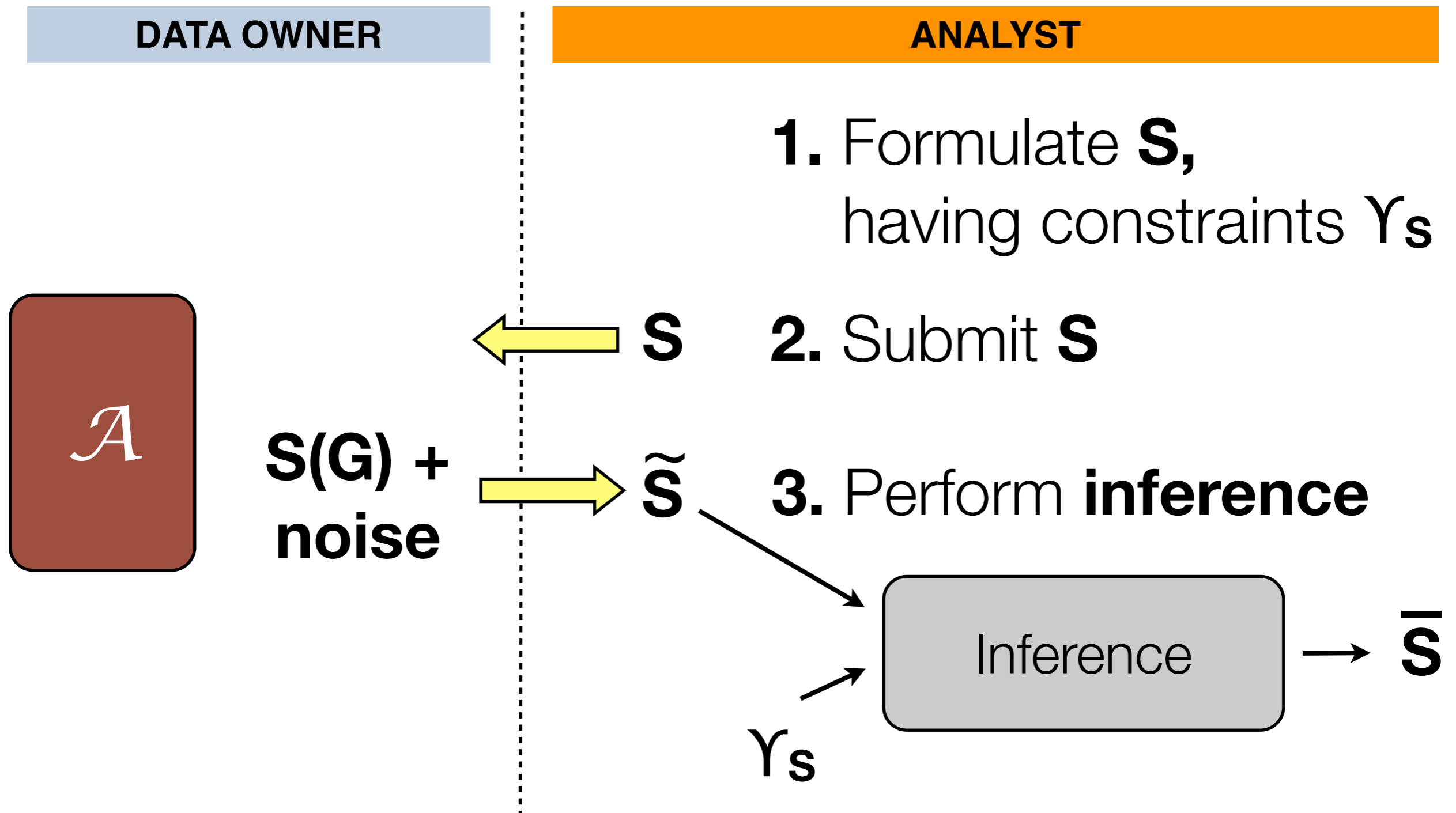


# Toward differentially-private synthetic data



- To realize the benefits of synthetic data, data owner can release noisy parameters of network model.
- Baseline: the degree distribution as network model
  - Deriving the power law parameter very accurate
  - Measuring clustering coefficient not constrained by deg. distr.

# A useful paradigm for improving accuracy



See [Hay, PVLDB 10]

# Questions?

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Additional details on our work may be found here:

- **[Hay, PVLDB 10]** M. Hay, V. Rastogi, G. Miklau, and D. Suciu. Boosting the accuracy of differentially-private queries through consistency. To appear, Proceedings of the VLDB Endowment (PVLDB), 2010.
- **[Hay, ICDM 09]** M. Hay, C. Li, G. Miklau, and D. Jensen. Accurate estimation of the degree distribution of private networks. In International Conference on Data Mining (ICDM) 2009.
- **[Rastogi, PODS 09]** V. Rastogi, M. Hay, G. Miklau, and D. Suciu. Relationship privacy: Output perturbation for queries with joins. In Principles of Database Systems (PODS), 2009.
- **[Hay, VLDB 08]** M. Hay, G. Miklau, D. Jensen, D. Towsley, and P. Weis. Resisting structural identification in anonymized social networks. In Proceedings of the VLDB Endowment (PVLDB), 2008.

# References

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- **[Backstrom, WWW 07]** L. Backstrom, C. Dwork, and J. Kleinberg. Wherefore art thou R3579X? Anonymized social networks hidden patterns and structural steganography. In WWW, 2007.
- **[Liu, SIGMOD 08]** K. Liu and E. Terzi. Towards identity anonymization on graphs. In SIGMOD, 2008.
- **[Zhou, ICDE 08]** B. Zhou and J. Pei. Preserving privacy in social networks against neighborhood attacks. In ICDE, 2008.
- **[Zou, VLDB 09]** L. Zou, L. Chen, and T. Ozsü. K-automorphism: A general framework for privacy preserving network publication. In Proceedings of VLDB Conference, 2009.
- **[Ying, SDM 2008]** X. Ying and X. Wu. Randomizing social networks: a spectrum preserving approach. In SIAM International Conference on Data Mining, 2008.
- **[Cormode, VLDB 08]** G. Cormode, D. Srivastava, T. Yu, and Q. Zhang. Anonymizing bipartite graph data using safe groupings. In VLDB Conference, 2008.

# References (con't)

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- **[Cormode, VLDB 09]** G. Cormode, D. Srivastava, S. Bhagat, and B. Krishnamurthy. Class-based graph anonymization for social network data. In VLDB Conference, 2009.
- **[Narayanan, OAKL 09]** A. Narayanan and V. Shmatikov. De-anonymizing social networks. In Security and Privacy, 2009.
- **[Campan, PinKDD 08]** A. Campan and T. M. Truta. A clustering approach for data and structural anonymity in social networks. In PinKDD, 2008.
- **[Rastogi, VLDB 07]** V. Rastogi, S. Hong, and D. Suciu. The boundary between privacy and utility in data publishing. In VLDB, pages 531–542, 2007.
- **[Dwork, TCC 06]** C. Dwork, F. McSherry, K. Nissim, and A. Smith. Calibrating noise to sensitivity in private data analysis. In Third Theory of Cryptography Conference, 2006.
- **[Nissim, STOC 07]** K. Nissim, S. Raskhodnikova, and A. Smith. Smooth sensitivity and sampling in private data analysis. In STOC, pages 75–84, 2007.