# CS345
# Data Mining

Page Rank Variants

# Review Page Rank

- Web graph encoded by matrix **M**
  - N£N matrix (N = number of web pages)
  - $M_{ij} = 1/|O(j)|$ iff there is a link from j to i
  - $M_{ij} = 0$ otherwise
  - O(j) = set of pages node i links to
- Define matrix **A** as follows
  - $A_{ij} = \beta M_{ij} + (1-\beta)/N$, where $0<\beta<1$
  - $1-\beta$ is the "tax" discussed in prior lecture
- Page rank **r** is first eigenvector of **A**
  - **Ar = r**

# Random walk interpretation

- At time 0, pick a page on the web uniformly at random to start the walk
- Suppose at time t, we are at page j
- At time t+1
  - With probability $\beta$, pick a page uniformly at random from O(j) and walk to it
  - With probability 1-$\beta$, pick a page on the web uniformly at random and teleport into it
- Page rank of page p = "steady state" probability that at any given time, the random walker is at page p

# Many random walkers

- Alternative, equivalent model
- Imagine a large number M of independent, identical random walkers (MÀN)
- At any point in time, let M(p) be the number of random walkers at page p
- The page rank of p is the fraction of random walkers that are expected to be at page p i.e., **E**[M(p)]/M.

# Problems with page rank

- Measures generic popularity of a page
  - Biased against topic-specific authorities
  - Ambiguous queries e.g., jaguar
  - This lecture
- Link spam
  - Creating artificial link topographies in order to boost page rank
  - Next lecture

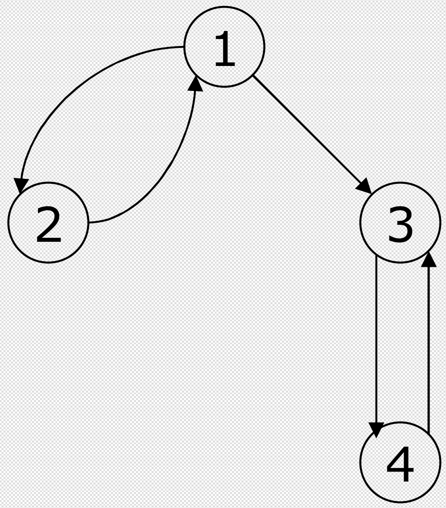
# Topic-Specific Page Rank

- Instead of generic popularity, can we measure popularity within a topic?
  - E.g., computer science, health
- Bias the random walk
  - When the random walker teleports, he picks a page from a set S of web pages
  - S contains only pages that are relevant to the topic
  - E.g., Open Directory (DMOZ) pages for a given topic (www.dmoz.org)
- Correspong to each teleport set S, we get a different rank vector $\mathbf{r_S}$

# Matrix formulation

- $A_{ij} = \beta M_{ij} + (1-\beta)/|S|$ if i 2 S
- $A_{ij} = \beta M_{ij}$ otherwise
- Show that **A** is stochastic
- We have weighted all pages in the teleport set S equally
  - Could also assign different weights to them

# Example




Suppose S = {1}, β = 0.8

| **Node** | **Iteration** | | | |
|---|---|---|---|---|
| | 0 | 1 | 2… | stable |
| 1 | 0.2 | 0.2 | 0.264 | 0.294 |
| 2 | 0 | 0.08 | 0.08 | 0.118 |
| 3 | 0 | 0.08 | 0.08 | 0.328 |
| 4 | 0 | 0 | 0.064 | 0.262 |

Note how we initialize the page rank vector differently from the unbiased page rank case.

# How well does TSPR work?

- Experimental results [Haveliwala 2000]
- Picked 16 topics
  - Teleport sets determined using DMOZ
  - E.g., arts, business, sports,…
- "Blind study" using volunteers
  - 35 test queries
  - Results ranked using Page Rank and TSPR of most closely related topic
  - E.g., bicycling using Sports ranking
  - In most cases volunteers preferred TSPR ranking

# Which topic ranking to use?

- User can pick from a menu
- Can use the context of the query
  - E.g., query is launched from a web page talking about a known topic
  - E.g., use Bayesian classification schemes to classify query into a topic (forthcoming lecture)
  - History of queries e.g., “basketball” followed by “jordan”
- User context e.g., user’s My Yahoo settings, bookmarks, …

# Scaling with topics and users

- Suppose we wanted to cover 1000's of topics
  - Need to compute 1000's of different rank vectors
  - Need to store and retrieve them efficiently at query time
  - For good performance vectors must fit in memory
- Even harder when we consider personalization
  - Each user has their own teleport vector
  - One page rank vector per user!

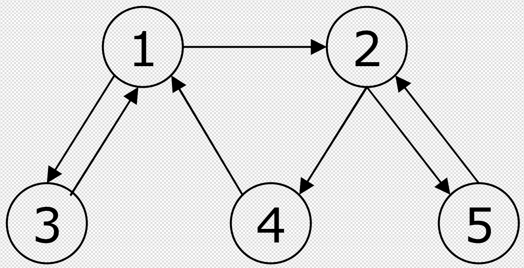
# Tricks

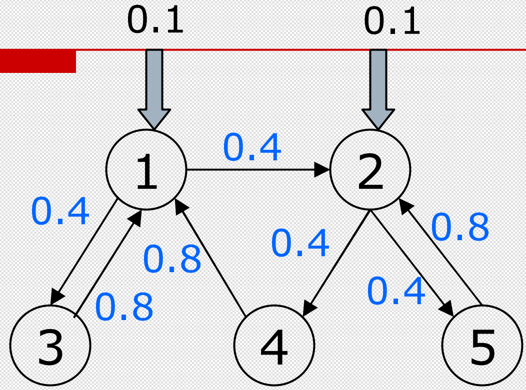
- Determine a set of basis vectors so that any rank vector is a linear combination of basis vectors
- Encode basis vectors compactly as partial vectors and a hubs skeleton
- At runtime perform a small amount of computation to derive desired rank vector elements

# Linearity Theorem

- Let S be a teleport set and $\mathbf{r}_S$ be the corresponding rank vector
- For page i2S, let $\mathbf{r}_i$ be the rank vector corresponding to the teleport set {i}
  - $\mathbf{r}_i$ is a vector with N entries
- $\mathbf{r}_S = (1/|S|) \sum_{i2S} \mathbf{r}_i$
- Why is linearity important?
  - Instead of $2^N$ biased page rank vectors we need to store N vectors

# Linearity example

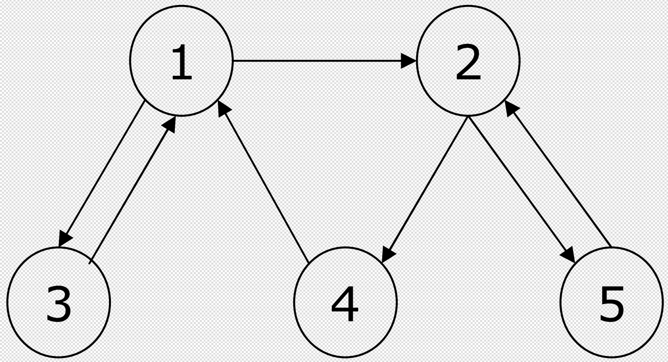



Let us compute $r_{\{1,2\}}$ for $\beta = 0.8$

| **Node** | **Iteration** | | | |
|---|---|---|---|---|
| | 0 | 1 | 2… | stable |
| 1 | 0.1 | 0.1 | 0.164 | 0.300 |
| 2 | 0.1 | 0.14 | 0.172 | 0.323 |
| 3 | 0 | 0.04 | 0.04 | 0.120 |
| 4 | 0 | 0.04 | 0.056 | 0.130 |
| 5 | 0 | 0.04 | 0.056 | 0.130 |

# Linearity example




| $r_{\{1,2\}}$ | $r_1$ | $r_2$ | $(r_1+r_2)/2$ |
|---|---|---|---|
| 0.300 | 0.407 | 0.192 | 0.300 |
| 0.323 | 0.239 | 0.407 | 0.323 |
| 0.120 | 0.163 | 0.077 | 0.120 |
| 0.130 | 0.096 | 0.163 | 0.130 |
| 0.130 | 0.096 | 0.163 | 0.130 |

# Intuition behind proof

- Let's use the many-random-walkers model with M random walkers
- Let us color a random walker with color i if his most recent teleport was to page i
- At time t, we expect M/|S| of the random walkers to be colored i
- At any page j, we would therefore expect to find $(M/|S|)r_i(j)$ random walkers colored i
- So total number of random walkers at page j = $(M/|S|)\sum_{i \in S} r_i(j)$

# Basis Vectors

- Suppose T = union of all teleport sets of interest
  - Call it the teleport universe
- We can compute the rank vector corresponding to any teleport set SµT as a linear combination of the vectors $\mathbf{r}_\mathbf{i}$ for i2T
- We call these vectors the basis vectors for T
- We can also compute rank vectors where we assign different weights to teleport pages

# Decomposition

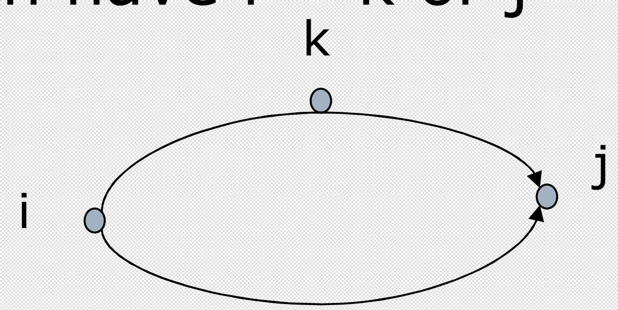
- Still too many basis vectors
  - E.g., |T| might be in the thousands
  - N|T| values
- Decompose basis vectors into partial vectors and hubs skeleton

# Tours

- Consider a random walker with teleport set {i}
  - Suppose walker is currently at node j
- The random walker's tour is the sequence of nodes on the walker's path since the last teleport
  - E.g., i,a,b,c,a,j
  - Nodes can repeat in tours – why?
- Interior nodes of the tour = {a,b,c,j}
- Start node = {i}, end node = {j}
  - A page can be both start node and interior node, etc

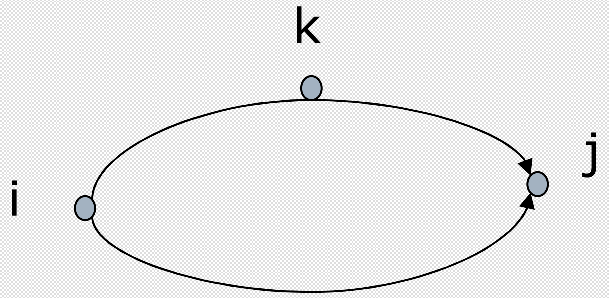
# Tour splitting

- Consider random walker with teleport set {i}, biased rank vector $r_i$
- $r_i(j)$ = probability random walker reaches j by following some tour with start node i and end node j
- Consider node k
  - Can have i = k or j = k

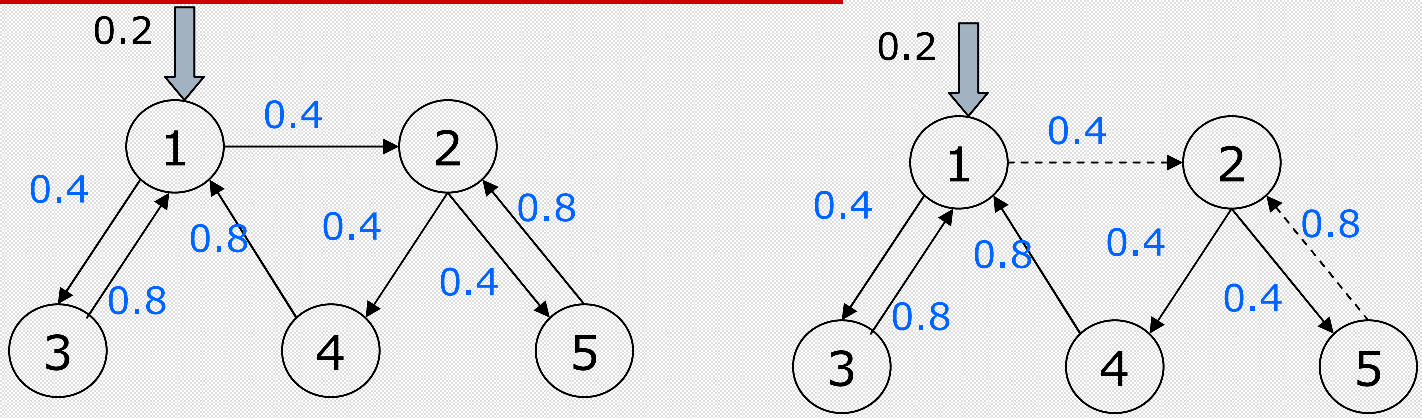


# Tour splitting

- Let $r_i^k(j)$ be the probability that random surfer reaches page j through a tour that includes page k as an interior node or end node
- Let $r_i^{\sim k}(j)$ be the probability that random surfer reaches page j through a tour that does not include k as an interior node or end node
- $r_i(j) = r_i^k(j) + r_i^{\sim k}(j)$



# Example

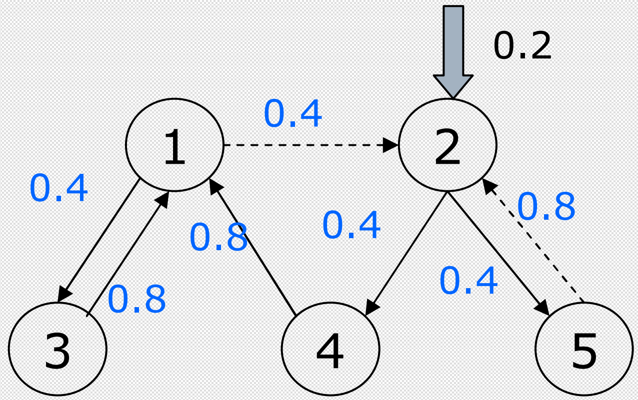



Let us compute $r_1^{\sim 2}$ for $\beta = 0.8$

| Node | Iteration | | | |
|---|---|---|---|---|
| | 0 | 1 | 2… | stable |
| 1 | 0.2 | 0.2 | 0.264 | 0.294 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0.08 | 0.08 | 0.118 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |

Note that many entries are zeros

# Example



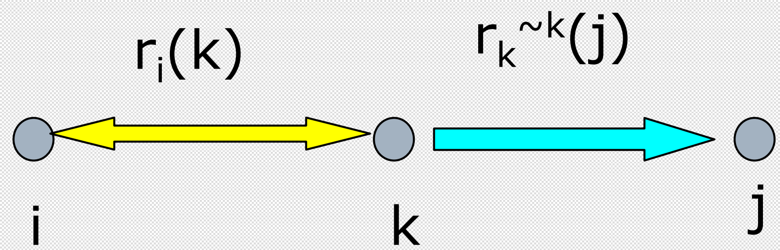

Let us compute $r_2^{\sim 2}$ for $\beta = 0.8$

| Node | Iteration | | | |
|---|---|---|---|---|
| | 0 | 1 | 2… | stable |
| 1 | 0 | 0 | 0.064 | 0.094 |
| 2 | 0.2 | 0.2 | 0.2 | 0.2 |
| 3 | 0 | 0 | 0 | 0.038 |
| 4 | 0 | 0.08 | 0.08 | 0.08 |
| 5 | 0 | 0.08 | 0.08 | 0.08 |

# Rank composition

- Notice:
  - $r_1^2(3) = r_1(3) - r_1^{\sim 2}(3)$
    $= 0.163 - 0.118 = 0.045$
  - $r_1(2) * r_2^{\sim 2}(3) = 0.239 * 0.038$
    $= 0.009$
    $= 0.2 * 0.045$
    $= (1-\beta)*r_1^2(3)$
  - $r_1^2(3) = r_1(2)\, r_2^{\sim 2}(3) / (1-\beta)$

# Rank composition

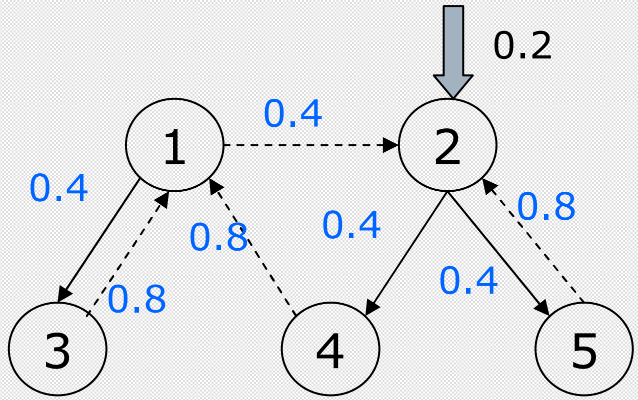



$$r_i^k(j) = r_i(k) r_k^{\sim k}(j)/(1-\beta)$$

# Hubs

- Instead of a single page k, we can use a set H of “hub” pages
  - Define $r_i^{\sim H}(j)$ as set of tours from i to j that do not include any node from H as interior nodes or end node

# Hubs example




H = {1,2}
$\beta = 0.8$

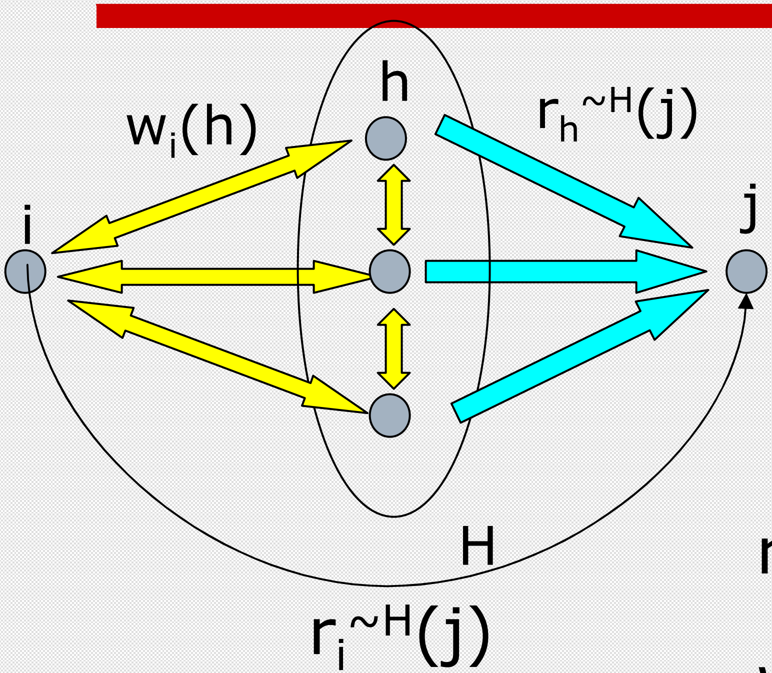
$r_2^{\sim H}$

| Node | Iteration | | |
|---|---|---|---|
| | 0 | 1 | stable |
| 1 | 0 | 0 | 0 |
| 2 | 0.2 | 0.2 | 0.2 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0.08 | 0.08 |
| 5 | 0 | 0.08 | 0.08 |

$r_1^{\sim H}$

| Node | Iteration | | |
|---|---|---|---|
| | 0 | 1 | stable |
| 1 | 0.2 | 0 | 0.2 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0.08 | 0.08 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |

# Rank composition with hubs



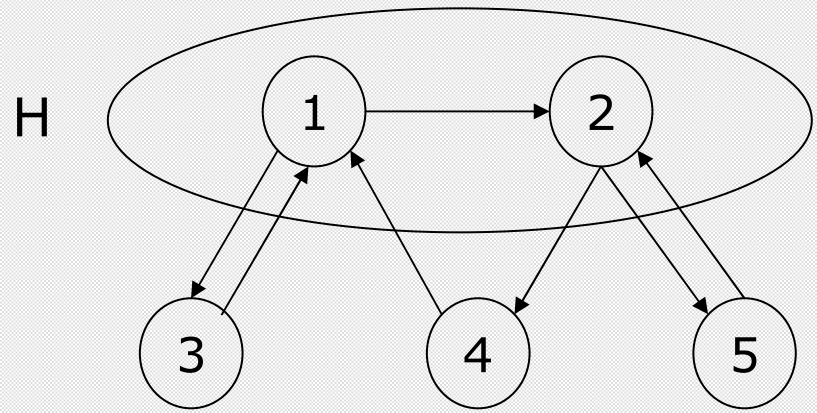

$$r_i(j) = r_i^{\sim H}(j) + r_i^{H}(j)$$

$$r_i^{H}(j) = \sum_{h2H} w_i(h) r_h^{\sim H}(j)/(1-\beta)$$

$$w_i(h) = r_i(h) \text{ if } i \neq h$$

$$w_i(h) = r_i(h) - (1-\beta) \text{ if } i = h$$

# Hubs rule example



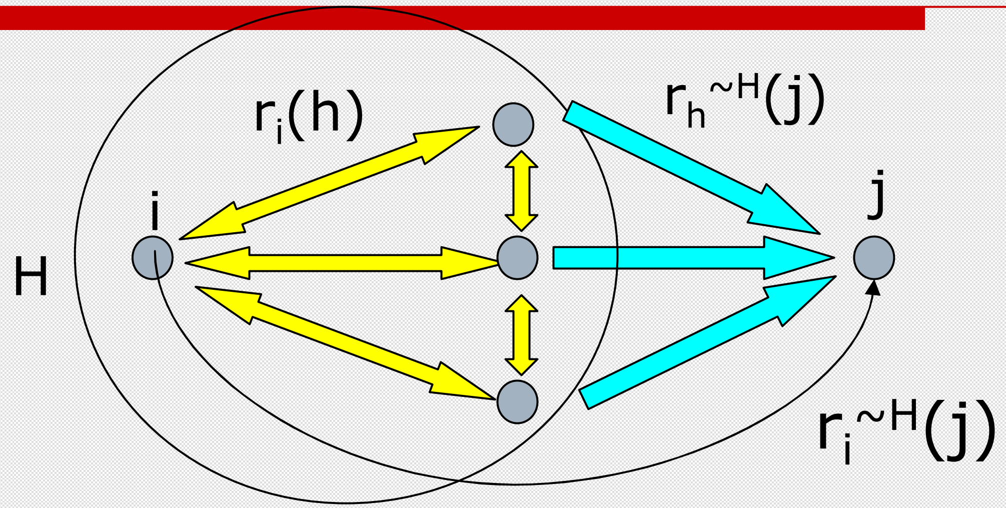

$H = \{1,2\}$
$\beta = 0.8$

$$r_2(3) = r_2^{\sim H}(3) + r_2^{H}(3) = 0 + r_2^{H}(3)$$
$$= [r_2(1)r_1^{\sim H}(3)]/0.2+[(r_2(2)-0.2)r_2^{\sim H}(3)]/0.2$$
$$= [0.192*0.08]/0.2+[(0.407-0.2)*0]/0.2$$
$$= 0.077$$

# Hubs

- Start with H = T, the teleport universe
- Add nodes to H such that given any pair of nodes i and j, there is a high probability that H separates i and j
  - i.e., $r_i^{\sim H}(j)$ is zero for most i,j pairs
- Observation: high page rank nodes are good separators and hence good hub nodes

# Hubs skeleton




- To compute $r_i(j)$ we need:
  - $r_i^{\sim H}(j)$ for all i2H, j2V
    - called the partial vector
    - Sparse
  - $r_i(h)$ for all h2H
    - called the hubs skeleton

# Storage reduction

- Say |T| = 1000, |H|=2000, N = 1 billion
- Store all basis vectors
  - 1000*1 billion = 1 trillion nonzero values
- Use partial vectors and hubs skeleton
  - Suppose each partial vector has N/200 nonzero entries
  - Partial vectors = 2000*N/200 = 10 billion nonzero values
  - Hubs skeleton = 2000*2000 = 4 million values
  - Total = approx 10 billion nonzero values
- Approximately 100x compression