Finding Similar Pairs

Divide-Compute-Merge

Locality-Sensitive Hashing

Applications

Finding Similar Pairs

- Suppose we have in main memory data representing a large number of objects.
  - May be the objects themselves (e.g., summaries of faces).
  - May be signatures as in minhashing.
- We want to compare each to each, finding those pairs that are sufficiently similar.

Candidate Generation From Minhash Signatures

- Pick a similarity threshold s, a fraction < 1.
- A pair of columns c and d is a *candidate pair* if their signatures agree in at least fraction s of the rows.
  - I.e., $M(i, c) = M(i, d)$ for at least fraction s values of i.

Other Notions of “Sufficiently Similar”

- For images, a pair of vectors is a candidate if they differ by at most a small amount t in at least s % of the components.
- For entity records, a pair is a candidate if the sum of similarity scores of corresponding components exceeds a threshold.

Checking All Pairs is Hard

- While the signatures of all columns may fit in main memory, comparing the signatures of all pairs of columns is quadratic in the number of columns.
- Example: 10^6 columns implies $5*10^{11}$ comparisons.
- At 1 microsecond/comparison: 6 days.

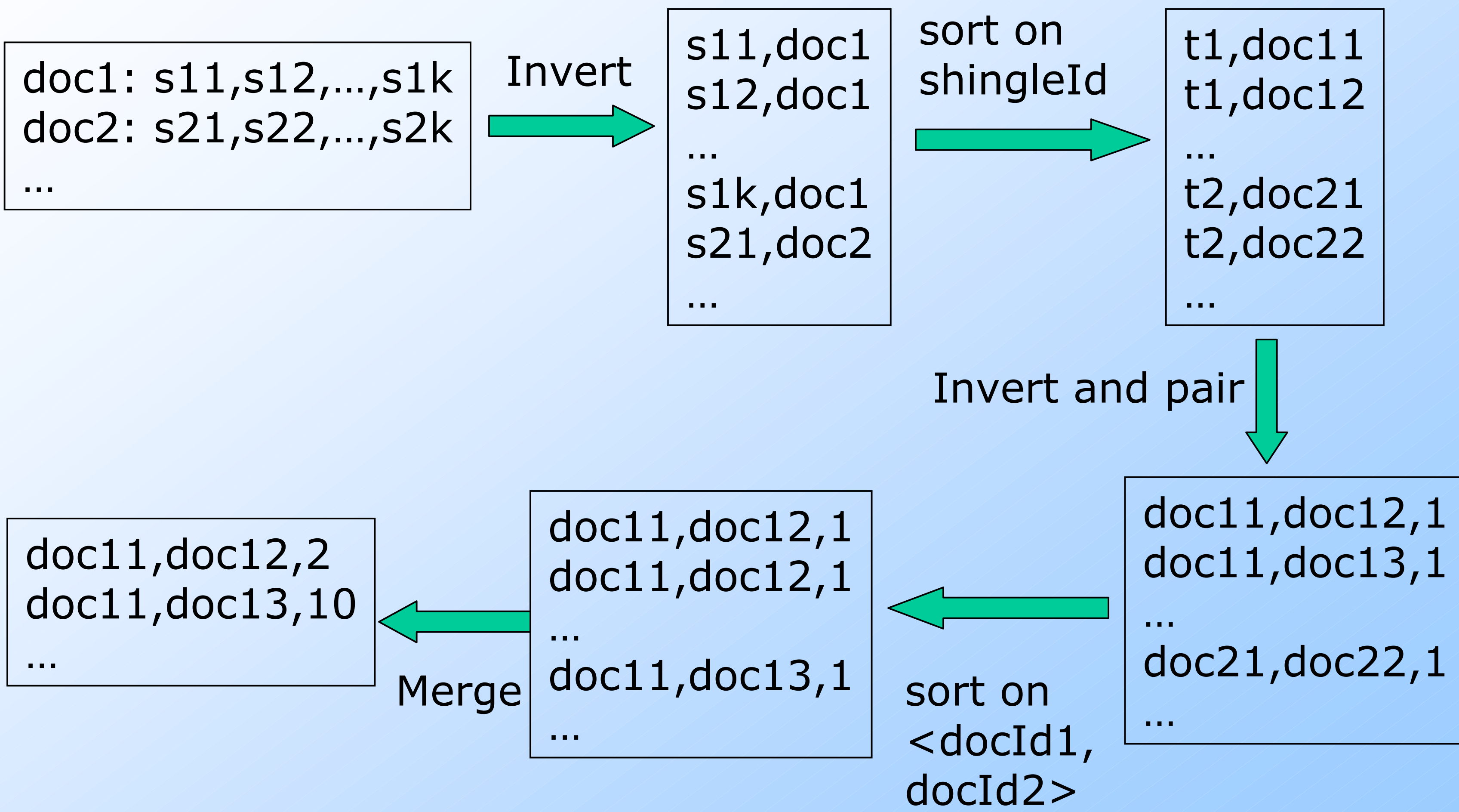
Solutions

1. *Divide-Compute-Merge* (DCM) uses external sorting, merging.
2. *Locality-Sensitive Hashing* (LSH) can be carried out in main memory, but admits some false negatives.

Divide-Compute-Merge

- Designed for "shingles" and docs.
  - Or other problems where data is presented by column.
- At each stage, divide data into batches that fit in main memory.
- Operate on individual batches and write out partial results to disk.
- Merge partial results from disk.

DCM Steps


doc1: s11,s12,…,s1k
doc2: s21,s22,…,s2k
…
Invert
s11,doc1
s12,doc1
…
s1k,doc1
s21,doc2
…
sort on
shingleId
t1,doc11
t1,doc12
…
t2,doc21
t2,doc22
…
Invert and pair
doc11,doc12,1
doc11,doc13,1
…
doc21,doc22,1
…
sort on
<docId1,
docId2>
doc11,doc12,1
doc11,doc12,1
…
doc11,doc13,1
…
Merge
doc11,doc12,2
doc11,doc13,10
…

DCM Summary

1. Start with the pairs <shingleId, docId>.
2. Sort by shingleId.
3. In a sequential scan, generate triplets <docId1, docId2, 1> for pairs of docs that share a shingle.
4. Sort on <docId1, docId2>.
5. Merge triplets with common docIds to generate triplets of the form <docId1,docId2,count>.
6. Output document pairs with count > threshold.

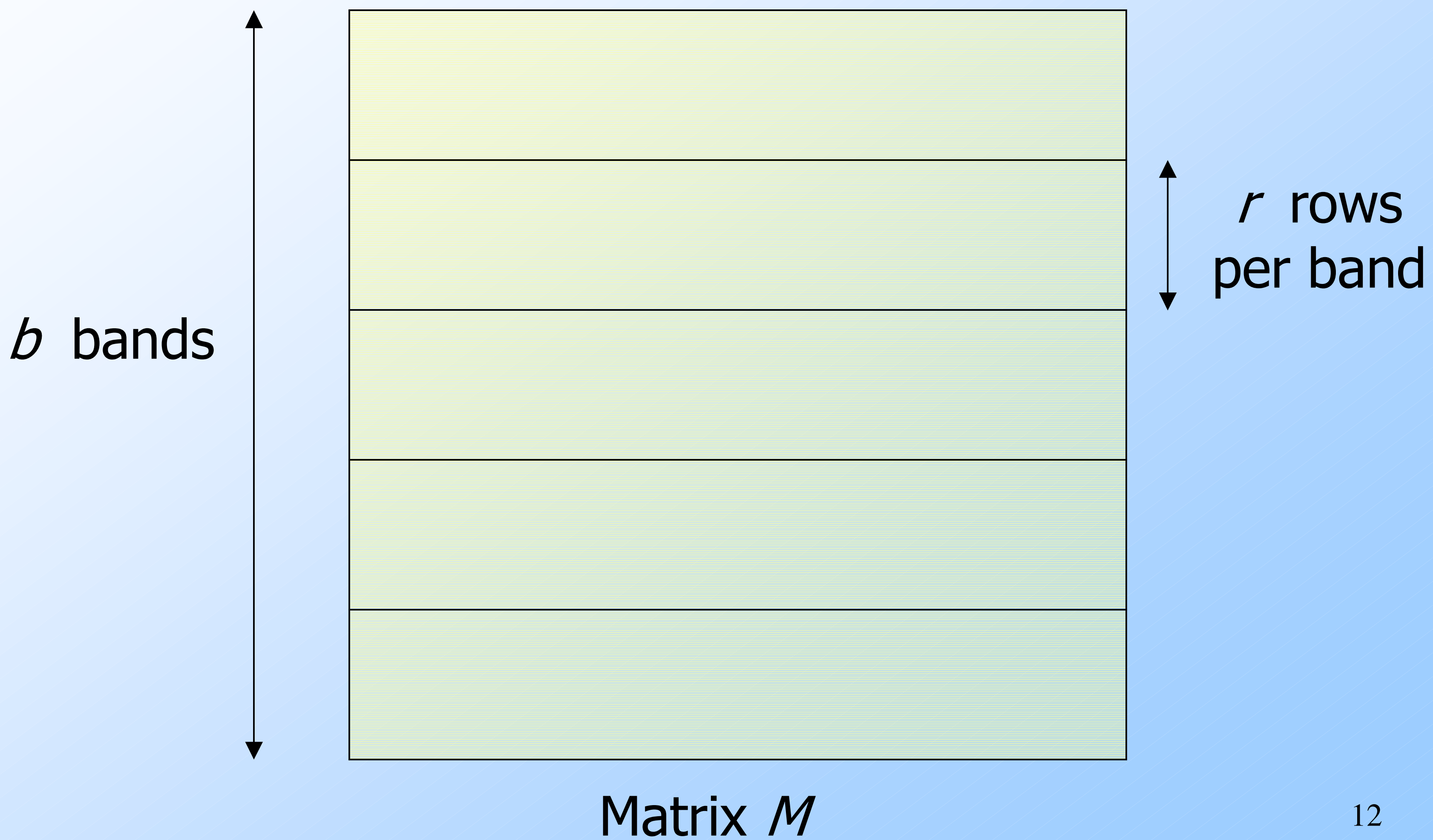
Some Optimizations

- “Invert and Pair” is the most expensive step.
- Speed it up by eliminating very common shingles.
  - “the”, “404 not found”, “<A HREF”, etc.
- Also, eliminate exact-duplicate docs first.

Locality-Sensitive Hashing

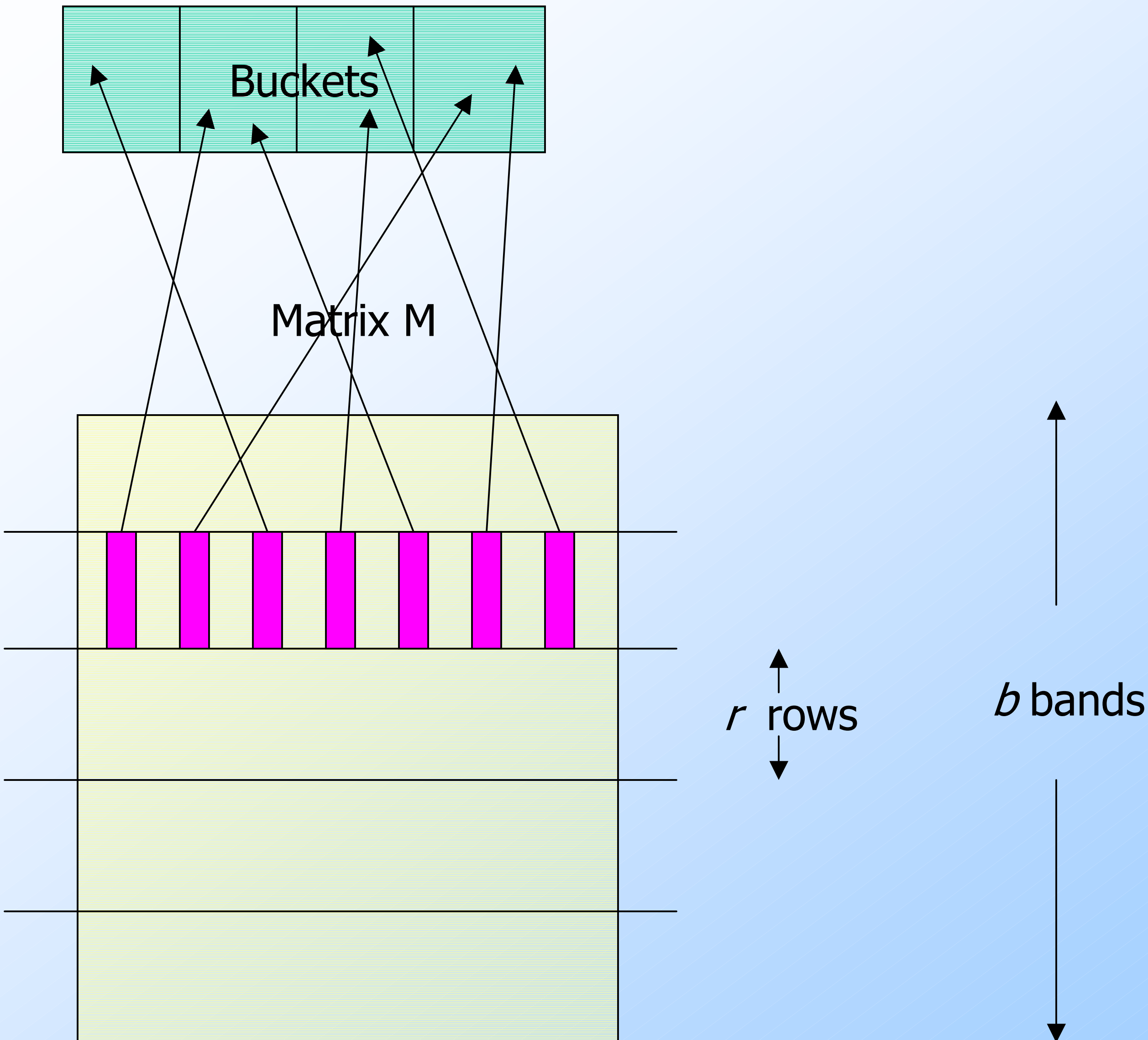
- Big idea: hash columns of signature matrix *M* several times.
- Arrange that (only) similar columns are likely to hash to the same bucket.
- Candidate pairs are those that hash at least once to the same bucket.

Partition Into Bands



Partition into Bands – (2)

- Divide matrix M into b bands of r rows.
- For each band, hash its portion of each column to a hash table with k buckets.
- *Candidate* column pairs are those that hash to the same bucket for ≥ 1 band.
- Tune b and r to catch most similar pairs, but few nonsimilar pairs.


Buckets
Matrix M
r rows
b bands

Simplifying Assumption

- There are enough buckets that columns are unlikely to hash to the same bucket unless they are identical in a particular band.
- Hereafter, we assume that "same bucket" means "identical."

Example

- Suppose 100,000 columns.
- Signatures of 100 integers.
- Therefore, signatures take 40Mb.
- But 5,000,000,000 pairs of signatures can take a while to compare.
- Choose 20 bands of 5 integers/band.

Suppose C_1, C_2 are 80% Similar

- Probability C_1, C_2 identical in one particular band: $(0.8)^5 = 0.328$.
- Probability C_1, C_2 are *not* similar in any of the 20 bands: $(1\text{-}0.328)^{20} = .00035$.
  - i.e., we miss about 1/3000th of the 80%-similar column pairs.

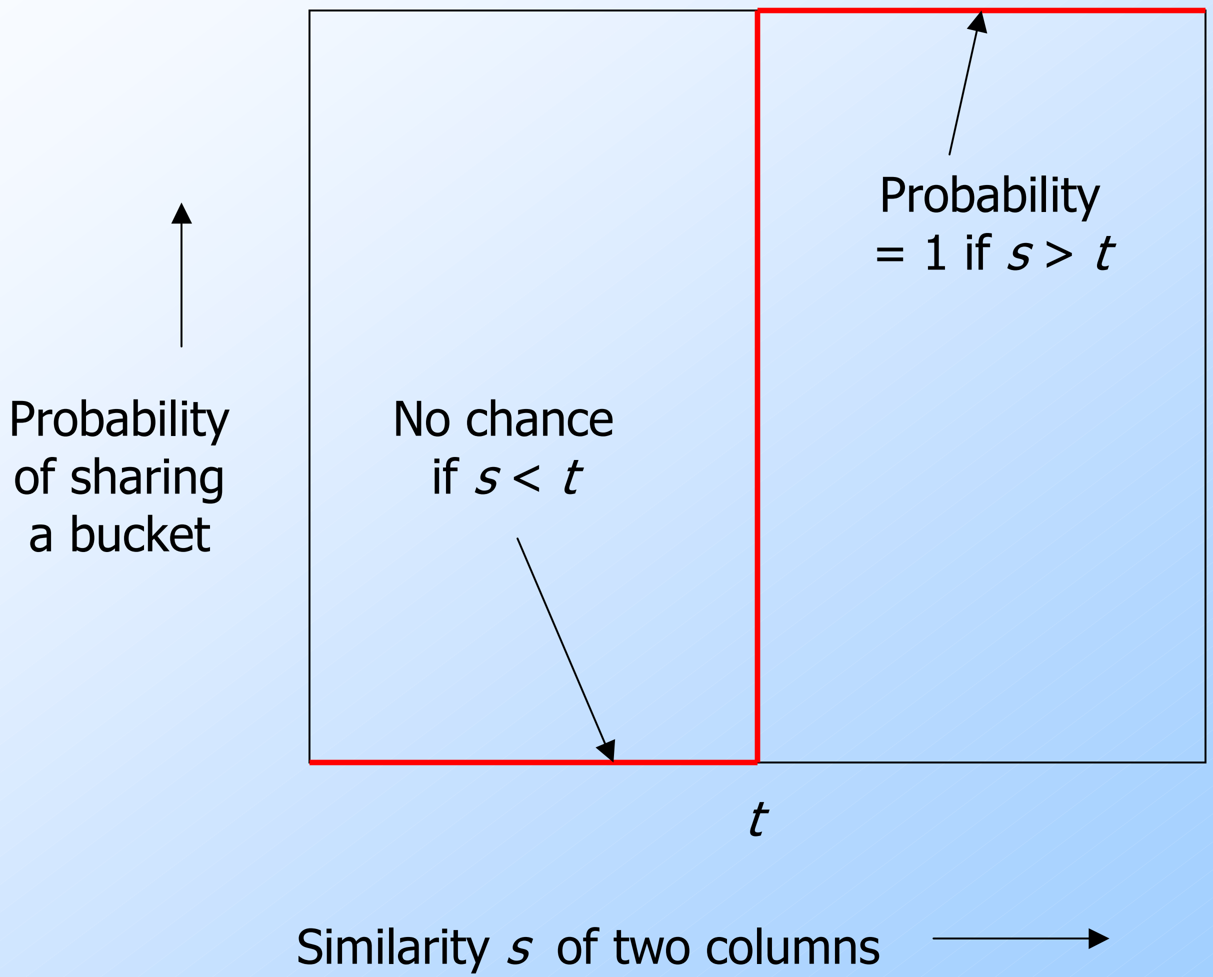
Suppose C_1, C_2 Only 40% Similar

- Probability C_1, C_2 identical in any one particular band: $(0.4)^5 = 0.01$.
- Probability C_1, C_2 identical in $\geq$ 1 of 20 bands: $\leq 20 * 0.01 = 0.2$.
- But false positives much lower for similarities << 40%.

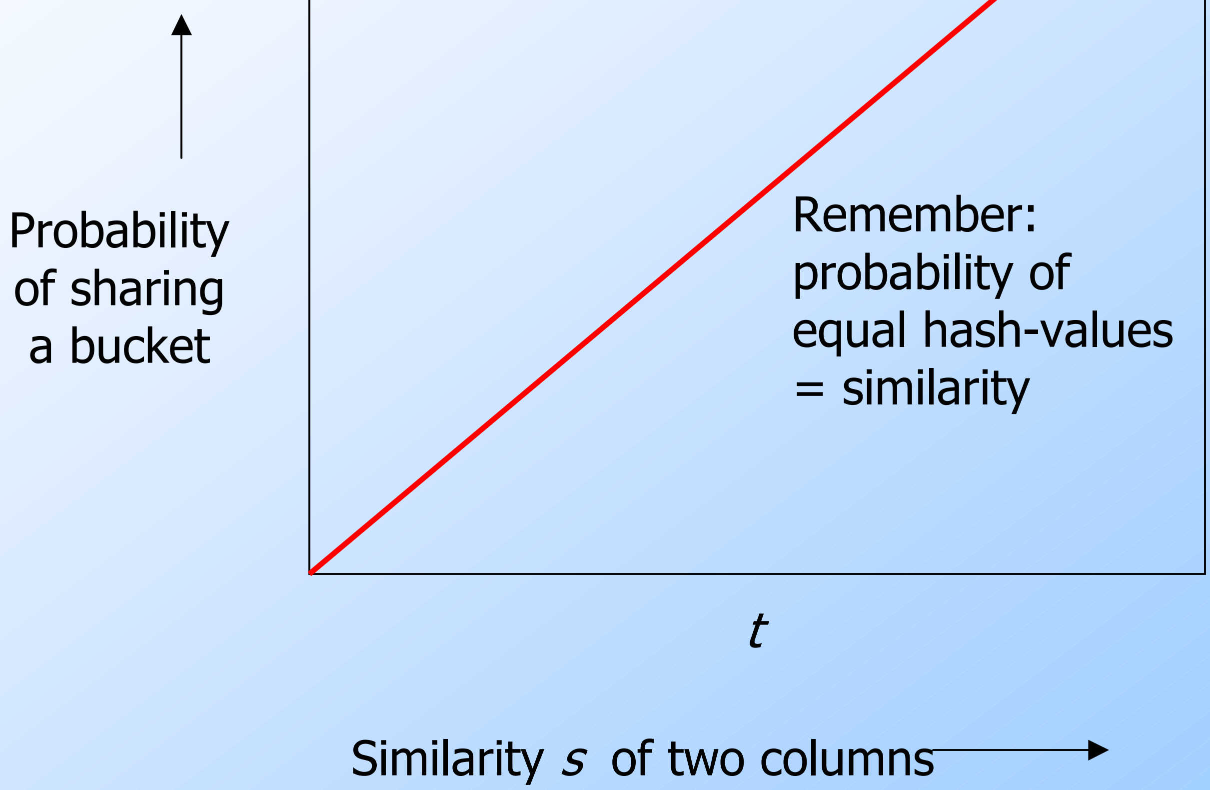
LSH Involves a Tradeoff

- Pick the number of minhashes, the number of bands, and the number of rows per band to balance false positives/negatives.
- Example: if we had fewer than 20 bands, the number of false positives would go down, but the number of false negatives would go up.

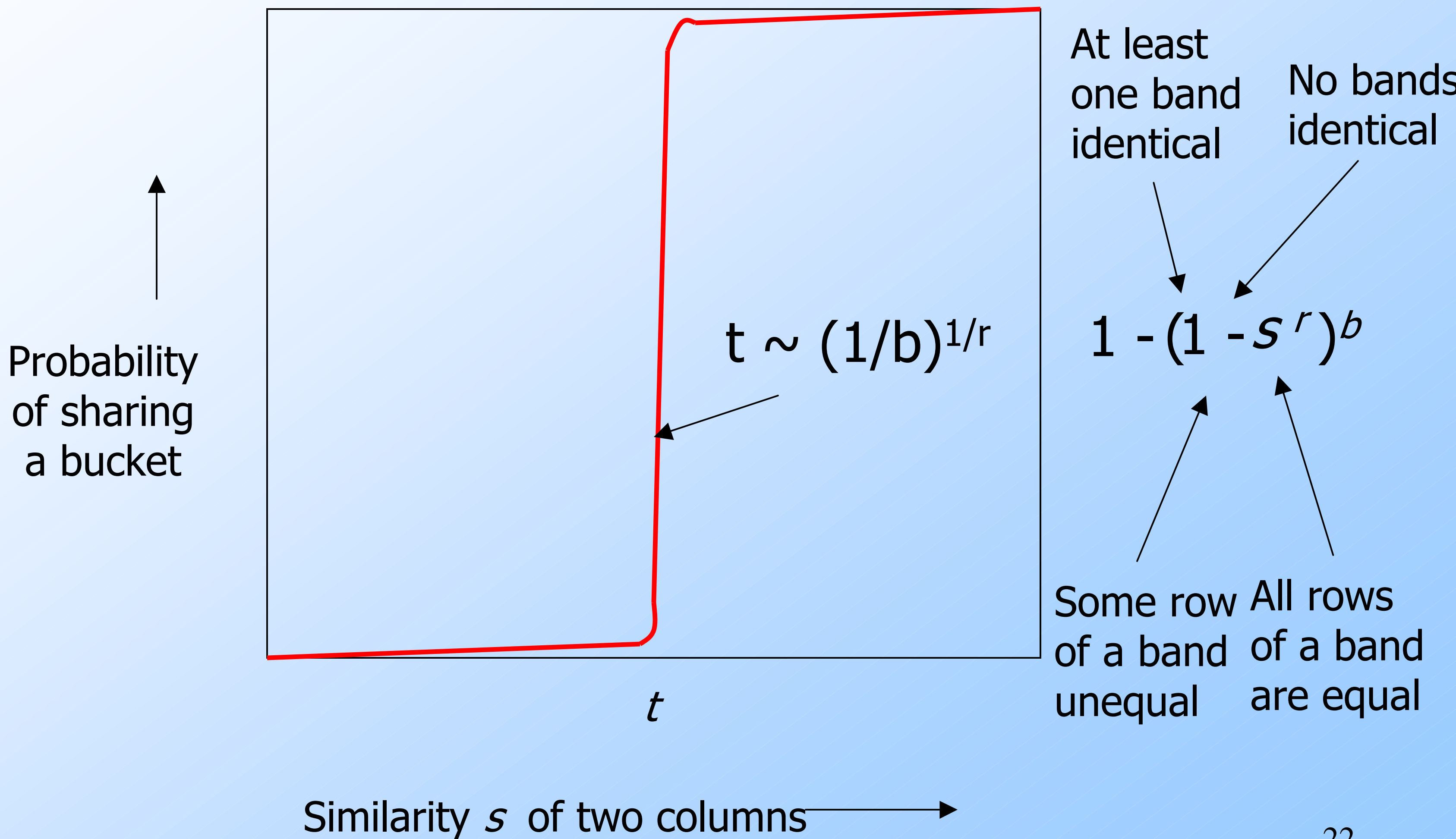
Analysis of LSH – What We Want



What One Row Gives You



What b Bands of r Rows Gives You



LSH Summary

- Tune to get almost all pairs with similar signatures, but eliminate most pairs that do not have similar signatures.
- Check in main memory that candidate pairs really do have similar signatures.
- Optional: In another pass through data, check that the remaining candidate pairs really are similar *columns* .

LSH for Other Applications

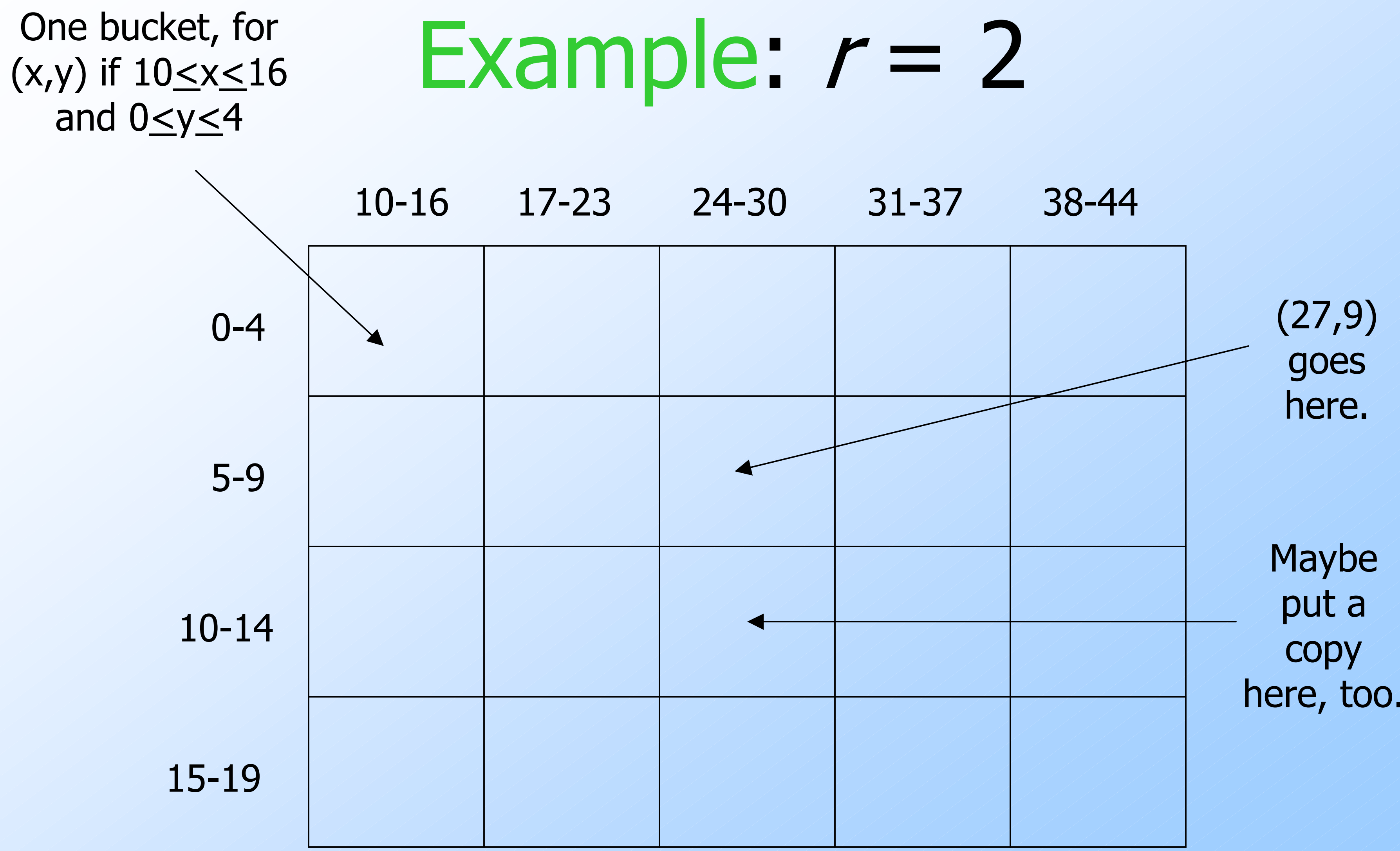
1. Face recognition from 1000 measurements/face.
2. Entity resolution from name-address-phone records.

- General principle: find many hash functions for elements; *candidate pairs* share a bucket for $\geq$ 1 hash.

Face-Recognition Hash Functions

1. Pick a set of r of the 1000 measurements.
2. Each bucket corresponds to a range of values for each of the r measurements.
3. Hash a vector to the bucket such that each of its r components is in-range.
4. Optional: if near the edge of a range, also hash to an adjacent bucket.

Example: $r = 2$



One bucket, for (x,y) if 10≤x≤16 and 0≤y≤4
10-16
17-23
24-30
31-37
38-44
0-4
5-9
10-14
15-19
(27,9) goes here.
Maybe put a copy here, too.

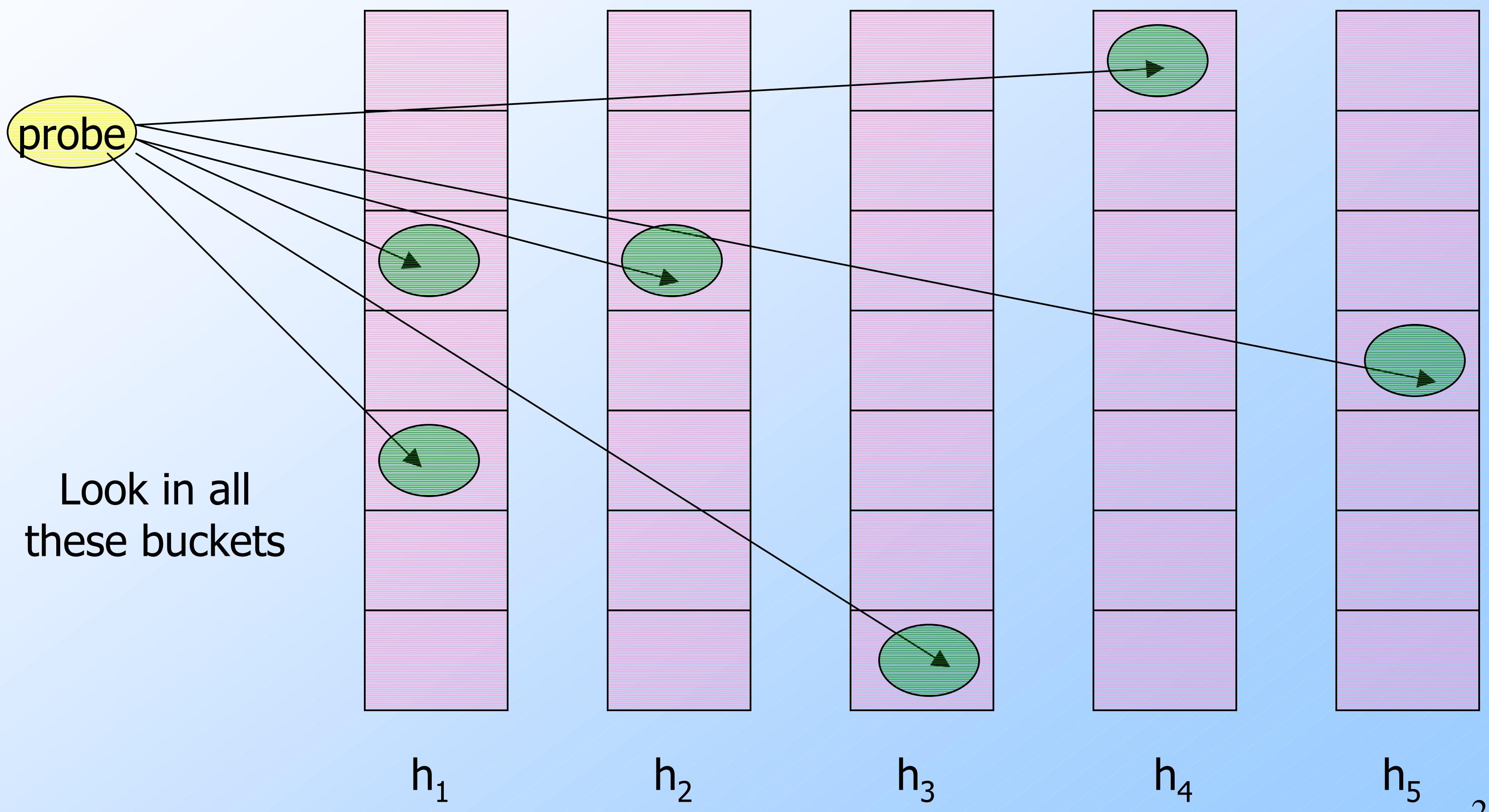
Many-One Face Lookup

- As for boolean matrices, use many different hash functions.
  - Each based on a different set of the 1000 measurements.
- Each bucket of each hash function points to the images that hash to that bucket.

Face Lookup – (2)

- Given a new image (the *probe*), hash it according to all the hash functions.
- Any member of any one of its buckets is a candidate.
- For each candidate, count the number of components in which the candidate and probe are close.
- Match if #components $\geq$ threshold.

Hashing the Probe



Many-Many Problem

- Make each pair of images that are in the same bucket according to any hash function be a candidate pair.
- Score each candidate pair as for the many-one problem.

Entity Resolution

- You don't have the convenient multidimensional view of data that you do for "face-recognition" or "similar-columns."
- We actually used an LSH-inspired simplification.

Matching Customer Records

- I once took a consulting job solving the following problem:
  - Company A agreed to solicit customers for Company B, for a fee.
  - They then had a parting of the ways, and argued over how many customers.
  - Neither recorded exactly which customers were involved.

Customer Records – (2)

- Company B had about 1 million records of all its customers.
- Company A had about 1 million records describing customers, some of which it had signed up for B.
- Records had name, address, and phone, but for various reasons, they could be different for the same person.

Customer Records – (3)

- Step 1: design a measure of how similar records are:
  - E.g., deduct points for small misspellings (“Jeffrey” vs. “Geoffery”), same phone, different area code.
- Step 2: score all pairs of records; report very similar records as matches.

Customer Records – (4)

- Problem: $(1 \text{ million})^2$ is too many pairs of records to score.
- Solution: A simple LSH.
  - Three hash functions: exact values of name, address, phone.
    - Compare iff records are identical in at least one.
  - Misses similar records with a small difference in all three fields.

Customer Records – Aside

- We were able to tell what values of the scoring function were reliable in an interesting way.
  - Identical records had a creation date difference of 10 days.
  - We only looked for records created within 90 days, so bogus matches had a 45-day average.

Aside – (2)

- By looking at the pool of matches with a fixed score, we could compute the average time-difference, say x, and deduce that fraction (45-x)/35 of them were valid matches.
- Alas, the lawyers didn't think the jury would understand.