Association Rules

Market Baskets

Frequent Itemsets

A-priori Algorithm

The Market-Basket Model

- A large set of *items*, e.g., things sold in a supermarket.
- A large set of *baskets*, each of which is a small set of the items, e.g., the things one customer buys on one day.

Support

- Simplest question: find sets of items that appear "frequently" in the baskets.
- *Support* for itemset I = the number of baskets containing all items in I.
- Given a support *threshold* s, sets of items that appear in $\geq s$ baskets are called *frequent itemsets*.

Example: Frequent Itemsets

- Items={milk, coke, pepsi, beer, juice}.
- Support = 3 baskets.

B_1 = {m, c, b}	B_2 = {m, p, j}
B_3 = {m, b}	B_4 = {c, j}
B_5 = {m, p, b}	B_6 = {m, c, b, j}
B_7 = {c, b, j}	B_8 = {b, c}

- Frequent itemsets: {m}, {c}, {b}, {j}, {m,b}, {b,c}, {c,j}.

Applications – (1)

- Real market baskets: chain stores keep terabytes of information about what customers buy together.
  - Tells how typical customers navigate stores, lets them position tempting items.
  - Suggests tie-in "tricks," e.g., run sale on diapers and raise the price of beer.
- High support needed, or no $$'s.

Applications – (2)

- Baskets = sentences; items = words in those sentences.
  - Lets us find words that appear together unusually frequently, i.e., linked concepts.
- Baskets = sentences, items = documents containing those sentences.
  - Items that appear together too often could represent plagiarism.

Applications – (3)

- Baskets = people; items = genes or blood-chemistry factors.
  - Has been used to detect combinations of genes that result in diabetes, e. g.
  - But requires extension: absence of an item needs to be observed as well as presence.

Many-Many Relationships

- "Market Baskets" is an abstraction that models any many-many relationship between two concepts: "items" and "baskets."
  - Items need not be "contained" in baskets.
- The only distinction is that we count co-occurrences of items, not baskets

Scale of Problem

- WalMart sells 100,000 items and can store billions of baskets.
- The Web has over 100,000,000 words and billions of pages.

Association Rules

- If-then rules about the contents of baskets.
- $\{i_1, i_2,\ldots,i_k\} \rightarrow j$ means: "if a basket contains all of $i_1,\ldots,i_k$ then it is *likely* to contain j."
- *Confidence* of this association rule is the probability of j given $i_1,\ldots,i_k$.

Example: Confidence

+ $B_1 = \{m, c, b\}$ $B_2 = \{m, p, j\}$
– $B_3 = \{m, b\}$ $B_4 = \{c, j\}$
– $B_5 = \{m, p, b\}$ + $B_6 = \{m, c, b, j\}$
$B_7 = \{c, b, j\}$ $B_8 = \{b, c\}$

- An association rule: $\{m, b\} \rightarrow c$.
  - Confidence = 2/4 = 50%.

Interest

- The *interest* of an association rule $X \rightarrow Y$ is the absolute value of the amount by which the confidence differs from the probability of Y being in a given basket.

Example: Interest

$B_1 = \{m, c, b\}$ $B_2 = \{m, p, j\}$
$B_3 = \{m, b\}$ $B_4 = \{c, j\}$
$B_5 = \{m, p, b\}$ $B_6 = \{m, c, b, j\}$
$B_7 = \{c, b, j\}$ $B_8 = \{b, c\}$

- For association rule $\{m, b\} \rightarrow c$, item *c* appears in 5/8 of the baskets.
- Interest = |2/4 - 5/8| = 1/8 --- not very interesting.

Relationships Among Measures

- Rules with high support and confidence may be useful even if they are not "interesting."
  - We don't care if buying bread *causes* people to buy milk, or whether simply a lot of people buy both bread and milk.
- But high interest suggests a cause that might be worth investigating.

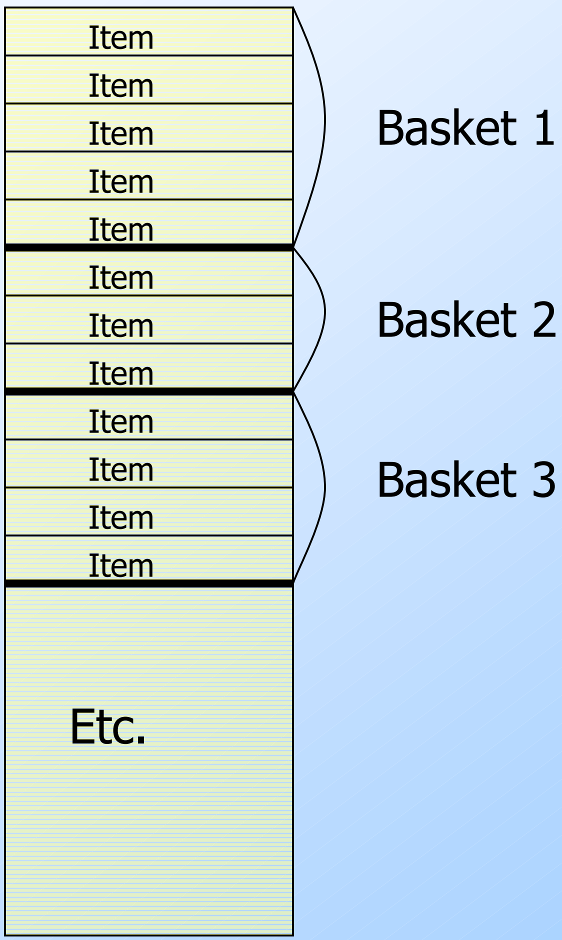
Finding Association Rules

- A typical question: "find all association rules with support $\geq s$ and confidence $\geq c$."
  - Note: "support" of an association rule is the support of the set of items it mentions.
- Hard part: finding the high-support (*frequent*) itemsets.
  - Checking the confidence of association rules involving those sets is relatively easy.

Computation Model

- Typically, data is kept in a flat file rather than a database system.
  - Stored on disk.
  - Stored basket-by-basket.
  - Expand baskets into pairs, triples, etc. as you read baskets.
    - Use k nested loops to generate all sets of size k.

File Organization



Computation Model – (2)

- The true cost of mining disk-resident data is usually the number of disk I/O's.
- In practice, association-rule algorithms read the data in *passes* – all baskets read in turn.
- Thus, we measure the cost by the number of passes an algorithm takes.

Main-Memory Bottleneck

- For many frequent-itemset algorithms, main memory is the critical resource.
  - As we read baskets, we need to count something, e.g., occurrences of pairs.
  - The number of different things we can count is limited by main memory.
  - Swapping counts in/out is a disaster.

Finding Frequent Pairs

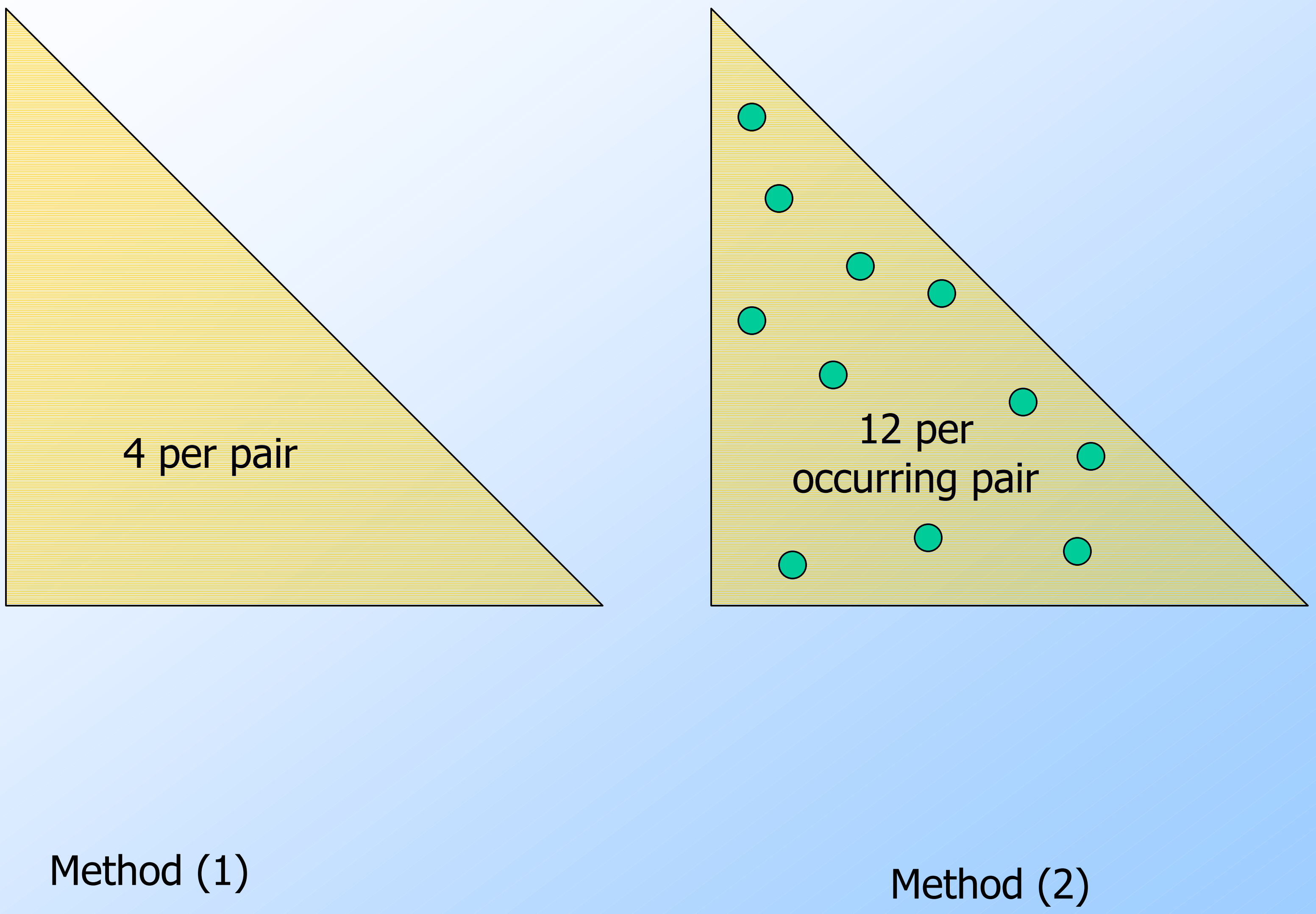
- The hardest problem often turns out to be finding the frequent pairs.
- We'll concentrate on how to do that, then discuss extensions to finding frequent triples, etc.

Naïve Algorithm

- Read file once, counting in main memory the occurrences of each pair.
  - From each basket of n items, generate its $n(n-1)/2$ pairs by two nested loops.
- Fails if $(\#\text{items})^2$ exceeds main memory.
  - Remember: #items can be 100K (Wal-Mart) or 10B (Web pages).

Details of Main-Memory Counting

- Two approaches:
  1. Count all pairs, using a triangular matrix.
  2. Keep a table of triples [*i*, *j*, *c*] = the count of the pair of items {*i*,*j* } is *c*.
- (1) requires only 4 bytes/pair.
  - Note: assume integers are 4 bytes.
- (2) requires 12 bytes, but only for those pairs with count > 0.


4 per pair
12 per
occurring pair
Method (1)
Method (2)

Triangular-Matrix Approach – (1)

- Number items 1, 2,…
- Requires table of size O(n).
- Keep pairs in the order {1,2}, {1,3},…, {1,n}, {2,3}, {2,4},…,{2,n}, {3,4},…, {3,n},…{n-1,n}.

Triangular-Matrix Approach – (2)

- Find pair $\{i, j\}$ at the position
  $(i-1)(n-i/2) + j - i$.
- Total number of pairs $n(n-1)/2$; total bytes about $2n^2$.

Details of Approach #2

- Total bytes used is about 12*p*, where *p* is the number of pairs that actually occur.
  - Beats triangular matrix if at most 1/3 of possible pairs actually occur.
- May require extra space for retrieval structure, e.g., a hash table.

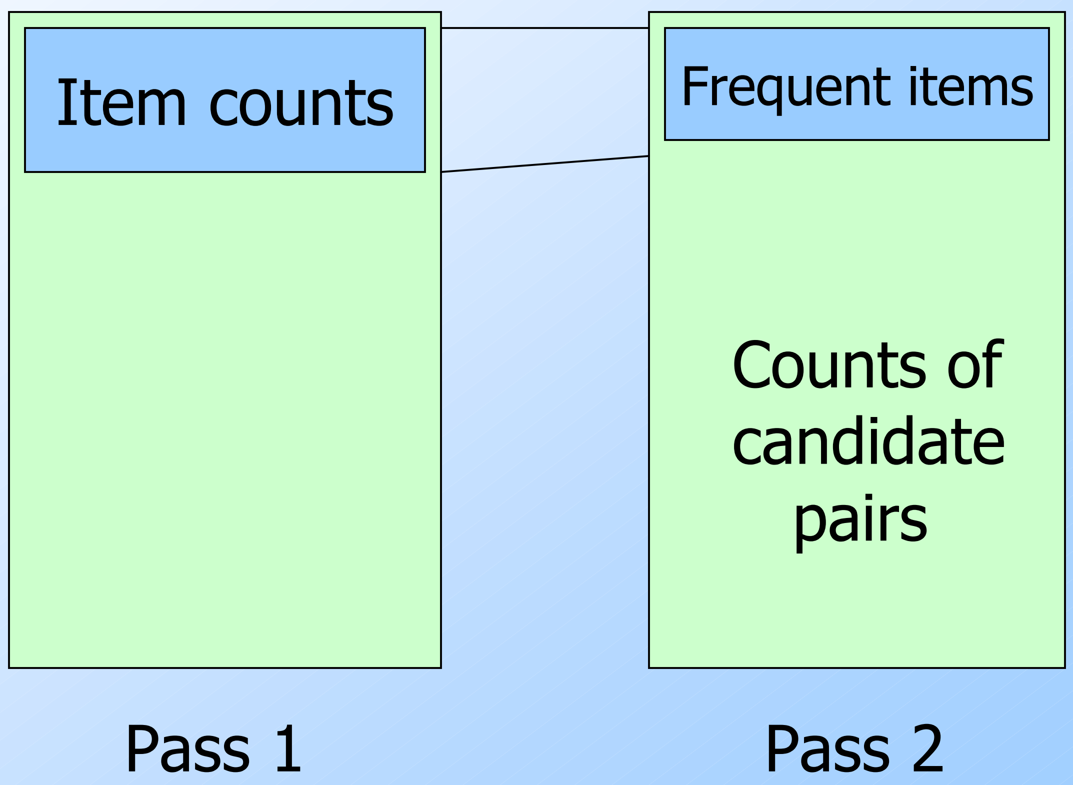
A-Priori Algorithm – (1)

- A two-pass approach called *a-priori* limits the need for main memory.
- Key idea: *monotonicity* : if a set of items appears at least s times, so does every subset.
  - Contrapositive for pairs: if item i does not appear in s baskets, then no pair including i can appear in s baskets.

A-Priori Algorithm – (2)

- Pass 1: Read baskets and count in main memory the occurrences of each item.
  - Requires only memory proportional to #items.
- Pass 2: Read baskets again and count in main memory only those pairs both of which were found in Pass 1 to be frequent.
  - Requires memory proportional to square of frequent items only.

Picture of A-Priori

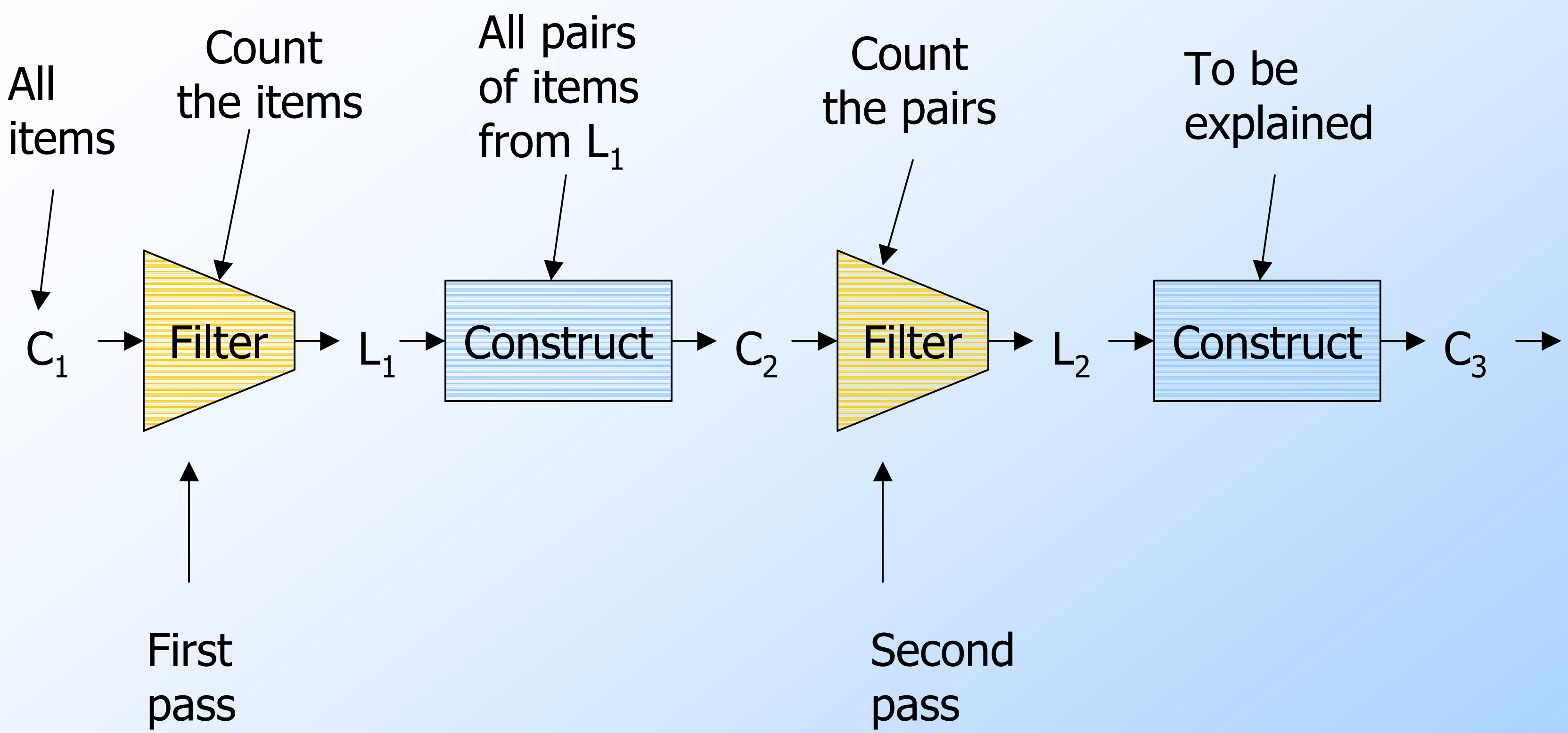


Detail for A-Priori

- You can use the triangular matrix method with *n* = number of frequent items.
  - Saves space compared with storing triples.
- Trick: number frequent items 1,2,… and keep a table relating new numbers to original item numbers.

Frequent Triples, Etc.

- For each k, we construct two sets of k–tuples:
  - C_k = *candidate* k - tuples = those that might be frequent sets (support $\geq s$) based on information from the pass for k–1.
  - L_k = the set of truly frequent k–tuples.


All items
Count the items
All pairs of items from L_1
Count the pairs
To be explained
C_1
Filter
L_1
Construct
C_2
Filter
L_2
Construct
C_3
First pass
Second pass

A-Priori for All Frequent Itemsets

- One pass for each *k*.
- Needs room in main memory to count each candidate *k* –tuple.
- For typical market-basket data and reasonable support (e.g., 1%), $k = 2$ requires the most memory.

Frequent Itemsets – (2)

- C_1 = all items
- L_1 = those counted on first pass to be frequent.
- C_2 = pairs, both chosen from L_1.
- In general, C_k = k–tuples, each k–1 of which is in $L_{k\text{-}1}$.
- L_k = members of C_k with support $\geq s$.