Stream Clustering

Extension of DGIM to More Complex Problems

Clustering a Stream

- Assume points enter in a stream.
- Maintain a sliding window of points.
- Queries ask for clusters of points within some suffix of the window.
- Important issue: where are the cluster centroids?

BDMO Approach

- BDMO = Babcock, Datar, Motwani, O'Callaghan.
- *k* –means based.
- Can use less than O(*N*) space for windows of size *N*.
- Generalizes trick of DGIM: buckets of increasing "weight."

Recall DGIM

- Maintains a sequence of buckets B_1, B_2, …
- Buckets have timestamps (most recent stream element in bucket).
- Sizes of buckets nondecreasing.
  - In DGIM size = power of 2.
- Either 1 or 2 of each size.

Alternative Combining Rule

- Instead of "combine the 2nd and 3rd of any one size" we could say:
- "Combine B_{i+1} and B_i if size($B_{i+1} \cup B_i$) < size($B_{i-1} \cup B_{i-2} \cup \ldots \cup B_1$)."
  - If B_{i+1}, B_i, and B_{i-1} are the same size, inequality must hold (almost).
  - If B_{i-1} is smaller, it cannot hold.

Buckets for Clustering

- In place of "size" (number of 1's) we use (an approximation to) the sum of the distances from all points to the centroid of their cluster.
- Merge consecutive buckets if the "size" of the merged bucket is less than the sum of the sizes of all later buckets.

Consequence of Merge Rule

- In a stable list of buckets, any two consecutive buckets are "bigger" than all smaller buckets.
- Thus, "sizes" grow exponentially.
- If there is a limit on total "size," then the number of buckets is O(log N).
  - N = window size.
  - E.g., all points are in a fixed hypercube.

Outline of Algorithm

1. What do buckets look like?
   - Clusters at various levels, represented by centroids.
2. How do we merge buckets?
   - Keep # of clusters at each level small.
3. What happens when we query?
   - Final clustering of all clusters of all relevant buckets.

Organization of Buckets

- Each bucket consists of clusters at some number of levels.
  - 4 levels in our examples.
- Clusters represented by:
  1. Location of centroid.
  2. *Weight* = number of points in the cluster.
  3. *Cost* = upper bound on sum of distances from member points to centroid.

Processing Buckets --- (1)

- Actions determined by N (window size) and k (desired number of clusters).
- Also uses a tuning parameter τ for which we use 1/4 to simplify.
  - $1/\tau$ is the number of levels of clusters.

Processing Buckets --- (2)

- Initialize a new bucket with *k* new points.
  - Each is a cluster at level 0.
- If the timestamp of the oldest bucket is outside the window, delete that bucket.

Level-0 Clusters

- A single point p is represented by $(p, 1, 0)$.
- That is:
  1. A point is its own centroid.
  2. The cluster has one point.
  3. The sum of distances to the centroid is 0.

Merging Buckets --- (1)

- Needed in two situations:
  1. We have to process a query, which requires us to (temporarily) merge some tail of the bucket sequence.
  2. We have just added a new (most recent) bucket and we need to check the rule about two consecutive buckets being "bigger" than all that follow.

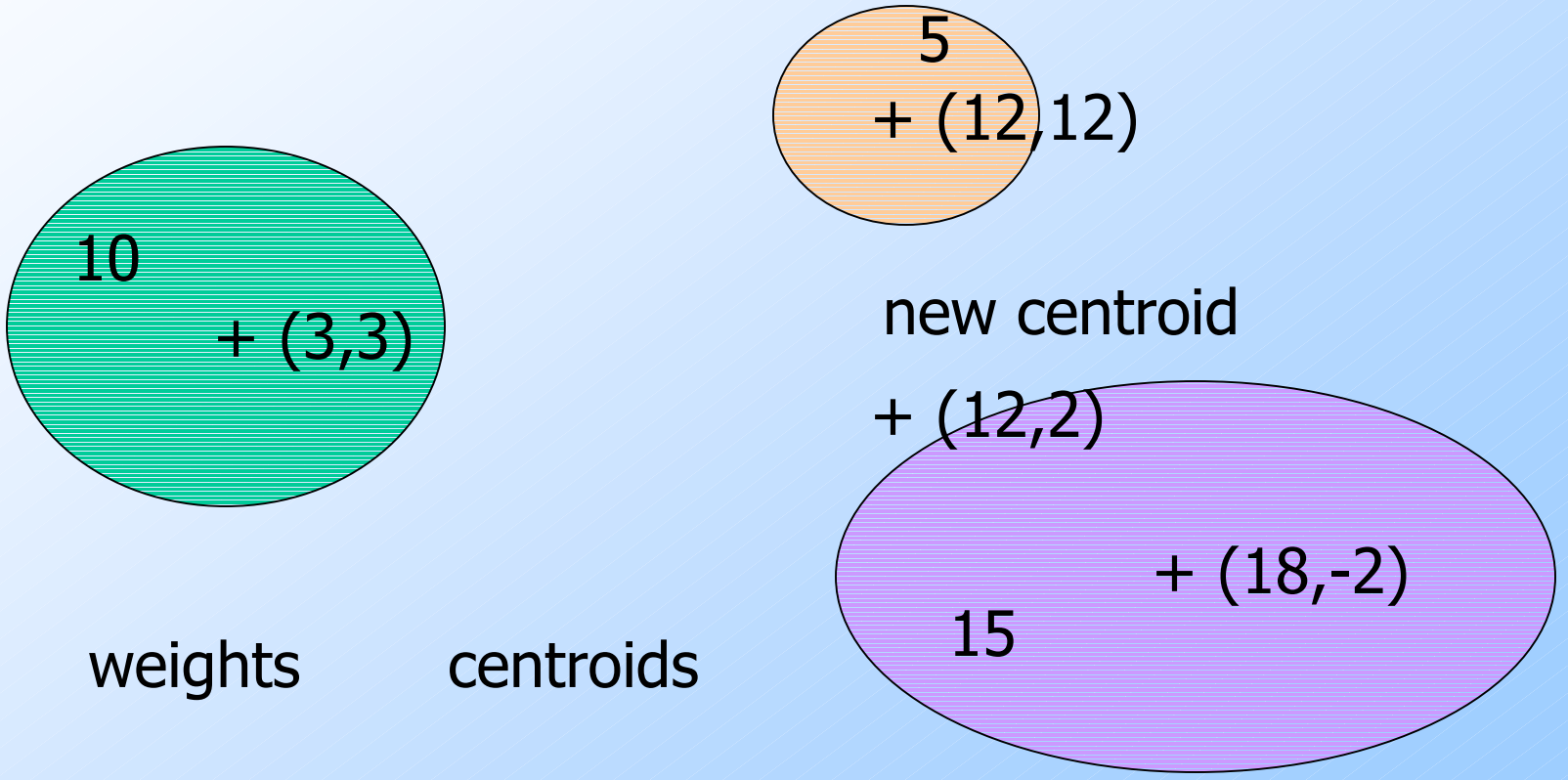
Merging Buckets --- (2)

- Step 1: Take the union of the clusters at each level.
- Step 2: If the number of clusters (points) at level 0 is now more than $N^{1/4}$, cluster them into k clusters.
  - These become clusters at level 1.
- Steps 3,…: Repeat, going up the levels, if needed.

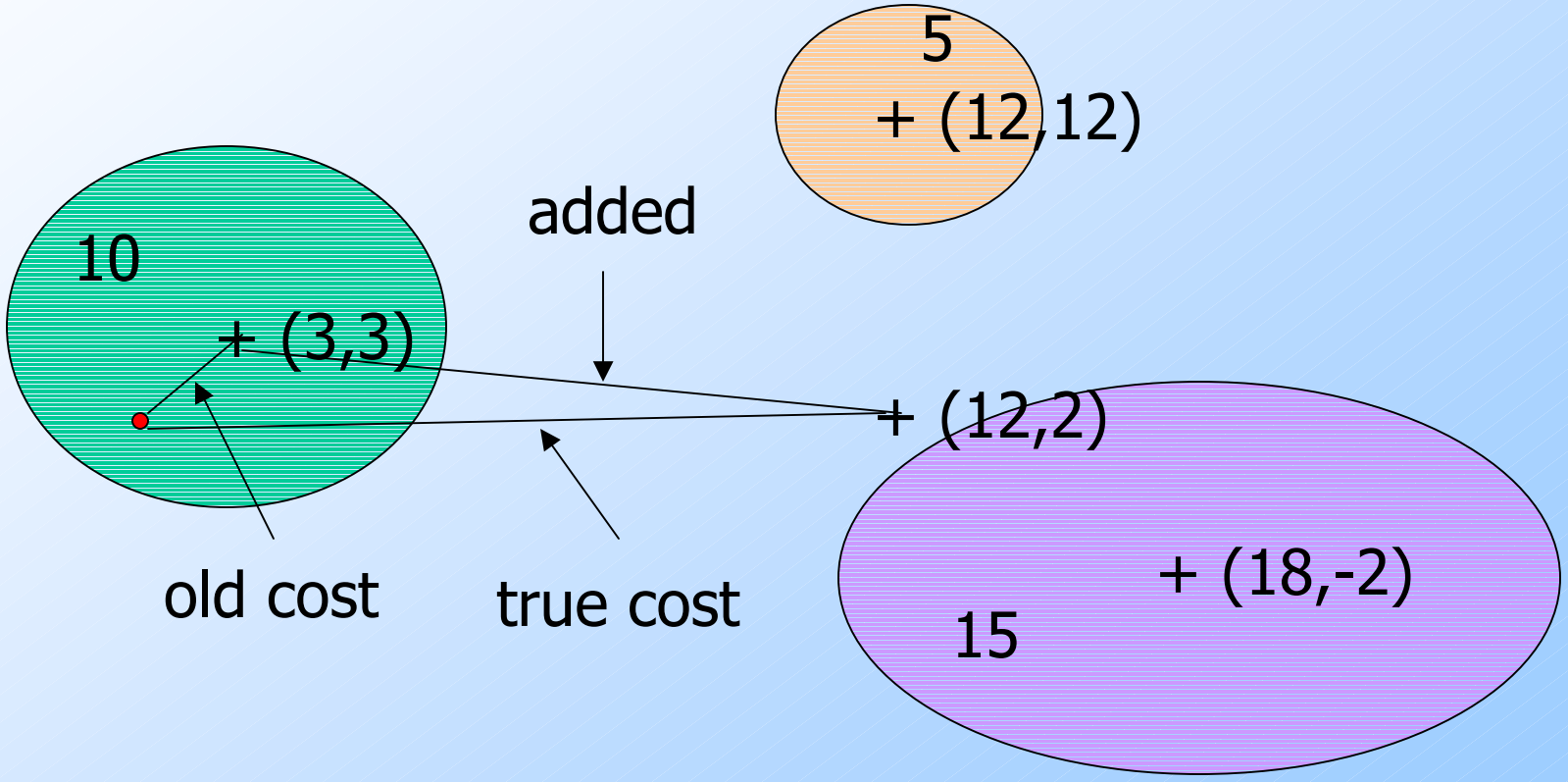
Representing New Clusters

- Centroid = weighted average of centroids of component clusters.
- Weight = sum of weights.
- Cost = sum over all component clusters of:
  1. Cost of component cluster.
  2. Weight of component times distance from its centroid to new centroid.

Example: New Centroid



Example: New Costs



Queries

- Find all the buckets within the range of the query.
  - The last bucket may be only partially within the range.
- Cluster all clusters at all levels into *k* clusters.
- Return the *k* centroids.

Error in Estimation

- Goal is to pick the *k* centroids that minimize the *true* cost (sum of distances from each point to its centroid).
- Since recorded "costs" are inexact, there can be a factor of 2 error at each level.
- Additional error because some of last bucket may not belong.
  - But fraction of spurious points is small (why?).

Effect of Cost-Errors

1. May alter when buckets get combined.
   - Not really important.
2. Produce suboptimal clustering at any stage of the algorithm.
   - The real measure of how bad the output is.

Speedup of Algorithm

- As given, algorithm is slow.
  - Each new bucket causes O(log N) bucket-merger problems.
- A faster version allows the first bucket to have not k, but $N^{1/2}$ (or in general $N^{2\tau}$) points.
  - A number of consequences, including slower queries, more space.