Locality-Sensitive Hashing

Basic Technique

Hamming-LSH

Applications

Finding Similar Pairs

- Suppose we have in main memory data representing a large number of objects.
  - May be the objects themselves (e.g., summaries of faces).
  - May be signatures as in minhashing.
- We want to compare each to each, finding those pairs that are sufficiently similar.

Candidate Generation From Minhash Signatures

- Pick a similarity threshold *s*, a fraction < 1.
- A pair of columns *c* and *d* is a *candidate pair* if their signatures agree in at least fraction *s* of the rows.
  - I.e., $M(i, c) = M(i, d)$ for at least fraction *s* values of *i*.

Candidate Generation --- (2)

- For images, a pair of vectors is a candidate if they differ by at most a small threshold t in at least s% of the components.
- For entity records, a pair is a candidate if the sum of similarity scores of corresponding components exceeds a threshold.

The Problem with Checking for Candidates

- While the signatures of all columns may fit in main memory, comparing the signatures of all pairs of columns is quadratic in the number of columns.
- Example: 10^6 columns implies $5*10^{11}$ comparisons.
- At 1 microsecond/comparison: 6 days.

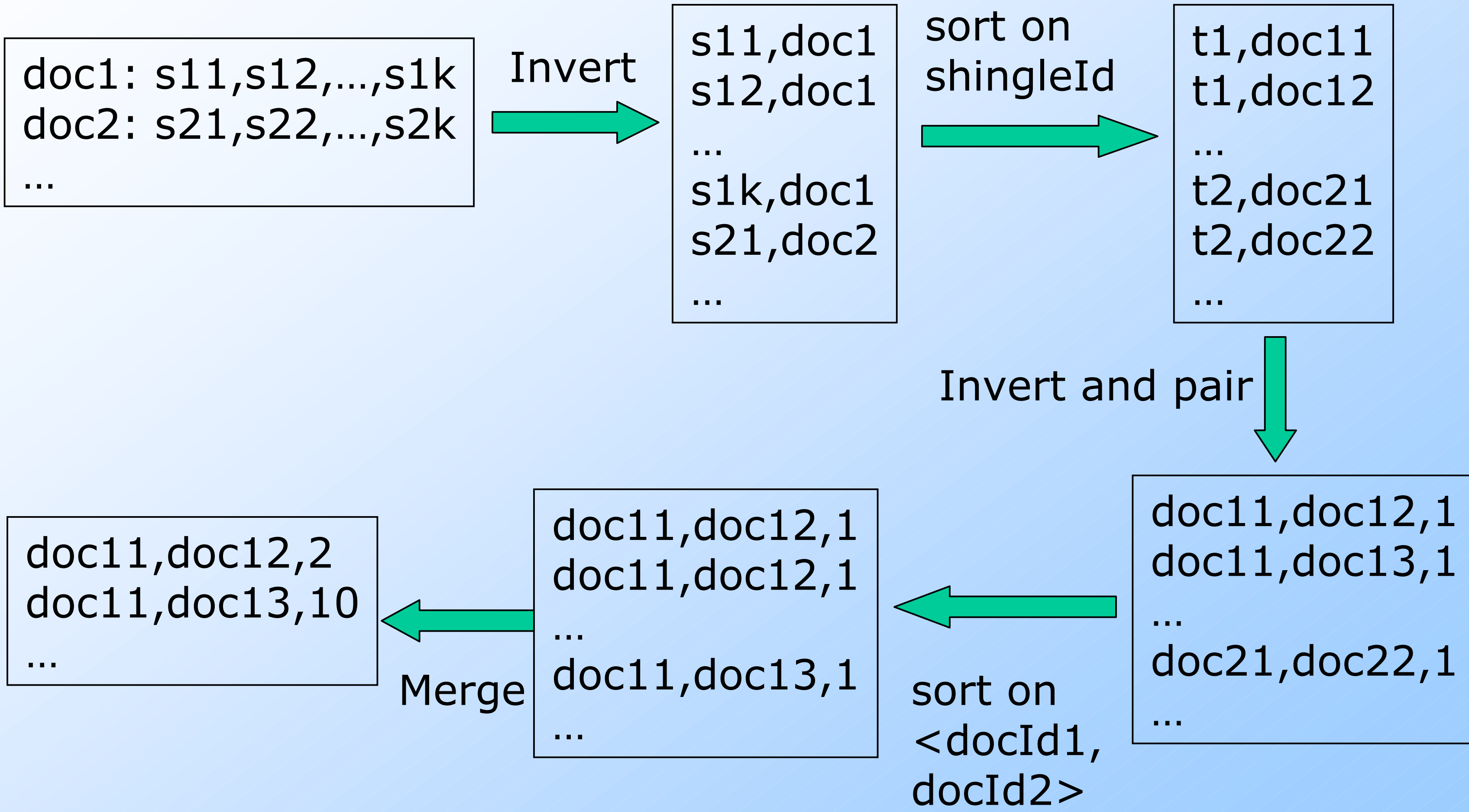
Solutions

1. *Divide-Compute-Merge* (DCM) uses external sorting, merging.
2. *Locality-Sensitive Hashing* (LSH) can be carried out in main memory, but admits some false negatives.
3. *Hamming LSH* --- a variant LSH method.

Divide-Compute-Merge

- Designed for “shingles” and docs.
- At each stage, divide data into batches that fit in main memory.
- Operate on individual batches and write out partial results to disk.
- Merge partial results from disk.

DCM Steps



doc1: s11,s12,…,s1k
doc2: s21,s22,…,s2k
…
Invert
s11,doc1
s12,doc1
…
s1k,doc1
s21,doc2
…
sort on shingleId
t1,doc11
t1,doc12
…
t2,doc21
t2,doc22
…
Invert and pair
doc11,doc12,1
doc11,doc13,1
…
doc21,doc22,1
…
sort on <docId1, docId2>
doc11,doc12,1
doc11,doc12,1
…
doc11,doc13,1
…
Merge
doc11,doc12,2
doc11,doc13,10
…

DCM Summary

1. Start with the pairs <shingleId, docId>.
2. Sort by shingleId.
3. In a sequential scan, generate triplets <docId1, docId2, 1> for pairs of docs that share a shingle.
4. Sort on <docId1, docId2>.
5. Merge triplets with common docIds to generate triplets of the form <docId1,docId2,count>.
6. Output document pairs with count > threshold.

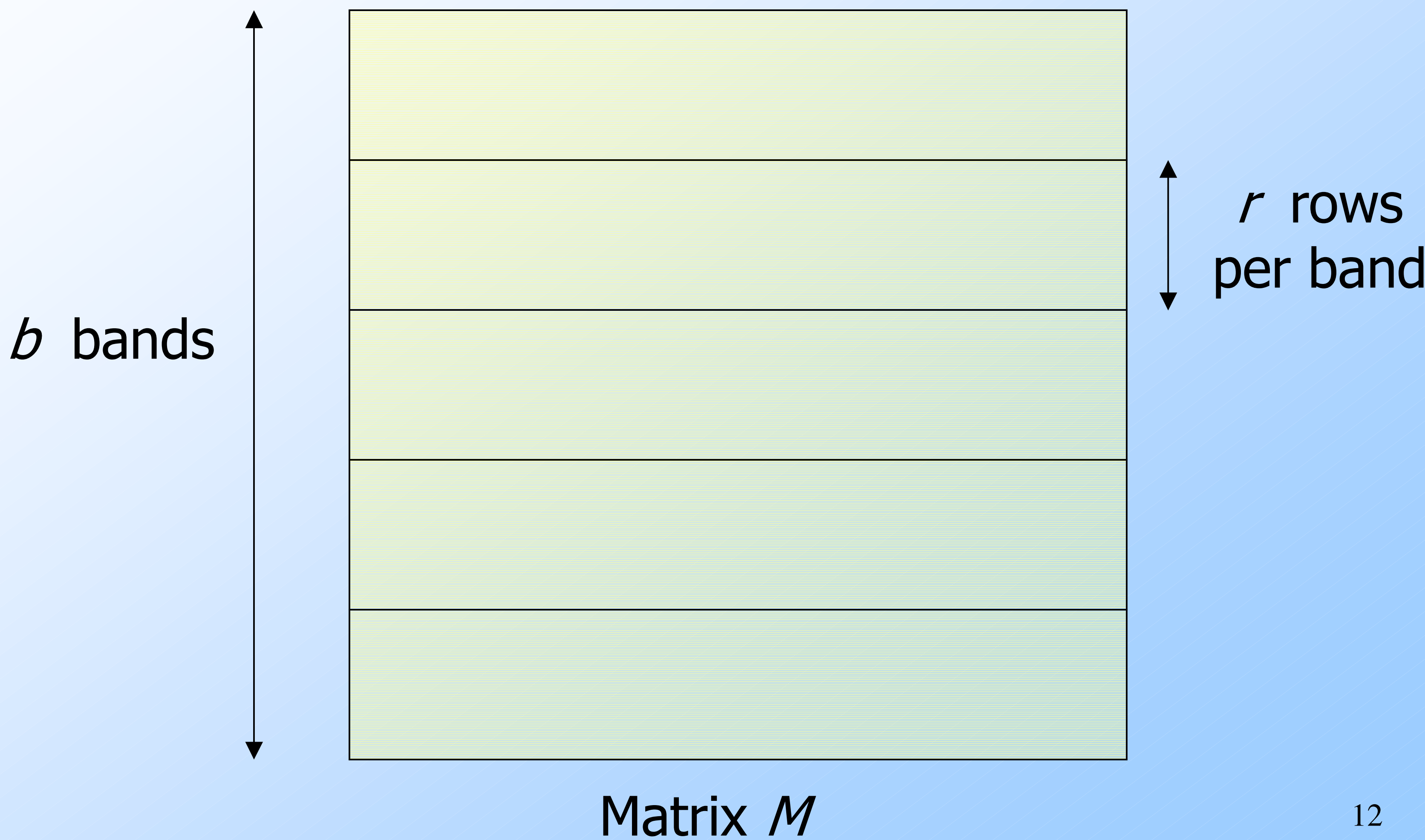
Some Optimizations

- "Invert and Pair" is the most expensive step.
- Speed it up by eliminating very common shingles.
  - "the", "404 not found", "<A HREF", etc.
- Also, eliminate exact-duplicate docs first.

Locality-Sensitive Hashing

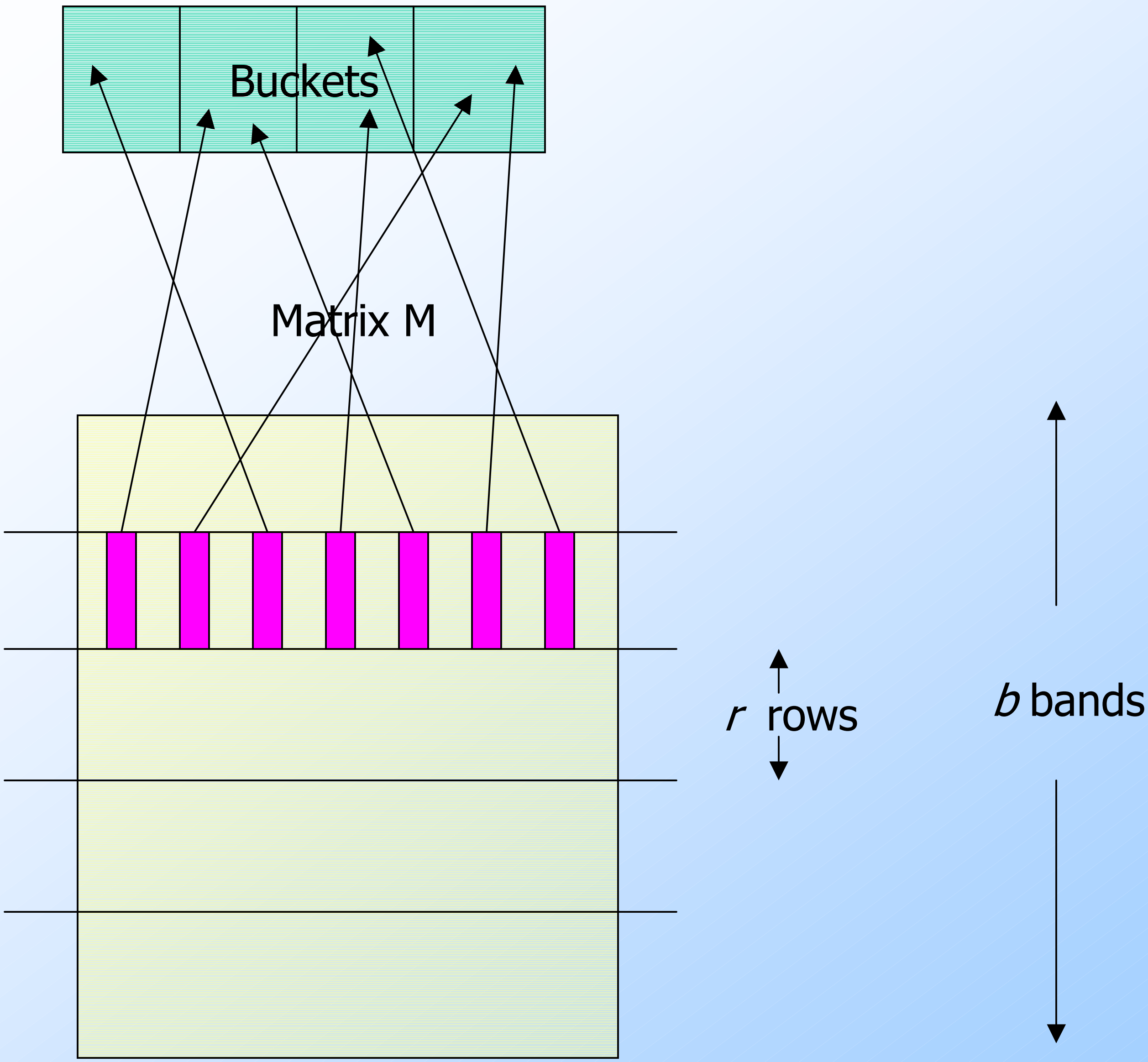
- Big idea: hash columns of signature matrix M several times.
- Arrange that (only) similar columns are likely to hash to the same bucket.
- Candidate pairs are those that hash at least once to the same bucket.

Partition Into Bands



Partition into Bands --- (2)

- Divide matrix *M* into *b* bands of *r* rows.
- For each band, hash its portion of each column to a hash table with *k* buckets.
- *Candidate* column pairs are those that hash to the same bucket for ≥ 1 band.
- Tune *b* and *r* to catch most similar pairs, but few nonsimilar pairs.


Buckets
Matrix M
r rows
b bands

Simplifying Assumption

- There are enough buckets that columns are unlikely to hash to the same bucket unless they are identical in a particular band.
- Hereafter, we assume that “same bucket” means “identical.”

Example

- Suppose 100,000 columns.
- Signatures of 100 integers.
- Therefore, signatures take 40Mb.
- But 5,000,000,000 pairs of signatures can take a while to compare.
- Choose 20 bands of 5 integers/band.

Suppose C_1, C_2 are 80% Similar

- Probability C_1, C_2 identical in one particular band: $(0.8)^5 = 0.328$.
- Probability C_1, C_2 are *not* similar in any of the 20 bands: $(1-0.328)^{20} = .00035$.
  - i.e., we miss about 1/3000th of the 80%-similar column pairs.

Suppose C_1, C_2 Only 40% Similar

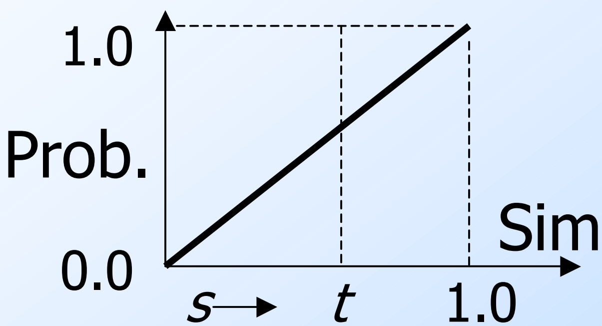
- Probability C_1, C_2 identical in any one particular band: $(0.4)^5 = 0.01$.
- Probability C_1, C_2 identical in $\geq$ 1 of 20 bands: $\leq 20 * 0.01 = 0.2$.
- But false positives much lower for similarities << 40%.

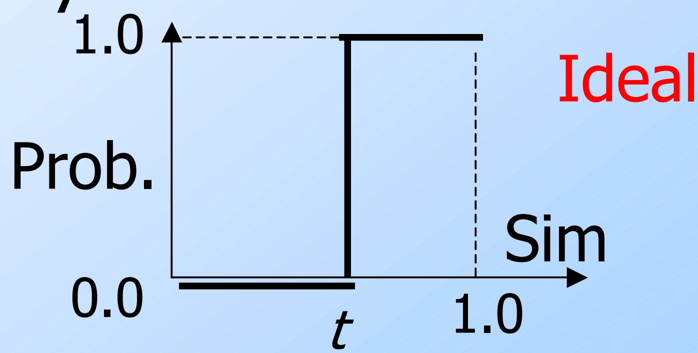
LSH Involves a Tradeoff

- Pick the number of minhashes, the number of bands, and the number of rows per band to balance false positives/negatives.
- Example: if we had fewer than 20 bands, the number of false positives would go down, but the number of false negatives would go up.

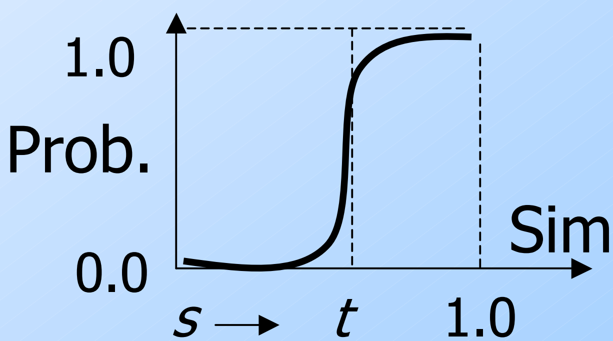
LSH --- Graphically

- Example Target: All pairs with *Sim* > *t*.
- Suppose we use only one hash function:





- Partition into bands gives us:



$$1 - (1 - s^r)^b$$

$$t \sim (1/b)^{1/r}$$

LSH Summary

- Tune to get almost all pairs with similar signatures, but eliminate most pairs that do not have similar signatures.
- Check in main memory that candidate pairs really do have similar signatures.
- Optional: In another pass through data, check that the remaining candidate pairs really are similar *columns* .

New Topic: Hamming LSH

- An alternative to minhash + LSH.
- Takes advantage of the fact that if columns are not sparse, random rows serve as a good signature.
- Trick: create data matrices of exponentially decreasing sizes, increasing densities.

Amplification of 1's

- *Hamming LSH* constructs a series of matrices, each with half as many rows, by OR-ing together pairs of rows.
- Candidate pairs from each matrix have (say) between 20% - 80% 1's and are similar in selected 100 rows.
  - 20%-80% OK for similarity thresholds $\geq$ 0.5.
    - Otherwise, two "similar" columns with widely differing numbers of 1's could fail to both be in range for at least one matrix.

Example

0	0	1	1
0	1	1	
1	0		
1	1		
0			
0			
1			
0			

Using Hamming LSH

- Construct the sequence of matrices.
  - If there are R rows, then $\log_2 R$ matrices.
  - Total work = twice that of reading the original matrix.
- Use standard LSH on a random selection of rows to identify similar columns in each matrix, but restricted to columns of "medium" density.

LSH for Other Applications

1. Face recognition from 1000 measurements/face.
2. Entity resolution from name-address-phone records.

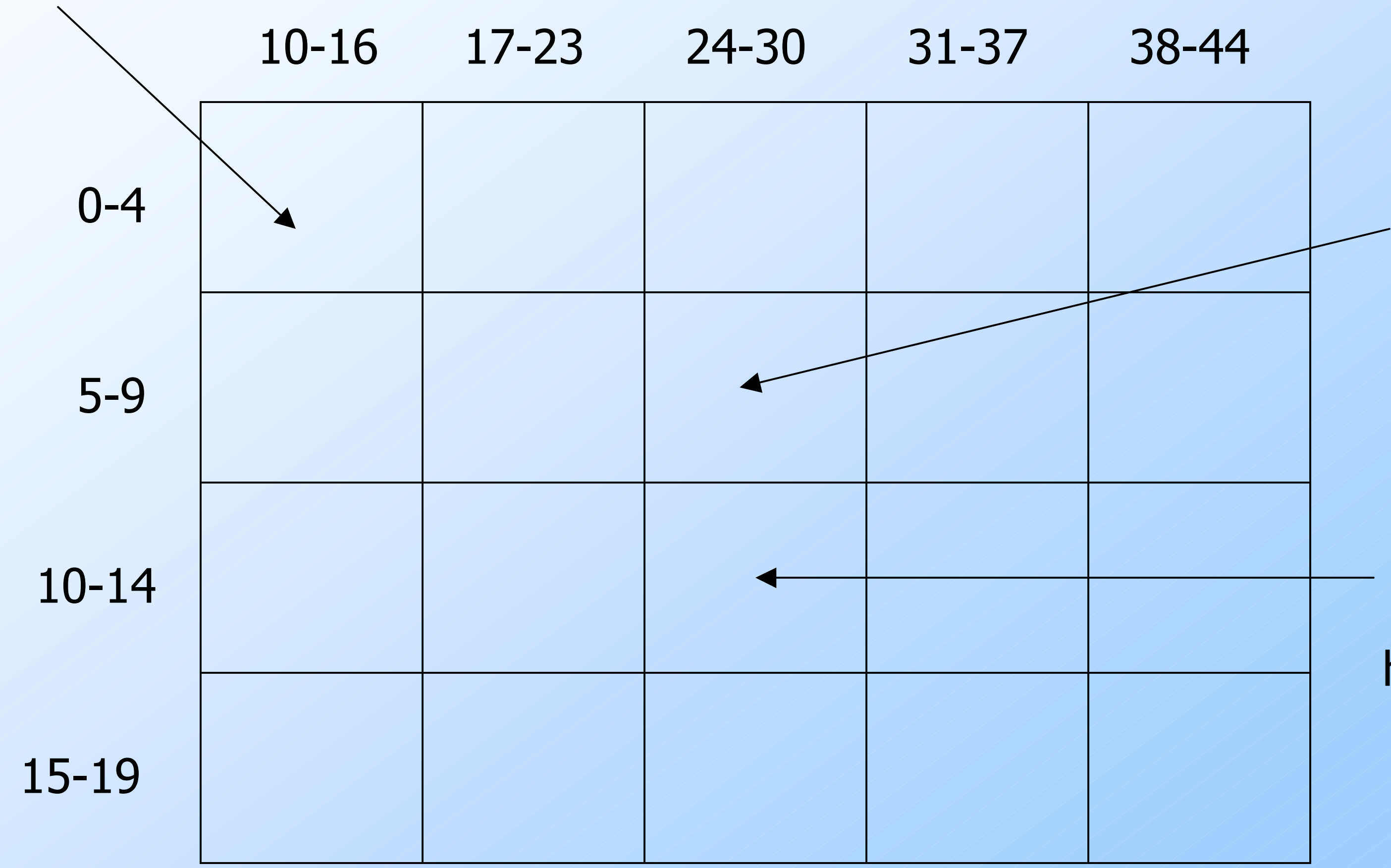
- General principle: find many hash functions for elements; *candidate pairs* share a bucket for $\geq$ 1 hash.

Face-Recognition Hash Functions

1. Pick a set of r of the 1000 measurements.
2. Each bucket corresponds to a range of values for each of the r measurements.
3. Hash a vector to the bucket such that each of its r components is in-range.
4. Optional: if near the edge of a range, also hash to an adjacent bucket.

Example: $r = 2$

One bucket, for (x,y) if $10 \leq x \leq 16$ and $0 \leq y \leq 4$



(27,9) goes here.

Maybe put a copy here, too.

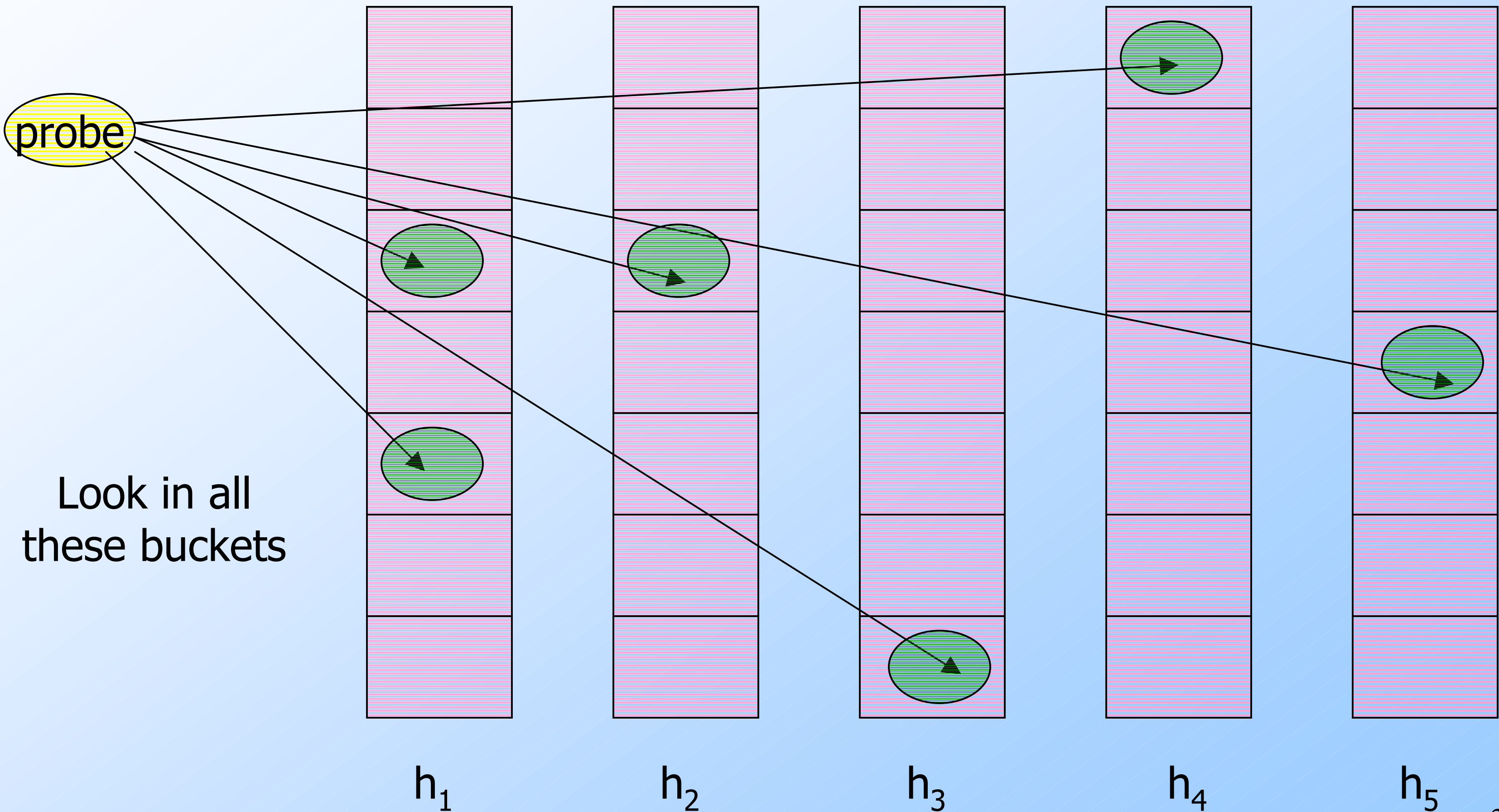
Many-One Face Lookup

- As for boolean matrices, use many different hash functions.
  - Each based on a different set of the 1000 measurements.
- Each bucket of each hash function points to the images that hash to that bucket.

Face Lookup --- (2)

- Given a new image (the *probe*), hash it according to all the hash functions.
- Any member of any one of its buckets is a candidate.
- For each candidate, count the number of components in which the candidate and probe are close.
- Match if #components $\geq$ threshold.

Hashing the Probe



Many-Many Problem

- Make each pair of images that are in the same bucket according to any hash function be a candidate pair.
- Score each candidate pair as for the many-one problem.

Entity Resolution

- You don’t have the convenient multidimensional view of data that you do for “face-recognition” or “similar-columns.”
- We actually used an LSH-inspired simplification.

Entity Resolution --- (2)

- Three hash functions:
  1. One bucket for each name string.
  2. One bucket for each address string.
  3. One bucket for each phone string.
- A pair is a candidate iff they mapped to the same bucket for at least one of the three hashes.