Mining Data Streams

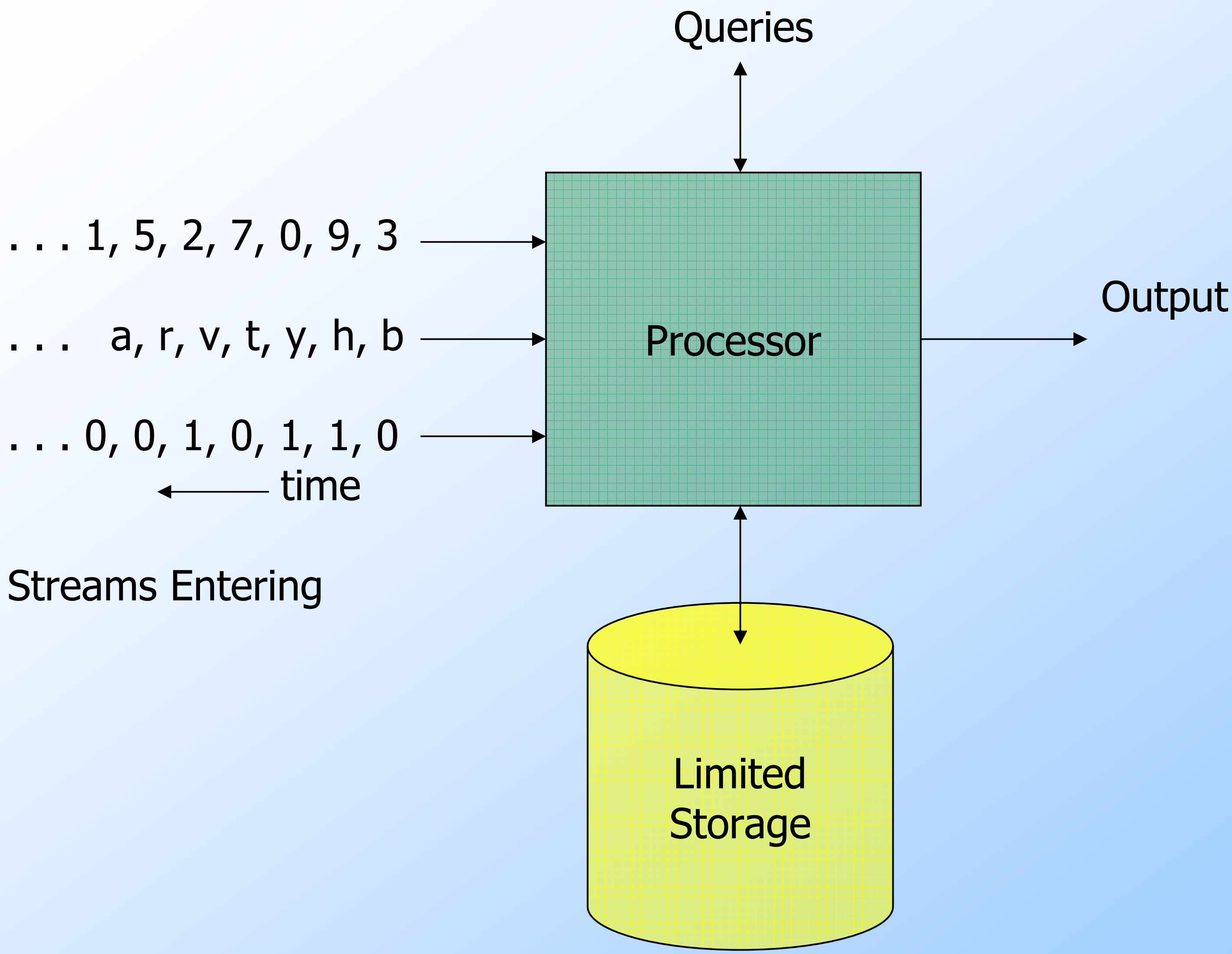
The Stream Model

Sliding Windows

Counting 1's

The Stream Model

- Data enters at a rapid rate from one or more input ports.
- The system cannot store the entire stream.
- How do you make critical calculations about the stream using a limited amount of (secondary) memory?


Queries
. . . 1, 5, 2, 7, 0, 9, 3
. . .   a, r, v, t, y, h, b
. . . 0, 0, 1, 0, 1, 1, 0
time
Processor
Output
Streams Entering
Limited Storage

Applications --- (1)

- In general, stream processing is important for applications where
  - New data arrives frequently.
  - Important queries tend to ask about the most recent data, or summaries of data.

Applications --- (2)

- Mining query streams.
  - Google wants to know what queries are more frequent today than yesterday.
- Mining click streams.
  - Yahoo wants to know which of its pages are getting an unusual number of hits in the past hour.

Applications --- (3)

- Sensors of all kinds need monitoring, especially when there are many sensors of the same type, feeding into a central controller, most of which are not sensing anything important at the moment.
- Telephone call records summarized into customer bills.

Applications --- (4)

- Intelligence-gathering.
  - Like “evil-doers visit hotels” at beginning of course, but much more data at a much faster rate.
    - Who calls whom?
    - Who accesses which Web pages?
    - Who buys what where?

Sliding Windows

- A useful model of stream processing is that queries are about a *window* of length N --- the N most recent elements received.
- Interesting case: N is still so large that it cannot be stored on disk.
  - Or, there are so many streams that windows for all cannot be stored.

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

← Past Future →

Counting Bits --- (1)

- Problem: given a stream of 0's and 1's, be prepared to answer queries of the form "how many 1's in the last k bits?" where $k \leq N$.
- Obvious solution: store the most recent N bits.
  - When new bit comes in, discard the $N+1^{st}$ bit.

Counting Bits --- (2)

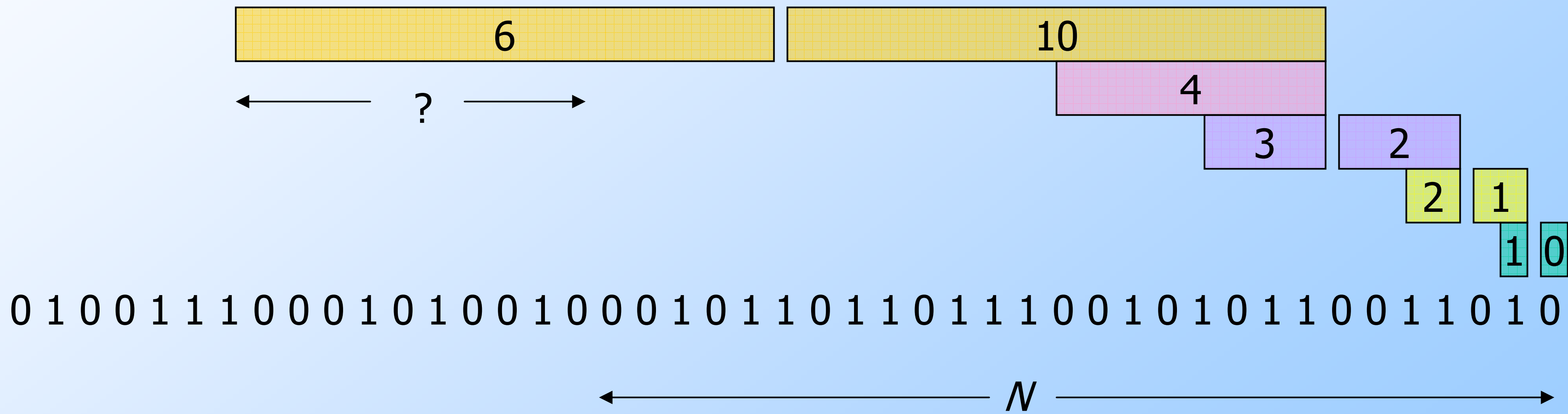
- You can't get an exact answer without storing the entire window.
- Real Problem: what if we cannot afford to store *N* bits?
  - E.g., we are processing 1 billion streams and *N* = 1 billion, but we're happy with an approximate answer.

Something That Doesn't (Quite) Work

- Summarize exponentially increasing regions of the stream, looking backward.
- Drop small regions when they are covered by completed larger regions.

Example

We can construct the count of the last *N* bits, except we're Not sure how many of the last 6 are included.



What's Good?

- Stores only $O(\log^2 N)$ bits.
- Easy update as more bits enter.
- Error in count no greater than the number of 1's in the "unknown" area.

What's Not So Good?

- As long as the 1's are fairly evenly distributed, the error due to the unknown region is small --- no more than 50%.
- But it could be that all the 1's are in the unknown area at the end.
- In that case, the error is unbounded.

Fixup

- Instead of summarizing fixed-length blocks, summarize blocks with specific numbers of 1's.
  - Let the block "sizes" (number of 1's) increase exponentially.
- When there are few 1's in the window, block sizes stay small, so errors are small.

DGIM* Method

- Store $O(\log^2 N)$ bits per stream.
- Gives approximate answer, never off by more than 50%.
  - Error factor can be reduced to any fraction > 0, with more complicated algorithm and proportionally more stored bits.

*Datar, Gionis, Indyk, and Motwani

Timestamps

- Each bit in the stream has a *timestamp*, starting 1, 2, …
- Record timestamps modulo N (the window size), so we can represent any relevant timestamp in $O(\log_2 N)$ bits.

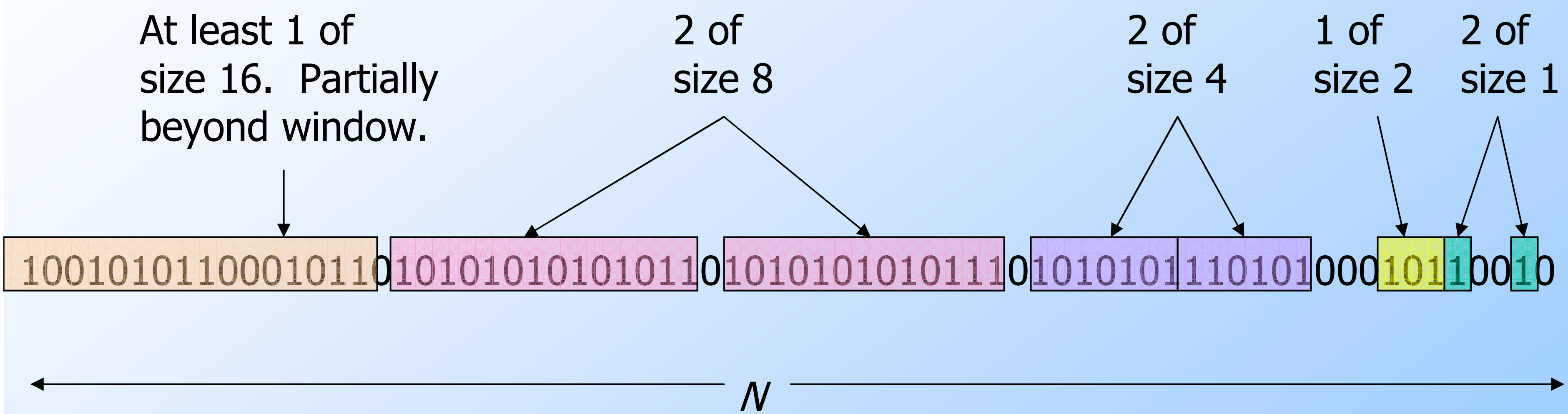
Buckets

- A *bucket* in the DGIM method is a record consisting of:
  1. The timestamp of its end [O(log N) bits].
  2. The number of 1's between its beginning and end [O(log log N) bits].
- Constraint on buckets: number of 1's must be a power of 2.
  - That explains the log log N in (2).

Representing a Stream by Buckets

- Either one or two buckets with the same power-of-2 number of 1's.
- Buckets do not overlap in timestamps.
- Buckets are sorted by *size* (# of 1's).
  - Earlier buckets are not smaller than later buckets.
- Buckets disappear when their end-time is > *N* time units in the past.

Example


At least 1 of size 16. Partially beyond window.
2 of size 8
2 of size 4
1 of size 2
2 of size 1
1001010110001011010101010101011010101010101110101010111010100010110010
N

Updating Buckets --- (1)

- When a new bit comes in, drop the last (oldest) bucket if its end-time is prior to N time units before the current time.
- If the current bit is 0, no other changes are needed.

Updating Buckets --- (2)

- If the current bit is 1:
  1. Create a new bucket of size 1, for just this bit.
     - End timestamp = current time.
  2. If there are now three buckets of size 1, combine the oldest two into a bucket of size 2.
  3. If there are now three buckets of size 2, combine the oldest two into a bucket of size 4.
  4. And so on…

Example

10010101100010110 1010101010101011 0 101010101010111 0 1010101 110101 000 101 1 00 1 0

0010101100010110 1010101010101011 0 101010101010111 0 1010101 110101 000 101 1 00 1 0 1

0010101100010110 1010101010101011 0 101010101010111 0 1010101 110101 000 101 1001 0 1

0101100010110 1010101010101011 0 101010101010111 0 1010101 110101 000 101 1001 0 1 1 0 1

0101100010110 1010101010101011 0 101010101010111 0 1010101 110101 000 101 1001 0 11 0 1

0101100010110 10101010101010110101010101010111 0 1010101110101 000 1011001 0 11 0 1

Querying

- To estimate the number of 1's in the most recent *N* bits:
  1. Sum the sizes of all buckets but the last.
  2. Add in half the size of the last bucket.
- Remember, we don't know how many 1's of the last bucket are still within the window.

Error Bound

- Suppose the last bucket has size 2^k.
- Then by assuming 2^{k-1} of its 1's are still within the window, we make an error of at most 2^{k-1}.
- Since there is at least one bucket of each of the sizes less than 2^k, the true sum is no less than 2^k-1.
- Thus, error at most 50%.

Extensions (For Thinking)

- Can we use the same trick to answer queries "How many 1's in the last k?" where $k < N$?
- Can we handle the case where the stream is not bits, but integers, and we want the sum of the last k?