Challenge Problem Set 4

May 25, 2010

1 Problem 1. (5 points)

- $C - R$ is context-free. We can express $C - R$ as $C \cap \bar{R}$. As regular languages are closed under complement, $\bar{R}$ is a regular language. Intersection of a regular language and a CFL is a CFL(refer to textbook) and hence $C - R$ is context-free.

- $R - C$ is not necessarily context-free. We can express $R - C$ as $R \cap \bar{C}$. If $R \cap \bar{C}$ is context free for every R it implies that $\bar{C}$ is context free(case $R = \sum^*$).We have a contradiction and hence $R - C$ is not necessarily context-free.

2 Problem 2. (5 points)

Consider language $L = \{0^i 1^j 2^j 3^{3i} | i, j \geq 1\}$. L is context free as we can represented by the following grammar.

$$S \rightarrow 0S333 | 0T333$$
$$T \rightarrow 1T2 | 12$$

Suppose that n is the pumping lemma constant. Let $z = 0^n 1^n 2^n$. Note that $z \in \text{half}(L)$ as $0^n 1^n 2^n 3^{3n} \in L$. It is straight forward(*discussed in weekly section*) to apply pumping lemma on $0^n 1^n 2^n$ and prove that there is no configuration to split z into $uvwxy$ such that the pumping lemma holds.Hence half(L) is not a context free language.