Challenge Problem Set 2

May 2, 2010

1 Problem 1. (5 points)

As per the given conditions, the 2's can only occur in the beginning of the sentence. So one of the possible representation is $2^*(1 + \epsilon)(01)^*(0 + \epsilon)$.
Grading - There are many equivalent regular expressions, all were considered. In case the expression was incorrect, we tried to mention either the illegal expressions generated or some legal expression missed.

2 Problem 2. (5 points)

First, we can convert the NFA A to a DFA B by using the subset construction, where $B = (Q, \Sigma, \delta, q_0, F)$. We have $L(A) = L(B)$. We then change the final states of B to nonfinal states and nonfinal states of B to final states. The language $L(C)$ of the resulting DFA $C = (Q, \Sigma, \delta, q_0, Q - F)$ is the complement of $L(A)$.

w is in $L(C)$ if and only if $\hat{\delta}(q_0, w)$ is in $Q - F$, which occurs if and only if w is not in $L(B)$. Since $L(B) = L(A)$, $L(C) = \overline{L(A)}$.

3 Problem 3. (5 points)

The idea is to construct a DFA that recognizes all those w for which ww is not recognizes by the given DFA A. If this DFA recognizes the empty language, the property holds, else it does not.

To construct such a DFA, we first consider the DFA's $A_i's$, one for each final state of A. Each A_i is identical to A, except for the fact that their

starting state is different, and one of the final states of A. We therefore have one A_i for each final state of A. We construct our objective DFA B, by taking the product DFA of A and all these $A_i's$.

Let q_i's be the final states of A, and q_0 the start state of A. The start state of B is defined as $(q_0, q_1, ..., q_m)$. A state $(q_i, p_1, p_2, ..., p_m)$ is defined as a final state of B if q_i is a final state of A and p_i is not a final state of A. Basically after reading an input w if A reaches q_i, it reaches p_i after reading w from q_i, by the definition of the automata A_i's.

Error Codes

- 3.1 : The given construction does not work. 3 points deducted.

- 3.2 : The solution involves going through all possible w for A. 4 points deducted

- 3.3 : The solution lacks minor details in the end. 1 point deducted