CS154 Midterm Examination

May 4, 2010, 2:15 - 3:30PM

Directions: Answer all 7 questions on this paper. The exam is open book and open notes. Any materials may be used.

Name: ____

I acknowledge and accept the Honor Code.

(signed) _____________________________________

Problem	**Points**
1	
2	
3	
4	
5	
6	
7	
Total	

Problem 1 (20 pts.) Let L be the language mentioned in the third Challenge-Problem set: the set of strings over alphabet {0, 1, 2} that do not have two consecutive identical symbols. That is, strings of L are any string in {0,1,2}* such that there is no occurrence of 00, no occurrence of 11, and no occurrence of 22. In the space below, design a DFA (transition table or transition diagram -- your choice) that accepts L.

For each of your states, give a brief description of what strings get you to that state.

State	Purpose

Problem 2 (15 pts.) Below is the transition table of a DFA.

	0	**1**
->A	E	B
* B	D	A
C	G	A
* D	G	E
E	A	D
F	B	E
* G	B	A

Find the distinguishable states by filling out the table below. However, place X's only for those pairs that are distinguishable by the basis step. For those discovered to be distinguishable during the induction, place numbers 1, 2, ... indicating the order in which you discovered these pairs to be distinguishable. *Note* that many different orders are correct. *Also note*: cells with # are not to be filled in.

	G	F	E	D	C	B
A						
B						#
C					#	#
D				#	#	#
E			#	#	#	#
F		#	#	#	#	#

Which sets of states are mutually equivalent?

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In the space below, draw the transition table for the minimum-state equivalent DFA.

Problem 3 (10 pts.) We wish to construct a DFA B from a given DFA A such that B accepts input string w if and only if A accepts w but does not accept www (Note there are three w's, not two). The construction is similar to that of a problem on the second Challenge-Problem set, but it involves keeping track of what all states of A do, not just the accepting states. Let A have states $q_1, q_2, \ldots, q_n$, where q_1 is the start state. The states of B are vectors of n states of A. The start state of B is $[q_1, q_2, \ldots, q_n]$, that is, the states of A in order. Let δ_A the transition function of A. Then the transition function δ_B is defined by

$$\delta_B([p_1, p_2, \ldots, p_n], a) = [\delta_A(p_1, a), \delta_A(p_2, a), \ldots, \delta_A(p_n, a)]$$

That is, B simulates A from each of its states, each reading the same input. To complete the construction, all that needs to be done is to define the final states of B. That's your job. In the space below, state the condition under which state $[p_1, p_2, \ldots, p_n]$ should be a final state of B.

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Problem 4 (20 pts.) A *wiggle string* is defined to be a nonempty string of 0's and 1's such that each 0 is followed by a 1, and each 1 is followed by a 0. A *0-wiggle string* is a wiggle string that begins and ends in 0, and a *1-wiggle string* is a wiggle string that begins and ends with 1. For example, 01010 is a 0-wiggle string, 1 is a 1-wiggle string, 0101 is a wiggle string that is neither a 0-wiggle string nor a 1-wiggle string, and 010010 is not a wiggle string. The language of the grammar G consisting of productions

S -> SAS | 0
A -> ASA | 1

(S is the start symbol) is the set of 0-wiggle strings, as you will prove. In this proof, you can assume the true fact that the concatenation of two wiggle strings is a wiggle string, as long as the first ends in a symbol different from the symbol by which the second begins. You do not have to state this fact in the proof. Give your proof by answering the following very specific questions.

Prove first that every 0-wiggle string is in L(G). To do so, we need to prove a more general statement inductively: (1) if w is a 0-wiggle string then S =>* w, and (2) if w is a 1-wiggle string, then A =>* w.

Remember: S -> SAS | 0   A -> ASA | 1

(a) On what is your induction?

(b) What is the basis case?

(c) Prove the basis.

(d) For the inductive part, first show that if w is a 0-wiggle string, then S =>* w.

The second part of the induction is that if w is a 1-wiggle string, then A =>* w. You do not have to provide this part, since its proof is essentially like that of (d), with 0 and 1 interchanged.

Conversely, to show that every string in L(G) is a 0-wiggle string, we shall show that (1) if S =>* w, then w is a 0-wiggle string, and (2) if A =>* w, then w is a 1-wiggle string.

(e) On what is your induction?

(f) What is the basis case?

(g) Prove the basis.

Remember: S -> SAS | 0   A -> ASA | 1

(h) For the inductive part, first show that if S =>* w, then w is a 0-wiggle string of length n.

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To complete the inductive part, you also need to show that if A =>* w, then w is a 1-wiggle string. You do not need to provide this part, since it is essentially (h), with 0 and 1 interchanged.

Problem 5 (15 pts.) The grammar from Problem 4:

S -> SAS | 0
A -> ASA | 1

(S is the start symbol) is ambiguous. Give an example of a string that has two or more parse trees.

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Show two of its parse trees in the space below.

Give an unambiguous grammar for the set of 0-wiggle strings. You need not justify your answer.

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Problem 6 (10 pts.) Let L be the language $\{a^i \mid i \text{ is a perfect square}\}$. That is, L contains the strings a, aaaa, aaaaaaaaa, aaaaaaaaaaaaaaaa, and so on. We want to prove that L is not context free. (*Hint*: Notice that as strings in L get larger, the difference in length between one string and the next longer string grows without limit.) Suppose n is the pumping-lemma constant for L. What string z do you choose to apply the pumping lemma?

z = ____________________________

Suppose "the adversary" picks a way to break z = uvwxy that satisfies the constraints of the pumping lemma. We hope to prove that there must be some string in L whose length is not a perfect square. In terms of u, v, w, x, and y, what string would you choose?

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Complete the proof that your chosen string is not in L.

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Problem 7 (10 pts.) Suppose we use the construction of a CFG G whose language is N(P) for a given PDA P, as given in the text or in the class slides. Let P have s states. Suppose also that $\delta(q, a, X) = \{(p, \beta)\}$, while $\delta(q, \varepsilon, X)$ is empty. Let β have length k. As a function of s and k, how many productions of grammar G have variable [qXp] on the left?

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