CS109A Problem Set #6

Due 4:30 PM, Monday, 2/27/95

1) (20 points) Define the types:

```
typedef struct NODE *pNODE;
struct NODE {
    int label;
    pNODE leftmostChild;
    pNODE rightSibling
}
```

Write a recursive C function that takes a pointer to a node and returns the largest of the labels in the subtree rooted at that node. (Exercise 5.4.2, FCS)

2) (20 points) Prove by structural induction that in any tree T, the number of leaves is 1 more than the number of nodes that have right siblings. (Exercise 5.5.4, FCS) *Important*: In order to be considered a structural induction

   a) You must not mention numerical properties of the tree such as the height of nodes or the number of nodes in subtrees.

   b) You must use the inductive hypothesis to say something about the children of a node.

3) (20 points) What is the probability that the integers 1 through 15, placed at random into an array A of length 15, will form a partially ordered tree, i.e., $A[i/2] > A[i]$ for $i = 2, 3, \ldots, 15$? (Special case of Exercise 5.9.8, FCS) *Hint*: You might think that each of the 14 tests is either true or false with probability 1/2, so the probability of all 14 being true is 2^{-14}. However, the tests are not independent, so the product rule does not apply. You need to count the number of orders (that's easy: 15!) and then count the number of orders that form a heap. For example, you know that $A[1]$ must be 1. You also know that whatever 7 integers are in the left subtree, $A[2]$ must hold the smallest of them, and so on. Happy counting!

4) (20 points) Write the function `comb`, computing $\binom{n}{m}$, in such a way that when we call `comb(n,m)` it prints n and m before printing the result. Print out suitable words so n, m, and $\binom{n}{m}$ are clearly distinguishable from one another. (Exercise 9.4, EMLP)

5) (20 points) Suppose $f(x, y, z)$ is a function. Give an example of a definition of f that would cause the argument of f to have the type `'a * 'a * int`. (Exercise 10.3(b), EMLP)