CS109B Second Class Quiz
Monday, May 22, 1995, 2:15–3:05PM

Directions

The exam is *open book*; any written materials may be used. Answer all 7 questions on the exam paper itself. The total number of points is 100. You may make additional comments at the bottoms of pages or at the end if you feel the need to clarify or justify your answers.

Do not forget to **sign the pledge** below.

I acknowledge and accept the honor code.

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Print your name here: ____

1	2	3	4
5	6	7	

Problem 1: (15 points) Stanford telephone numbers, internally, consist of five digits, the first of which is either 3, 5, or 7. In addition, there is a hyphen after the first digit. Write a grammar with a syntactic category <internal>, whose language is the set of strings that are legal internal Stanford telephone numbers. You may use whatever "helper" syntactic categories you like. However, *digit* is NOT a terminal; the terminals are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and - (the hyphen).

An external Stanford number is 72, followed by the internal number. Add productions to your grammar above so that the syntactic category <external> has as its language the set of strings that are legal Stanford external numbers.

Problem 2: (15 points) It happens that in each of the following grammars, the language $L(<S>)$ is a regular set. Write regular expressions for each of these languages.

a) $<S> \rightarrow \mathbf{a} <S> \mid \mathbf{b} <S> \mid \mathbf{c}$

b) $<S> \rightarrow \mathbf{a} <T> <T>$
$<T> \rightarrow <T> \mathbf{a} \mid \mathbf{b}$

c) $<S> \rightarrow <S> \mathbf{a}\ \mathbf{a} <S> \mid \mathbf{b}$

Problem 3: (15 points) Below is a grammar that can be parsed by recursive descent.

(1) $<S> \rightarrow <A> <A>$

(2) $<A> \rightarrow 0 <A>$
(3) $<A> \rightarrow 1$

We can write for this grammar ML functions `s` and `a`, each of which takes a list of 0's and 1's as an argument and either raises the exception `Fail` or finds a prefix of its argument list that is in the language $L(<S>)$ or $L(<A>)$, respectively, and returns the list that remains after removing this prefix.

a) How do you decide which production to use for $<S>$?

b) How do you decide which production to use for $<A>$?

c) Write the function `s` here. You may assume exception `Fail` is already defined.

d) Write the function `a` here. Again assume `Fail` is defined.

Problem 4: (10 points) A certain vending machine has two slots for coins. It reads each slot using the following code:

No coin	00
Nickel	01
Dime	10
Quarter	11

Suppose logical variables pq represent the code for the first slot, and rs represent the code for the second slot. Write, in the Karnaugh map below, entried for the function that represents the condition that whatever is in the two slots has total value at least 15 cents. *You need not convert your Karnaugh map into a logical expression.*

pq \ rs	00	01	11	10
00				
01				
11				
10				

Problem 5: (15 points) The ML function `cat` below takes arguments `x` and `y`, which must be lists, and produces the concatenation of `x` and `y`, with `y` preceding `x`.

```
fun cat x y = y@x;
```

a) Write a val-statement (statement beginning `val` and the name of a variable whose value is being defined) to partially instantiate `cat`, producing a function `append01` that takes any integer list L as an argument and produces L`@[0,1]`.

b) Write a val-statement to partially instantiate `cat`, producing a function `append23` that takes any integer list L as an argument and produces L`@[2,3]`.

c) Write a val-statement that composes your functions from (a) and (b) to produce a function `append0123` that takes any integer list L as an argument and produces L`@[0,1,2,3]`. (Do not partially instantiate `cat` to get this function.)

Problem 6: (15 points) For the Karnaugh map below:

pq \ rs	00	01	11	10
00	1	1		
01	1			1
11			1	1
10		1	1	1

a) Circle all the prime implicants on the diagram itself.

b) Write below the product-of-literals for each of the prime implicants (in any order).

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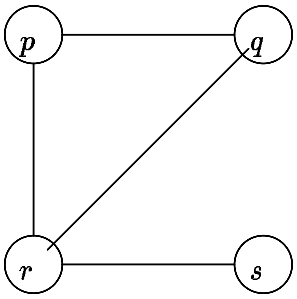
c) Find from your answer to (b) a minimal set of prime implicants that covers all the 1's on the map.

Problem 7: (15 points) Below is a graph with four nodes p, q, r, and s.



We can associate with these nodes propositional variables of the same name. Then, we can associate a set of nodes with any truth-assignment for these variables; the set is those nodes for which the corresponding propositional variables are assigned value 1 (true). For instance, the truth-assignment $p = q = r = 1$, $s = 0$ corresponds to the set of nodes $\{p, q, r\}$.

a) Write a logical expression of variables p, q, r, and s that is true if and only if the truth-assignment corresponds to a set that is a node cover (set of nodes such that each edge has at least one end in this set) for this graph.

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b) Write a logical expression of variables p, q, r, and s that is true if and only if the truth-assignment corresponds to a set that is an independent set (set of nodes such that no two are connected by an edge) for this graph.
