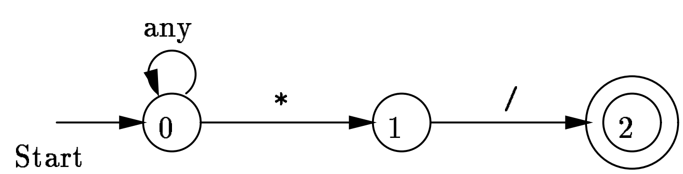
CS109B Notes for Lecture 4/21/95

Nondeterministic Automata Looking for Substrings

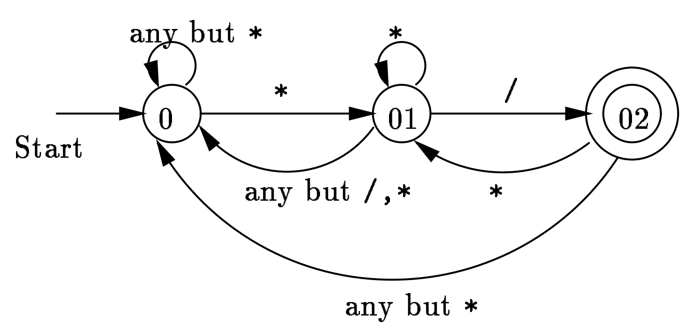
We can build an NFA to recognize a string that ends in any given substring $a_1 a_2 \cdots a_n$ if we:

1. Have a start state s_0 that goes to itself on any input.
   - □ I.e., you can always "guess" that the substring has not yet begun, even if the input is a_1.
2. For $i = 1, 2, \ldots, n$, s_{i-1} goes to s_i on input a_i.
3. s_n is the accepting state.

Example: Strings that end in `*/`.



- Careful how you use this automaton: when it accepts, the job is done and you do not continue searching for a later occurrence of `*/`.
- We can convert to a DFA as follows:



Class Problem

Describe a NFA that accepts those strings of 0's and 1's such that the 10th position from the end is 1.

- Note this automaton's input has no "end-marker." At all times it accepts if 10 inputs ago it received a 1.

Now, describe a DFA that recognizes the same language. How many states do your automata have?

Regular Expressions

- An algebraic notation for describing the *regular sets* (= sets of strings accepted by a FA).
  - ☐ Note that the subset construction tells us that NFA's and DFA's accept the same sets of strings.
  - ☐ A set of strings is a *language*.
- The RE's use three operators: union, concatenation, and "closure."
- $L(R)$ = the language represented by RE R.

Why Regular Expressions?

An important notation for expressing character-string patterns. Used in many UNIX commands, e.g., grep, lex, editors, and (in somewhat different form) the shell.

Operands

- Constants, which are symbols a standing for the language $\{a\}$ consisting of one string; that string is of length 1 and has the symbol a in its lone position.
- Variables, standing for unknown languages.
- The special symbols $\emptyset$ standing for the empty language and ϵ standing for $\{\epsilon\}$ (the set containing only the empty string).
  - ☐ Note that $\emptyset \neq \{\epsilon\}$.

Concatenation

If R and S are RE's, then RS (= *concatenation* of R and S) denotes the language $L(RS) = \{rs \mid r$ is in R and s is in $S\}$.

- In general, the language of RS is formed by concatenating a string from R and a string from S in all possible combinations.
- Special case: $a_1 a_2 \cdots a_n$ (concatenation of n RE's, each a single symbol) denotes one-string language $\{a_1 a_2 \cdots a_n\}$.

Union

If R and S are RE's then $L(R \mid S) = L(R) \cup L(S)$.

Example: Let $R = (a \mid b)(ab \mid ba)$. What is $L(R)$?

- $L(a \mid b) = \{a, b\}$.
- $L(ab \mid ba) = \{ab, ba\}$.
- $L(R) = \{a, b\}\{ab, ba\} = \{aab, aba, bab, bba\}$.

Closure

If R is an RE, then $L(R^*)$ denotes $\{\epsilon\} \cup L(R) \cup L(RR) \cup L(RRR) \cup \cdots$.

- That is, the union of zero or more strings chosen arbitrarily from R.

Example: $L\big((a \mid b)^*\big)$ = set of all strings of a's and b's.

Example: $L(a^* b^*)$ = set of all strings of a's and b's where the a's precede the b's.

Class Problem

Write a regular expression denoting the set of strings of 0's and 1's such that the 10th position from the right end is 1.